

國中數學領域

雙語教學資源手冊 英語授課用語

A Reference Handbook for **Junior High School** Bilingual Teachers
in the Domain of **Mathematics**: Instructional Language in English

〔八年級下學期〕





目次 Table of Contents

單元一	數列.....	1
單元二	等差級數.....	5
單元三	變數與函數.....	10
單元四	線型函數與圖形.....	14
單元五	內角與外角.....	19
單元六	尺規作圖與三角形的全等.....	24
單元七	全等三角形的應用.....	29
單元八	三角形的邊角關係.....	32
單元九	平行線與截角性質.....	38
單元十	平行四邊形.....	42
單元十一	特殊四邊形與梯形.....	46

單元一 數列

Sequence of Number

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

在這章我們會跟學生介紹等差數列、等比數列的相關名詞，如首項、公差、公比、項數、首項、末項等，並藉由推導出它們之間的關係，從而使學生了解公式，之後再介紹等差中項及等比中項。

■ 詞彙 Vocabulary

單字	中文	單字	中文
sequence of number	數列	arithmetic sequence	等差數列
term	項	common difference	公差
number of terms	項數	arithmetic mean	等差中項
initial term	首項	geometric sequence	等比數列
last term	末項	common ratio	公比
general term	一般項	geometric mean	等比中項

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

① _____, and the arithmetic mean is _____.

例句：We know that three numbers form the arithmetic sequence, **and the arithmetic mean is 7**. Find the sum of these three numbers.

已知三數成等差數列，且其等差中項為 7，求此三數的和。

② _____ of an arithmetic sequence is _____ the common difference is _____, find _____.

例句：We know that the 16th term **of an arithmetic sequence is 10** and **the common difference is 23**, **find** the initial term of this arithmetic sequence.

已知一個等差數列的第 16 項為 10，公差為 23，求此等差數列的首項。

■ 問題講解 Explanation of Problems

☞ 說明 ☞

Questions are used for better understanding arithmetic sequence and geometric sequence.
我們利用題目使學生更加地了解等差數列以及等比數列。

☞ 運算問題的講解 ☞

例題一

說明：藉由實際範例理解並應用等差中項。

To make students understand arithmetic mean and how to use it through practical examples

(英文) We know that 8 is the arithmetic mean of a and b , and $2a - 3b = -38$

Please find the values of a and b .

(中文) 已知 8 是 a 與 b 的等差中項，且 $2a - 3b = -38$ ，求 a 、 b 的值。

(翰林版課本)

Teacher: Let's read the question. 8 is the arithmetic mean of a and b , and $2a - 3b = -38$. Please find the values of a and b . Do you remember the formula of an arithmetic mean?

Student: An arithmetic mean equals its previous term plus its next term then divided by 2.

Teacher: That's right. In this question, since 8 is the arithmetic mean of a and b , we can set up the equation $8 = \frac{a+b}{2}$.

Therefore, $a + b = 16$. Besides, we know that $2a - 3b = -38$, the following simultaneous equation can be set up:

$$\begin{cases} 2a - 3b = -38 \\ a + b = 16 \end{cases}$$

Could you please solve the equation and tell me the values of a and b ? You should be able to do this because you've learned it in 7th grade.

Student: $a = 2, b = 14$.

Teacher: Great! So the answer for this question is $a = 2$, and $b = 14$.

老師：我們先來看一下題目，題目說「8 是 a 與 b 的等差中項，且 $2a - 3b = -38$ ，問 a 、 b 的值是多少」。那麼等差中項的公式是什麼？

學生：中項等於中項的前一項加中項的後一項再除 2。

老師：對，所以這題因為 8 是 a 與 b 的等差中項，因此 $8 = \frac{a+b}{2}$ 。故 $a + b = 16$ 。

再結合題目給的 $2a - 3b = -38$ ，我們就可以列出聯立方程式

$$\begin{cases} 2a - 3b = -38 \\ a + b = 16 \end{cases}$$

那請同學幫我解聯立並告訴我 a 、 b 是多少？這是國一的內容應該要會喔。

學生： $a = 2, b = 14$ 。

老師：很好，所以這題的答案就是 $a = 2, b = 14$ 。

例題二

說明：藉由題目使學生更加了等比數列的相關名詞並應用。

To make students understand the related terms of geometric sequence better and know how to operate through questions.

(英文) We know that the first term of a geometric sequence is 256, and the common ratio is $\frac{3}{4}$. Which term is 81 in this sequence?

(中文) 已知一個等比數列的首項為 256，公比為 $\frac{3}{4}$ ，求 81 是此數列的第幾項？

(翰林版課本)

Teacher: We know that the formula of a geometric sequence is $a_n = a_1 \times r^{n-1}$.

We can find the number of n once we substitute the initial term, the common ratio, and the term of n for the formula. It says that the initial term is 256, the common ratio is $\frac{3}{4}$, which term is 81?

Teacher: This means that $a_1 = 256$, $r = \frac{3}{4}$, and $a_n = 81$.

Teacher: Next, we just substitute these numbers for the formula and calculate for the result,

which is $81 = 256 \times (\frac{3}{4})^{n-1}$. Let's organize the equation. The process would be

$(\frac{3}{4})^{n-1} = \frac{81}{256} = \frac{3^4}{4^4} = (\frac{3}{4})^4$. Thus, we will learn that $n - 1 = 4$, therefore $n = 5$.

Teacher: So the fifth term is 81.

老師：首先我們知道等比數列第 n 項的公式為 $a_n = a_1 \times r^{n-1}$ ，因此只要將首項、公比和第 n 項找出來並代入公式便可得知 n 是多少了。

現在題目告訴我們首項是 256、公比為 $\frac{3}{4}$ ，問 81 是第幾項。

老師：這也就告訴我們 $a_1 = 256$ 、 $r = \frac{3}{4}$ 、 $a_n = 81$ 。

老師：接下來我們直接代進公式 $a_n = a_1 \times r^{n-1}$ 去算就可以了，代進去後我們會得到

$81 = 256 \times (\frac{3}{4})^{n-1}$ ，再化簡一下便可知 $(\frac{3}{4})^{n-1} = \frac{81}{256} = \frac{3^4}{4^4} = (\frac{3}{4})^4$ 。

因此 $n - 1 = 4$ ， $n = 5$ 。

老師：所以 81 是此數列的第 5 項。



單元二 等差級數

Arithmetic Series

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

我們在這章讓學生知道求和的方法並建立等差級數的觀念，再進行推導，得出等差級數求和公式，並應用於解題。

■ 詞彙 Vocabulary

單字	中文	單字	中文
series	級數	arithmetic series	等差級數

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

- ① _____, find the sum of the previous terms of this arithmetic series.

例句：If you know that the initial term of the arithmetic series is 3 and the common difference is -2 , **find the sum of the previous 12 terms of this arithmetic series.**

已知一個等差級數的首項為 3，公差為 -2 ，求此等差級數前 12 項的和。

■ 問題講解 Explanation of Problems

說明

Arithmetic series is used for students finding results faster.

我們利用等差級數使學生可以更快地計算出答案。

運算問題的講解

例題一

說明：藉由實際範例理解並應用等差級數求和公式。

To make students understand the sum formula of the arithmetic series and how to use it through practical examples.

(英文) Please find the sum of the arithmetic series: $81 + 74 + 67 + 60 + \dots + (-10)$.

(中文) 求等差級數 $81 + 74 + 67 + 60 + \dots + (-10)$ 的和。

(翰林版課本)

Teacher: Quick review, what is the sum of an arithmetic series, S_n ?

Student: n multiply the sum of the initial term and the last term then divided by 2.

Teacher: Great! It is asking for the sum of the arithmetic series

$81 + 74 + 67 + 60 + \dots + (-10)$.

What are the initial term and the last term of the sequence?

Student: They are 81 and (-10) .

Teacher: Very good! But we don't know what n is yet, we need the formula we learned in the last segment, $a_n = a_1 + (n - 1) \times d$, to find out what n is. Can anyone tell me what the common difference is?

Student: It is -7 .

Teacher: Correct, it is -7 . So if we substitute the common difference, the initial term, and the last term for the formula $a_n = a_1 + (n - 1) \times d$, we will get that $(-10) = 81 + (n - 1) \times (-7)$. Organize it, then we'll know that $n = 14$.

Final step, substitute them for the sum formula of the arithmetic series we reviewed at the beginning to get $S_{14} = \frac{14 \times [81 + (-10)]}{2}$. To find the answer, we just need to calculate it. What is the answer for the question?

Student: It's 497.

Teacher: That's right!

老師：我們先來複習一下，等差級數和 S_n 會等於什麼？

學生： n 乘上首項加末項的和再除 2。

老師：很好！現在題目要求等差級數 $81 + 74 + 67 + 60 + \dots + (-10)$ 的和。那麼這一題的首項和末項是多少呢？

學生：81 和 (-10) 。

老師：非常好！但我們還不知 n 是多少。所以我們還需要用到上一小節教的 $a_n = a_1 + (n - 1) \times d$ 來算出 n 是多少。那麼有同學能告訴我公差 d 是多少嗎？

學生： -7 。

老師：對，就是 -7 。所以我們把公差、首項和末項代進 $a_n = a_1 + (n - 1) \times d$ 便可得到 $(-10) = 81 + (n - 1) \times (-7)$ ，化簡一下我們就可以知道 $n = 14$ 。

最後再代入我們一開始複習的等差級數和公式，便可得到 $S_{14} = \frac{14 \times [81 + (-10)]}{2}$ ，

再把他整理計算就是答案了。因此這題的答案是多少？

學生：497。

老師：答對了！

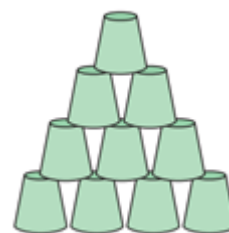
例題二

說明：藉由實際應用使學生更加熟練、理解等差級數求和公式。

Students will understand better and get familiar with the sum formula of the arithmetic series through practice.

(英文) Chen was in charge of cleaning all the 105 cups after a party. He would like to dry the cups, so he put the cups upside-down like it is the picture on the right for neat-looking and saving some space. Here is the regulation: each layer has one more cup than the upper layer from the bottom to the top, and there was only one cup on the top. If the 105 cups were all used, and there were no cups left, how many cups were there at the bottom layer?

(中文) 同樂會結束後，小城負責洗現場所有的 105 個杯子，洗完後他想把杯子裡的水瀝乾，且為了美觀與節省空間，他將杯子倒著放並排成如右圖。排放的規律如下：由下往上每一層少一個杯子，最高層僅剩一個杯子。如果 105 個杯子剛好排完，最底層需要排幾個杯子？



(翰林版課本)

Teacher: “Each layer has one more cup than the upper layer from the bottom to the top.” From this, we can learn that the common difference is 1. As for “there was only one cup on the top,” we can assume that the initial term is 1. Finally, “the 105 cups were all used” tells us that the sum of the series is 105. We are supposed to figure out how many cups were there at the bottom layer.

Teacher: Let’s assume that there were n layers, and there were a_n cups at the n^{th} layer, which was the bottom layer. We learn that $a_n = n$ from a_n equals $1 + (n - 1) = n$.

Teacher: Next, we substitute the numbers for the sum formula of the arithmetic series,

$$S_n = \frac{n[a_1 + a_n]}{2}, \text{ we get } 105 = \frac{n[1 + n]}{2}.$$

Organize the equation, we’ll get $n^2 + n - 210 = 0$. To find the answer, we just need to solve the quadratic equation in one variable. What is the number of n ?

Student: It's 14 or -15.

Teacher: That's right. But remember, the number of cups cannot be negative, n can only be 14. Therefore, there were 14 cups at the bottom layer.

老師：題目說由下往上每一層少一個杯子，從這一句話我們可以得出公差是 1。然後最高層僅剩一個杯子，因此我們假設首項是 1。題目最後還說 105 個杯子剛好排完，這句話就意味著這個級數的總和是 105。現在題目問最底層需要排幾個杯子。

老師：設總共有 n 層且第 n 層也就是最後一層有 a_n 個杯子。
因為 a_n 會等於 $1 + (n - 1) = n$ ，故 $a_n = n$ 。

老師：之後我們再代入等差級數和的公式 $S_n = \frac{n[a_1 + a_n]}{2}$ ，便能得到 $105 = \frac{n[1 + n]}{2}$ ，再整理一下， $n^2 + n - 210 = 0$ 。所以我們只要把這個一元二次方程式解出來就可以得到答案了。那麼解出來 n 會是多少？

學生：14 或 -15。

老師：沒錯，但是要注意喔～杯子是不會有負數。因此 n 只能等於 14，故最底層有 14 個杯子。

單元三 變數與函數

Variable and Function

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

對於學生函數是一個新的名詞與觀念，在這章節的前面我們先利用兩相異變數的對應關係介紹函數，在教學時可以不用一直停留在「判別是否為函數」上或者是利用對應關係來「判別是否為函數」。

而在函數值的部分，我們先從代數運算來使學生了解函數值的意義，再利用日常生活中的例子來訓練學生如何將應用問題轉換成函數的列式與函數值。

■ 詞彙 Vocabulary

單字	中文	單字	中文
variable	變數	dependent variable	應變數
function	函數	independent variables	自變數

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Let x be _____ and y be _____.

例句：Let x be the name of the item and y be the price of the item, is y a function of x ?

設 x 表示物品名稱， y 表示該物品的價錢，則 y 是否為 x 的函數？

② Write the relationship _____.

例句：Write the relationship between x and y .

寫出 x 與 y 的關係式。

③ _____ are equivalent in _____.

例句：If the functions $y = 4x - 7$ and $y = -6x - 2$ are equivalent in $x = k$, find the value of k .

若函數 $y = 4x - 7$ 與 $y = -6x - 2$ ，在 $x = k$ 的函數值相等，求 k 的值。

■ 問題講解 Explanation of Problems**說明**

Students will understand functions and find the value of the functions through questions.

利用題目使學生了解函數的關係並求出函數值。

運算問題的講解**例題一**

說明：使學生了解函數值。

to make students understand functions.

(英文) If the functions $y = 2x - 1$ and $y = -2x + 7$ are equivalent in $x = k$, find the value of k .

(中文) 若函數 $y = 2x - 1$ 與函數 $y = -2x + 7$ ，在 $x = k$ 的函數值相等，求 k 的值。

(翰林版課本)

Teacher: It is said that in $x = k$, the functions $y = 2x - 1$ and $y = -2x + 7$ are equivalent. If we substitute k for the x into the functions $y = 2x - 1$ and $y = -2x + 7$, what will we get?

Student: $y = 2k - 1$ and $y = -2k + 7$.

Teacher: That's right. We will get that $y = 2k - 1$ and $y = -2k + 7$. Next, since they are equivalent, we can make them $2k - 1 = -2k + 7$ directly. Then the answer shows up. How much is k ?

Student: $k = 2$.

Teacher: Correct, k is 2.

老師：題目上說當 $x = k$ 時 $y = 2x - 1$ 與 $y = -2x + 7$ 的函數值相等，所以我們先把 $x = k$ 代入到 $y = 2x - 1$ 與 $y = -2x + 7$ 中，請問我們會得到什麼？

學生： $y = 2k - 1$ 和 $y = -2k + 7$ 。

老師：沒錯，我們會得到 $y = 2k - 1$ 以及 $y = -2k + 7$ 。接下來，因為他們的函數值相等，所以我們可以直接列出 $2k - 1 = -2k + 7$ 。之後我們便能求出答案，那麼 k 值是多少？

學生： $k = 2$ 。

老師：答對了， k 值等於 2。

例題二

說明：使學生了解函數關係並求出函數值。

to make students understand functions and find the value of the functions.

(英文) A regular hexagon of side x has perimeter y , and y is a function of x . Please write down relation between x and y and find the values of y when $x = 8$ and $x = 5$.

(中文) 若邊長為 x 的正六邊形，其周長為 y ，則 y 是 x 的函數，請寫出 x 與 y 的關係式並求出當 $x = 8$ 及 $x = 5$ 時的函數值。

(翰林版課本)

Teacher: There are two questions in it. Let's begin with the first question: write down the relation of x and y . Let me ask you, how do you find the perimeter of a regular hexagon?

Student: The side times 6.

Teacher: Great, the side times 6. Therefore, the perimeter y equals the side x times 6. Which means the relation of x and y is $y = 6x$.

Teacher: Next, find the values of the functions $x = 5$ and $x = 8$ through this relation.

We can just find the answer by substituting 5 and 8 for the relation we got from the first question. How much is y when $x = 5$?

Student: It is 30.

Teacher: Exactly. When $x = 5$, y would be $6 \times 5 = 30$. Then, what is y when $x = 8$?

Student: 48.

Teacher: Yes, when $x = 8$, y would be $6 \times 8 = 48$.

老師：這一題總共問了我們兩個問題，我們先看第一個問題要寫出 x 與 y 的關係式。
那麼請問同學正六邊形的周長要怎麼算？

學生：邊長乘以 6。

老師：很好，邊長乘以 6 因此周長 y 會等於邊長 x 乘以 6，也就是說 x 與 y 的關係式為 $y = 6x$ 。

老師：接著我們再利用這個關係式來回答第二個問題求出 $x = 5$ 和 $x = 8$ 的函數值。
也就是我們直接將 5 和 8 代入第一題得到的關係式就是答案了，那麼當 $x = 5$ 時， y 是多少呢？

學生：30。

老師：沒錯。當 $x = 5$ 時， y 會是 $6 \times 5 = 30$ 。那麼當 $x = 8$ 時， y 又是多少？

學生：48。

老師：答對了。當 $x = 8$ 時， y 會是 $6 \times 8 = 48$ 。

單元四 線型函數與圖形

Linear Function and Graphics

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

在本節先用 $y = ax + b$ 介紹線性函數並定義一次函數與常數函數，之後利用生活中線性函數的例子講解其求法，並衍伸函數圖形的應用。在函數圖形的部分，我們先讓學生從描點開始慢慢的得到線性函數的圖形，在這裡老師也可以利用其他的數學軟體使學生會繪製函數圖形。

■ 詞彙 Vocabulary

單字	中文	單字	中文
linear function	一次函數	constant function	常數函數
linear term	一次項	linear function	線性函數
constant term	常數項		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

① Draw _____ on the coordinate plane.

例句：Draw the graph of the linear function $y = 3x - 2$ on the coordinate plane.

在坐標平面上畫出一次函數 $y = 3x - 2$ 的圖形。

② _____ pass through the points _____.

例句：We know a linear function which **passes through the points** $(2, -4)$ and $(-1, 5)$, and find the coordinates of the intersection of this graph with the y -axis.

已知一個線型函數，其圖形通過 $(2, -4)$ 與 $(-1, 5)$ 兩點，求此圖形與 y 軸的交點坐標。

■ 問題講解 Explanation of Problems**說明**

Students will understand how to surmise a linear function through points and solve application questions of a linear function.

使學生了解如何利用點反推線性函數及線性函數的應用。

運算問題的講解**例題一**

說明：使學生熟悉線性函數的應用。

Students will be familiar with the application of linear functions.

(英文) Customers complained that the prices were too high in Xiao Hua bookstore, so the owner decided to adjust the price. He adjusted the prices by a linear function $y = ax$. For example, a notebook was adjusted to \$28/ book from \$40/ book.

How much is a pair of 80-dollar scissors after adjusting? How much was a 140-dollar novel before adjusting? (from Han Lin)

(中文) 曉華書局因被顧客報怨價格太高，所以店長決定要調整價格。店長將店內每樣東西的價格用線型函數 $y = ax$ 進行調整成新的價格，例如：一本 40 元的筆記本經過調整後變成 28 元一本，則一把 80 元的剪刀經過調整後是多少元？調整後為 140 元的小說原價又是多少？（翰林版課本）

Teacher: Before answering the question, we need to find the value of a . Let's assume that the original price was x , the adjusted price is y . And the question also says that there was a notebook adjusted to \$28 from \$40, so we know that $x = 40$ and

$y = 28$. Substitute the numbers for the relation $y = ax$, then we'll get a . Could you tell me how much a is?

Student: a is $\frac{7}{10}$.

Teacher: That's right. So, the linear function in this question is $y = \frac{7}{10}x$.

Teacher: Since we already know the linear function, let's move to the scissors with the original price \$80. How much are the scissors with the adjusted price? This means that we can substitute $x = 80$ for $y = \frac{7}{10}x$. Then the relation becomes

$$y = \frac{7}{10} \times 80 = 56. \text{ Therefore, the answer for the first question is \$56.}$$

Teacher: Let's move on to the second question. We've learned that the adjusted price of a novel is \$140, and we need to find out the original price. Actually, we do this in the way we just did for the first question. The only difference is that we substitute the number of y . So, we know the answer from substitute $y = 140$ for $y = \frac{7}{10}x$. Please tell me what x is.

Student: x is 200.

Teacher: Yes, so the answer for the second question is \$200.

老師：在回答這題的問題前，我們需要先把 a 求出來，因此我們先假設原來的價格為 x 元，調整後的價格為 y 元。又因為題目告訴我們有一本筆記本原價為 40 元，經調整後變成 28 元，也就是說 $x = 40$ 、 $y = 28$ 再將他們代入 $y = ax$ 便可以得到 a 。那麼各位同學 a 為多少？

學生： $\frac{7}{10}$ 。

老師：沒錯，所以這題的線性函數為 $y = \frac{7}{10}x$ 。

老師：得知這題的線性函數後，我們該來看第一個問題一把80元的剪刀經過調整後是多少元。也就是 x 會等於80，再將其代入 $y = \frac{7}{10}x$ ，就寫成

$$y = \frac{7}{10} \times 80 = 56. \text{ 故第一個問題的答案便是56元。}$$

老師：回答完第一個問題，現在來看第二個問題告訴我們調整後的價格是140元問原價為多少。這其實和第一個問題的作法是一樣，只是第二個問題是要代 y 。

故 y 值為 140，再將其代入 $y = \frac{7}{10}x$ ，便是答案。那麼同學們 x 是多少？

學生：200。

老師：對，所以第二個問題的答案就是 200 元。

例題二

說明：讓學生利用代點求線性函數並以此求得函數值。

Students will find the linear function through the points and find the values.

(英文) It is known that doing some exercise for x minutes and the burned calories y have a relationship in linear function $y = ax + b$. If doing this exercise for 30 minutes, 440 kcals will be burned; doing it for 80, 1040 kcals will be burned, how many calories will be burned by doing this exercise for one hour? Doing how many minutes will burned 1160 kcals? (from Han Lin)

(中文) 已知某項運動的運動時間 x 分鐘與所消耗熱量 y 大卡存在線型函數關係為 $y = ax + b$ 。若此項運動 30 分鐘可消耗 440 大卡，運動 80 分鐘可消耗 1040 大卡的熱量，請問運動 1 小時能消耗多少熱量？想消耗 1160 大卡的熱量需要運動幾分鐘？（翰林版課本）

Teacher: What function does $y = ax + b$ belong to?

Student: Linear function.

Teacher: Good. It says that doing this exercise for 30 minutes burns 440 kcals and doing this exercise for 80 minutes burns 1160 kcals. So, the graph of this function must pass the two points, (30, 440) and (80, 1040). We just substitute the two points for $y = ax + b$ and solve the simultaneous equations, then we can get the function. What are a and b respectively?

Student: $a = 12, b = 80$.

Teacher: Great! So, the function would be $y = 12x + 80$. Let's read the question: how many calories are burned by doing the exercise for one hour and doing it for how long burns 1160 kcals? This means that the first question gives us x , we need to find y ; and the second question is reverse. Firstly, the unit of time is minute, we are going to transfer one hour to 60 minutes. Therefore, the x in the first question is 60. Let's substitute $x = 60$ for $y = 12x + 80$, how much is y , then?

What would x be when we substitute $y = 1160$ for the relation?

Student: $y = 800$ and $x = 90$.

Teacher: Correct. So, 800 kcals will be burned by doing this exercise for one hour and 1160 kcals for 90 minutes.

老師： $y = ax + b$ 是什麼函數啊？

學生： 一次函數。

老師： 很好，而題目告訴我們運動 30 分鐘消耗 440 大卡、運動 80 分鐘消耗 1160 大卡。故此一次函數的圖形必過 (30, 440)、(80, 1040) 這兩個點，因此我們只用將這兩點代入 $y = ax + b$ 並解聯立便可得到此函數了。

那麼解出來 a 和 b 為多少？

學生： $a = 12$ ， $b = 80$ 。

老師： 很好，所以這個函數便是 $y = 12x + 80$ 。那麼我們來看這個題目一小時可以消耗多少體力以及消耗 1160 大卡需要運動多久。也就是說第一個問題是給我們 x 要我們求 y 而第二個問題是顛倒過來的。

首先記得我們 x 的單位是分鐘，所以我們要先將一小時換成 60 分鐘，也就是說，第一個問題的 x 等於 60。那麼我們將 $x = 60$ 代入 $y = 12x + 80$ ，那麼 y 是多少？而將 $y = 1160$ 代入 $y = 12x + 80$ ， x 又會是多少？

學生： $y = 800$ 、 $x = 90$ 。

老師： 沒錯。因此運動一小時我們可以消耗 800 大卡，而消耗 1160 大卡需要運動 90 分鐘。

單元五 內角與外角

Interior Angle and Exterior Angle

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

在教授此章節前可依學生程度決定是否複習正三角形、等腰三角形及直角三角形。之後在教學生了解並運用兩個角的關係（互餘、互補、對頂角等）。了解角的關係後，我們再利用三角形內角和導出並使學生明白三角形的外角定理、外角和以及多邊形的內角和。

■ 詞彙 Vocabulary

單字	中文	單字	中文
supplementary angles	補角	interior angle	內角
supplementary	互補	diagonal	對角線
complementary angles	餘角	convex polygon	凸多邊形
complementary	互餘	concave polygon	凹多邊形
vertical opposite angle	對頂角	regular polygon	正 n 邊形
exterior angle	外角	n -sided polygon	n 邊形
interior opposite angle	內對角		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

① _____ intersects _____ at _____.

例句：The segment BC intersects the segment AD at point E . The angle $A = 50^\circ$, the angle $B = 60^\circ$, and the angle $C = 30^\circ$. Find the angle D .

\overline{BC} 與 \overline{AD} 相交於 E 點， $\angle A = 50^\circ$ ， $\angle B = 60^\circ$ ， $\angle C = 30^\circ$ ，求 $\angle D$ 。

■ 問題講解 Explanation of Problems

說明

Students can figure out the answer faster by making use of the relationship between the interior angle and exterior one, the exterior angle theorem, and the sum of the interior angles of polygons.

利用內外角的關係、外角定理以及多邊形的內角和使學生更快計算出答案。

運算問題的講解

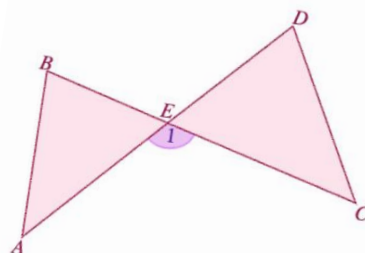
例題一

說明：使學生活用三角形的外角定理以及內角與外角的關係。

Students can use the exterior angle theorem and the relationship between the interior angle and exterior one flexibly.

(英文) In the diagram on the right, \overline{BC} and \overline{AD} intersect at point E . $\angle A = 35^\circ$, $\angle B = 75^\circ$ and $\angle C = 40^\circ$. Find $\angle 1$ and $\angle D$.

(中文) 如圖， \overline{BC} 與 \overline{AD} 相交於 E 點， $\angle A = 35^\circ$ ， $\angle B = 75^\circ$ ， $\angle C = 40^\circ$ ，求 $\angle 1$ 、 $\angle D$ 。



(翰林版課本)

Teacher: Let's work through $\angle 1$ first. What do you see from this drawing?

Student: $\angle 1$, $\angle AEB$ and $\angle DEC$ have a common vertex E .

Teacher: That is correct. That tells us that $\angle 1$ is an exterior angle of both $\triangle AEB$ and $\triangle CDE$. Based on the exterior angle theorem, we can say $\angle 1$ is equal to $\angle A$ plus $\angle B$. It's $\angle 1 = 35^\circ + 75^\circ = 110^\circ$.

Teacher: Now let's move on to $\angle D$. We also see that $\angle 1$ is an exterior angle of $\triangle CDE$, so once again what we can make out of $\angle 1$ given the exterior angle theorem?

Student: $\angle 1 = \angle C + \angle D$.

Teacher: Excellent! $\angle 1$ is equal to $\angle C$ plus $\angle D$. We can put it in this way,
 $110^\circ = 40^\circ + \angle D$, and $\angle D = 70^\circ$. The answer to this question is $\angle 1 = 110^\circ$ and $\angle D = 70^\circ$.

老師：我們先來解 $\angle 1$ 。首先我們來看這張圖，請問各位同學你們能觀察到什麼？

學生： $\angle 1$ 分別和 $\angle AEB$ 、 $\angle DEC$ 共線。

老師：沒錯，也就是說 $\angle 1$ 是 $\triangle AEB$ 和 $\triangle CDE$ 的一個外角。因此我們可以根據外角定理得知 $\angle 1$ 等於 $\angle A$ 加上 $\angle B$ ，故 $\angle 1 = 35^\circ + 75^\circ = 110^\circ$ 。

老師：接著我們來看 $\angle D$ ，剛剛我們有觀察到 $\angle 1$ 是 $\triangle CDE$ 的一個外角，因此我們再次根據外角定理可以得到什麼？

學生： $\angle 1 = \angle C + \angle D$ 。

老師：對， $\angle 1$ 會等於 $\angle C$ 加上 $\angle D$ 。所以，我們可以列式 $110^\circ = 40^\circ + \angle D$ 。經過計算後，我們可以得到 $\angle D = 70^\circ$ 。故這題的答案 $\angle 1 = 110^\circ$ ， $\angle D = 70^\circ$ 。

例題二

說明：使學生利用多邊形的內角和計算正多邊形的內角。

Students can make use of the interior angles of polygons and figure out measures of the interior angles of regular polygons.

(英文) What is the measure of an interior angle and exterior one of a regular 12-sided polygon.

(中文) 分別求正十二邊形的一個內角與一個外角的度數。

(翰林版課本)

Teacher: Before we go for the question, let's do a review first. What is the sum of the interior angles of a n -sided polygon?

Student: $(n - 2) \times 180^\circ$.

Teacher: That is right. The sum of the interior angles of a n -sided polygon is $(n - 2) \times 180^\circ$ because each interior angle of a regular n -sided polygon is equal sized. What is the measure of an interior angle in the polygon?

Student: $\frac{(n-2) \times 180^\circ}{n}$.

Teacher: Exactly! The measure of each interior angle of a regular n -sided polygon is $\frac{(n-2) \times 180^\circ}{n}$. Now, the question asks us to find the measure of an interior angle and an exterior one of the given polygon. The n here is 12. We can figure out an interior angle of it, and that is $\frac{(12-2) \times 180^\circ}{12} = 150^\circ$.

Teacher: And because the measure of every interior angle of a n -sided polygon is equal, that applies to the exterior angles too. Also, because an interior angle is supplementary to an exterior one, the measure of an exterior angle is going to be 180° minus the measure of an interior angle. What is it?

Student: 30° .

Teacher: Good job! $180^\circ - 150^\circ = 30^\circ$ and that is the measure of an exterior angle of a regular 12-sided polygon. So, in this polygon, the measure of an interior angle is 150° and the measure of an exterior one is 30° .

老師：在看題目前，我們先來複習一下 n 邊形的內角和為多少？

學生： $(n - 2) \times 180^\circ$ 。

老師：對， n 邊形的內角和為 $(n - 2) \times 180^\circ$ 。而因為正 n 邊形的每一個內角都一樣大，所以正 n 邊形的每個內角都等於多少呢？

學生： $\frac{(n-2) \times 180^\circ}{n}$ 。

老師：很好，正 n 邊形的每個內角就是為 $\frac{(n-2) \times 180^\circ}{n}$ 。現在題目問我們的是正十二邊形的一個內角和一個外角。所以我們要算一個內角只要把 $n = 12$ 代入，內角的度數就會是 $\frac{(12-2) \times 180^\circ}{12} = 150^\circ$ 。

老師：而因為正 n 邊形的每個內角的度數都一樣，所以正 n 邊形的每個外角的度數也都一樣，又因為內角和外角互補，因此我們只需用 180° 減掉內角的度數就是外角的度數。故外角等於多少？

學生： 30° 。

老師：答對了，外角等於 $180^\circ - 150^\circ = 30^\circ$ 。故正十二邊形的一個內角為 150° ，一個外角為 30° 。

單元六 尺規作圖與三角形的全等

Geometric Construction and Congruent Triangles

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

首先我們利用尺規作圖使學生了解三角形全等的意義，並作簡單的計算與推理。在此章節我們做簡單的推理時，只需要理解其原理即可，直到九上會再次提及幾何證明。

■ 詞彙 Vocabulary

單字	中文	單字	中文
geometric construction	尺規作圖	corresponding side	對應邊
perpendicular bisector	中垂線(垂直平分線)	corresponding angle	對應角
angle bisector	角平分線	corresponding point	對應點
Bisector of an angle	分角線	congruence property	全等性質
identical	全等		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

① Determine _____ is congruent to _____.

例句：Determine which of the following triangles is congruent to triangle ACD ?

判斷 $\triangle ACD$ 與下列哪一個三角形全等？

■ 問題講解 Explanation of Problems

說明

Students can know how to use a straightedge and compass to draw and prove two triangles congruent.

使學生知道如何用尺和圓規來繪圖以及證明兩三角形是全等。

運算問題的講解

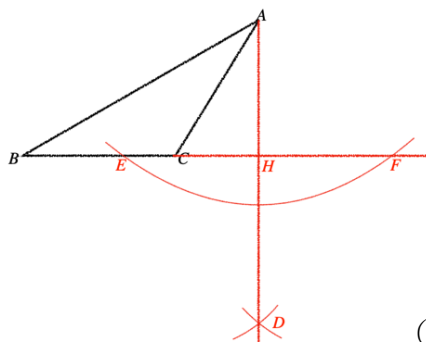
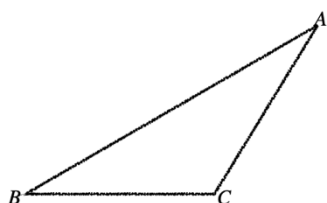
例題一

說明：利用題目使學生熟悉尺規作圖。

Students can get familiar with geometric constructions by solving questions.

(英文) In the figure on the right, construct an altitude to segment \overline{BC} of obtuse $\triangle ABC$

(中文) 如圖， $\triangle ABC$ 為鈍角三角形，利用尺規作圖畫出 \overline{BC} 上的高。



(翰林版課本)

Teacher: We are going to need our compasses and straightedges to draw the line requested. Please take them out first, class. Now the question asks us to draw the altitude to \overline{BC} . In other words, construct a perpendicular segment from vertex A to the opposite side \overline{BC} .

Teacher: First, we need to extend side \overline{BC} beyond point C. Place the pointy end of the compass on vertex A as the center, and make sure we set the radius large enough so it can intersect the extension line at two different places. The ideal length of the radius can be set somewhere between \overline{AC} and \overline{AB} . Now, swing an arc and mark the two intersections as E and F respectively.

Teacher: Now, those two points of the intersection, E and F, will be used as the center alternately to draw two arcs with a fixed radius; but, this radius needs to be wider

than $\frac{1}{2} \times \overline{EF}$ for the two arcs to intersect. All right, we have our intersecting point and let's call it point D .

Teacher: Next, we connect vertex A with point D over there, and label the third intersecting point on the extension line \overline{BC} as H . \overline{AH} is our altitude.

老師：我們這題要來做尺規作圖，請同學們先把尺和圓規拿出來。現在題目要我們作 \overline{BC} 上的高，換句話說就是作 \overline{BC} 的垂線且要過 A 點。

老師：首先我們先將 \overline{BC} 往 C 的方向延長，再以 A 點為圓心，取適當長為半徑畫弧，此半徑長最好是介在 \overline{AC} 和 \overline{AB} 之間。畫好後我們將交在 \overline{BC} 延長線上的兩點分別標示為 E 、 F 。

老師：現在我們分別以 E 、 F 兩點為圓心，並以相同的半徑畫弧，但這個半徑的長度需要大於 $\frac{1}{2} \times \overline{EF}$ 。畫完後我們可以看到兩弧會相交於一點，我們將這個交點記做 D 。

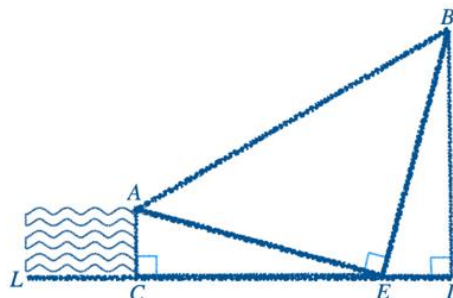
老師：接下來我們將 A 、 D 兩點連接起來，並將其交於 \overline{BC} 的延長線的點記做於 H 。而 \overline{AH} 便是我們所求的。

例題二

說明：藉由實際應用使學生更加熟練、理解三角形全等有關的內容。

Students will understand better and get familiar with the content related to triangle congruence. through practice.

(英文) DayDream Villa built a waterslide in their swimming pool with the slide length of \overline{AB} and four supporting side-stands (as illustrated). Two of them, \overline{AC} and \overline{BD} , are perpendicular to base L . \overline{AE} and \overline{BE} are equal in length and connected at E . Point C, D, E are secured on base L , and $\overline{AE} \perp \overline{BE}$. Are $\triangle ACE$ and $\triangle EDB$ congruent? Give reasons



(中文) 如圖，傻白甜渡假村在自家的游泳區建了一個滑水道，水道長為 \overline{AB} ，共有四個支架，兩根支架 \overline{AC} 、 \overline{BD} 均垂直於 L 兩根等長的支架 \overline{AE} 、 \overline{BE} 固定於 E 點，其中 C 、 D 、 E 三點都在直線 L 上且 $\overline{AE} \perp \overline{BE}$ 。

請問 $\triangle ACE$ 與 $\triangle EDB$ 是否全等？請說明你的理由。

(翰林素養題題本)

Teacher: Let's label $\angle CAE$ as $\angle 1$, $\angle CEA$ as $\angle 2$, and $\angle DEB$ as $\angle 3$. $\angle 1 + \angle 2 = 90^\circ$. Do you know why?

Student: Because the sum of all the interior angles of a triangle is 180° .

Teacher: That is correct. Because the sum of all the interior angles of a triangle is 180° and $\angle ACE = 90^\circ$, $\angle 1 + \angle 2 = 90^\circ$. Do you know why $\angle 2 + \angle 3 = 90^\circ$ too?

Student: Because $\angle AEB = 90^\circ$.

Teacher: Good job. Because $\angle 2 + \angle 3 + \angle AEB = 180^\circ$ and $\angle AEB = 90^\circ$, $\angle 2 + \angle 3 = 90^\circ$. And, either $\angle 1 + \angle 2$ or $\angle 2 + \angle 3$ is equal to 90° , so $\angle 1 = \angle 3$.

Teacher: Let's get straight up with what we have now. First, we know that $\angle ACE = \angle EDB = 90^\circ$ and $\overline{AE} = \overline{BE}$. We also found $\angle 1 = \angle 3$, so given these pieces of evidence, we can say $\triangle ACE$ and $\triangle EDB$ are congruent. Do you know what property is that based on? Is it based on the *RHS* congruence property?

Student: No, it's based on the *AAS* congruence property.

Teacher: Why not the *RHS* congruence property? Why *AAS*?

Student: Because we know nothing about whether the legs of both triangles are equal, it is not based on the *RHS* congruence property. Yet, we do know that these two triangles have two angles in equal measure and that either of their corresponding sides is the same, so it is based on the *AAS* congruence property.

Teacher: That is 100% correct. The so-called *RHS* congruence property refers to the hypotenuse and either side of two right triangles are equal, but we have no clue about whether they are equal in these two triangles. So, they do not fit into the *RHS* congruence property. On the other hand, the *AAS* congruence property describes two angles of two triangles are the same and either of their corresponding sides is equal. In this sense, we can reason out that $\triangle ACE$ is congruent to $\triangle EDB$ because of the *AAS* congruence property.

老師：我們先將 $\angle CAE$ 記做 $\angle 1$ 、 $\angle CEA$ 記做 $\angle 2$ 、 $\angle DEB$ 記做 $\angle 3$ 。

而 $\angle 1 + \angle 2 = 90^\circ$ ，你們知道為什麼嗎？

學生：因為三角形的內角和等於 180° 。

老師：沒錯，因為三角形的內角和等於 180° 且 $\angle ACE = 90^\circ$ ，所以 $\angle 1 + \angle 2 = 90^\circ$ 。那你們知道為啥 $\angle 2 + \angle 3$ 也等於 90° 嗎？

學生：因為 $\angle AEB = 90^\circ$ 。

老師：很好，因為 $\angle 2 + \angle 3 + \angle AEB = 180^\circ$ 且 $\angle AEB = 90^\circ$ ，因此 $\angle 2 + \angle 3 = 90^\circ$ 。又因為 $\angle 1 + \angle 2$ 與 $\angle 2 + \angle 3$ 都等於 90° ，故 $\angle 1 = \angle 3$ 。

老師：我們現在來整理一下我們有哪些的線索。首先是題目給的 $\angle ACE = \angle EDB = 90^\circ$ 以及 $\overline{AE} = \overline{BE}$ ，還有我們推出來的 $\angle 1 = \angle 3$ 。故在 $\triangle ACE$ 和 $\triangle EDB$ 中，因為 $\angle ACE = \angle EDB = 90^\circ$ 、 $\angle 1 = \angle 3$ 、 $\overline{AE} = \overline{BE}$ ，所以 $\triangle ACE \cong \triangle EDB$ 。那麼你們知道是根據什麼性質嗎？是 *RHS* 全等性質嗎？

學生：不是，是 *AAS* 全等性質。

老師：為什麼不是 *RHS* 全等性質？又為什麼是 *AAS* 全等性質？

學生：因為不知道股邊有沒有相等，所以不是 *RHS* 全等性質。因為我們知道有兩個角是相等的且其中一個角的對應邊是相等的，所以是 *AAS* 全等性質。

老師：回答的十分的完美。所謂的 *RHS* 全等性質是指兩個直角三角形的斜邊和其中一個股邊相等。而這題我們並不知道它們的股是否相等，故不是 *RHS* 全等性質。而 *AAS* 全等性質是指兩個三角形中有兩個角是分別相等的且其中一個角的對應邊是相等的，因此這題最後是根據 *AAS* 全等性質才能得出 $\triangle ACE$ 與 $\triangle EDB$ 為全等三角形。

單元七 全等三角形的應用

Applications of Congruent Triangles

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

利用三角形全等性質使學生理解角平分線、中垂線的性質與判別，同時也利用三角形全等性質說明等腰三角形的頂角平分線垂直平分底邊，並運用這些性質進行相關的計算及應用。

■ 詞彙 Vocabulary

單字	中文	單字	中文
isosceles triangle	等腰三角形	equilateral	正三角形

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

- ① _____ the midpoint of _____. Extend _____ to intersect the extension of _____ at _____.

例句: E is **the midpoint of** the segment BC . **Extend** the segment AE **to intersect the extension of** the segment DC **at** the point F . If the length of segment $AB = 6$, what is the length of the segment AF ?

E 是 \overline{BC} 的中點，延長 \overline{AE} 交 \overline{DC} 的延長線於 F 點。若 $\overline{AB} = 6$ ，則 \overline{AF} 的長是多少？

■ 問題講解 Explanation of Problems

說明

Students can get familiar with different types of bisectors and properties of special triangles so as to figure out answers faster.

使學生熟悉各種平分線和特殊三角形的性質，並更快地計算出答案。

運算問題的講解

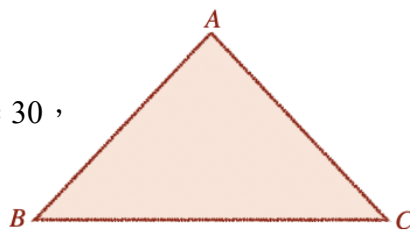
例題一

說明：使學生了解等腰三角形的頂角平分線垂直平分底邊，運用這些性質進行相關的計算。

Students can understand that the vertex bisector of an isosceles triangle is a line that is perpendicular to and bisects the base. Students can use this property to carry out relevant calculations.

(英文) In the diagram of $\triangle ABC$ on the right, $\overline{AB} = \overline{AC} = 17$ and $\overline{BC} = 30$. Find the length of the altitude to \overline{BC} .

(中文) 如圖， $\triangle ABC$ 中， $\overline{AB} = \overline{AC} = 17$ ， $\overline{BC} = 30$ ，求 \overline{BC} 上的高。



(翰林版課本)

Teacher: Now, the question asks us to find the length of the altitude to \overline{BC} , so we need to draw the altitude first and label this line segment as \overline{AD} . We also know this triangle has two sides congruent. Who knows what triangle it can be categorized into?

Student: An isosceles triangle.

Teacher: Exactly! This is an isosceles triangle. What property of the altitude to the base segment does it have?

Student: It is the perpendicular bisector of the base.

Teacher: Yes, it is the perpendicular bisector of the base. That also tells us this altitude will divide \overline{BC} into two equal segments, so we can say $\overline{BD} = \frac{1}{2} \times 30 = 15$. Now, because $\overline{AD} \perp \overline{BC}$, what type of triangle is $\triangle ABD$?

Student: A right triangle

Teacher: That is correct. It's a right triangle. We now know that in $\triangle ADB$, the length of $\overline{AB} = 17$, $\overline{BD} = 15$, what theorem can we apply if we want to figure out the length of the third side?

Student: The Pythagorean theorem.

Teacher: Excellent! So, $\overline{AD} = \sqrt{17^2 - 15^2} = 8$, and that is the length of \overline{BC} .

老師：現在題目要我們求 \overline{BC} 上高的長度，所以我們先把 \overline{BC} 上的高畫出來並將這條線記做 \overline{AD} 。那麼各位同學我們知道在這個三角形中有兩個邊相等，請問這是一個什麼三角形？

學生：等腰三角形。

老師：沒錯，這是一個等腰三角形。那麼等腰三角形底邊的高有啥性質呢？

學生：會垂直平分底邊。

老師：對，會垂直平分底邊。也就是說 \overline{BD} 的長度會是 \overline{BC} 的一半，

故 $\overline{BD} = \frac{1}{2} \times 30 = 15$ 。接下來，因為 $\overline{AD} \perp \overline{BC}$ ，所以 $\triangle ABD$ 是什麼三角形？

學生：直角三角形。

老師：很好，就是直角三角形。我們現在知道 $\triangle ADB$ 的斜邊 $\overline{AB} = 17$ 、其中一條股邊 $\overline{BD} = 15$ ，那麼我們想求另一條股邊能用什麼方法求？

學生：畢氏定理。

老師：非常棒，所以 $\overline{AD} = \sqrt{17^2 - 15^2} = 8$ 。故 \overline{BC} 上的高等於 8。

例題二

說明：利用題目帶領學生運用正三角形的相關公式。

Students can apply relevant theorems of the equilateral triangle by solving questions.

(英文) In an equilateral triangle, the length of its height is $5\sqrt{3}$ cm. Find its area.

(中文) 有一個正三角形的高為 $5\sqrt{3}$ 公分，求此正三角形的面積。(翰林版課本)

Teacher: Before we go for it, let's review what we have learned. Say, in an equilateral triangle, the length of the side segment is a cm. What are the length of its height and the area?

Student: The length of the height is $\frac{\sqrt{3}}{2}a$ and the area is $\frac{\sqrt{3}}{4}a^2$.

Teacher: That is correct. If either one does not ring a bell to you, you can try to verify them. Now, let's read the question. It tells us the length of the height of this equilateral triangle is $5\sqrt{3}$ and asks for the area of it. Let's say the length of its side is a cm and we already know the height of this equilateral triangle is measured $\frac{\sqrt{3}}{2}a$, so we can see this expression in equation $\frac{\sqrt{3}}{2} \times a = 5\sqrt{3}$. Now, we simplify this expression and can get the measure of $a = 10$.

Teacher: Now, we have the measure of the side length, and in our previous review, we know the area of an equilateral triangle is $\frac{\sqrt{3}}{4}a^2$ and the value of a . Now, we only need to simplify this expression and we will be done.
It's your turn to figure it out.

Teacher: Class, are you finished?

Student: Yes.

Teacher: Let's randomly pick a student from the class and invite him/her to write his/her answer down on the blackboard. No. xx, who is no. xx?

Student: It's him, xxx!

Teacher: Okay, xxx please come over here to write down your answer.

Teacher: xxx great job! $\frac{\sqrt{3}}{4} \times 10^2 = \frac{\sqrt{3}}{4} \times 100 = 25\sqrt{3}$, so the area of this equilateral triangle is $25\sqrt{3}$ cm².

老師：在接這題之前我們先來複習剛剛教的內容，假設一個正三角形的邊長為 a 公分，那麼他的高和面積各為多少？

學生： $\frac{\sqrt{3}}{2}a$ 、 $\frac{\sqrt{3}}{4}a^2$ 。

老師：對，如果忘了可以試著自己推導一下。我們現在來看這題，我們知道了這正三角形的高為 $5\sqrt{3}$ 問面積是多少。

所以我們先來假設這個正三角形的邊長為 a 公分，而我們剛剛複習過正三角形的高為 $\frac{\sqrt{3}}{2}a$ 。

因此我們可以列式 $\frac{\sqrt{3}}{2} \times a = 5\sqrt{3}$ ，計算一下我們就可以得到 $a = 10$ 。

學生：我們現在有了邊長，而我們剛剛也複習過正三角形的面積為 $\frac{\sqrt{3}}{4}a^2$ ，所以我們只要把邊長代入式子就行了。現在請同學們幫忙算一下。

老師：同學們，算好了嗎？

學生：好了。

老師：那我們現在來抽籤請同學上黑板寫，xx 號，是誰？

學生：是他，xxx。

老師：好，那請 xxx 同學上來寫吧。

老師：xxx，你寫得非常的好， $\frac{\sqrt{3}}{4} \times 10^2 = \frac{\sqrt{3}}{4} \times 100 = 25\sqrt{3}$ 。所以這個正三角形的面積為 $25\sqrt{3}$ 平方公分。

單元八 三角形的邊角關係

Properties of Angles and Sides of Triangles

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

在這章節中，讓學生理解當一個三角形有兩邊不等時，大邊對大角，大角對大邊，以及一個三角形的兩邊之和恆大於第三邊，並利用所學的內容判斷在何種條件下會形成一個三角形。

■ 詞彙 Vocabulary

單字	中文	單字	中文
hinge theorem	樞紐定理		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

① Compare the relationship between _____.

例句：Compare the relationship between the magnitude of angle1 and angle2.

比較 $\angle 1$ 、 $\angle 2$ 的大小關係。

■ 問題講解 Explanation of Problems

說明

Students can get familiar with the relationships between the side angles of a triangle by solving questions.

利用題目使學生熟悉三角形的邊角關係。

運算問題的講解

例題一

說明：利用題目使學生了解一個三角形中，當有兩邊不等時，大邊對大角。

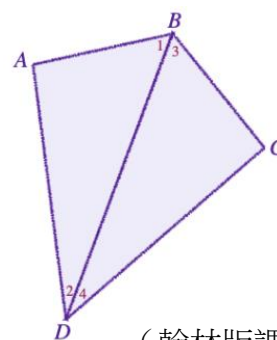
Students can understand when any two side-lengths in a triangle are not congruent, the side opposite the larger angle will be longer than the other.

(英文) In the diagram of quadrilateral $ABCD$ on the right, $\overline{AB} = 8$, $\overline{BC} = 8$, $\overline{CD} = 14$, and $\overline{DA} = 12$. Answer the questions below: Prove the relationship between $\angle ABC$ and $\angle ADC$. Explain your reasoning.

(中文) 如圖，四邊形 $ABCD$ 中， $\overline{AB} = 8$ ， $\overline{BC} = 8$ ， $\overline{CD} = 14$ ， $\overline{DA} = 12$ 。

回答下列問題：

寫出 $\angle ABC$ 和 $\angle ADC$ 的大小關係，並說明其理由。



(翰林版課本)

Teacher: A triangle property tells us that when a triangle has two unequal sides, the side opposite the larger angle will be longer than the side opposite the smaller angle.

Teacher: So that gives us two things. In $\triangle ABD$, because $\overline{DA} > \overline{AB}$, $\angle 1 > \angle 2$. We can also say in $\triangle BCD$, because $\overline{CD} > \overline{BC}$, $\angle 3 > \angle 4$.

Teacher: By combining them together, we can come to $\angle 1 + \angle 3 > \angle 2 + \angle 4$ and $\angle ABC = \angle 1 + \angle 3 > \angle 2 + \angle 4 = \angle ADC$. $\angle ABC > \angle ADC$.

老師：在一個三角形中有個性質-當兩邊不等時，大邊對大角，小角對小邊。

老師：所以在 $\triangle ABD$ 中，因為 $\overline{DA} > \overline{AB}$ ，所以 $\angle 1 > \angle 2$ ，這是第一個條件。而在 $\triangle BCD$ 中，因為 $\overline{CD} > \overline{BC}$ ，所以 $\angle 3 > \angle 4$ ，這是第二個條件。

老師：綜合這兩個條件，我們可以得到 $\angle 1 + \angle 3 > \angle 2 + \angle 4$ ，因此 $\angle ABC = \angle 1 + \angle 3 > \angle 2 + \angle 4 = \angle ADC$ 。故 $\angle ABC > \angle ADC$ 。

例題二

說明：利用三角形的不等關係做簡單的計算與推理。

Simple calculations and reasoning can be achieved by using the triangle inequality theorem.

(英文) Today, Coco, Bean, and Peppa are game for a competition. Each chooses different routes (as illustrated). The rule says that the travel pace of the three will be the same and that they are required to kick off the road at the same time to see who is the first one to arrive the fast-food restaurant (point C). Given the illustration shown, both Coco and Bean decide to start from the Observatory (point A), while Peppa will be from the Gym. If $\angle B > \angle A$ and $\overline{AD} = \overline{BD}$, who will win this game? Please explain your reasoning.

Coco: $A \rightarrow D \rightarrow C$

Bean: $A \rightarrow C$

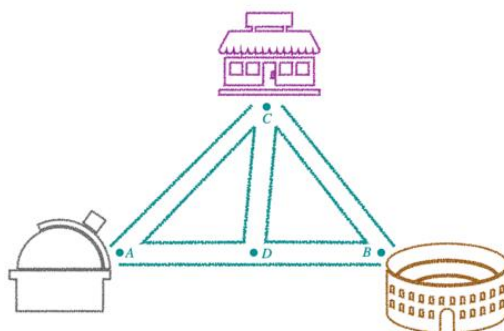
Peppa: $B \rightarrow C$

(中文) 今天小玟、小荳和小豬想來比個賽，它們分別選了三條路徑（如下）且規定三人都要以相同速率並同時出發來比賽看誰先到速食店(C 點處)。如圖，小玟和小荳選擇從天文台出發天文台 (A 點處)，而小豬選擇從體育館 (B 點處)，若 $\angle B > \angle A$ ，且 $\overline{AD} = \overline{BD}$ ，則他們三人中誰是贏家？請說明你的原因。

小玟： $A \rightarrow D \rightarrow C$

小荳： $A \rightarrow C$

小豬： $B \rightarrow C$



(素養題題本)

Teacher: This question is really long, but we can simply put it in this way: it asks us which

route is the shortest, $\overline{AD} + \overline{DC}$, \overline{AC} or \overline{BC} ? Let's review what we have at hand first. We already know $\angle B > \angle A$, and because the side opposite the greater angle is the longer side in a triangle, which side is longer, \overline{AC} or \overline{BC} ?

Student: \overline{AC} is longer.

Teacher: That is right. \overline{AC} is longer than \overline{BC} and we can also view this polygon ACD as a triangle. Given that property, what if we add two sides of this triangle together?

Student: The length of them combined is greater than that of the third side.

Teacher: Exactly! The length of them combined is going to be greater than that of the third side. So, in $\triangle ACD$, we can say $\overline{AD} + \overline{DC} > \overline{AC}$.

Teacher: Now, we can put everything we have known so far together. That will tell us that $\overline{AD} + \overline{DC} > \overline{AC} > \overline{BC}$. Who do you think is the winner?

Student: It's Peppa.

Teacher: Good job, the answer to this question is Peppa.

老師：雖然這題題目十分的冗長，但其實我們可以把牠看成是在問 $\overline{AD} + \overline{DC}$ 、 \overline{AC} 、 \overline{BC} 哪個最短？那我們先來整理一下我們目前有的條件，題目說 $\angle B > \angle A$ ，而因為大角對大邊的關係，所以 \overline{AC} 、 \overline{BC} 哪個比較長呢？

學生： \overline{AC} 比較長。

老師：對， \overline{AC} 會大於 \overline{BC} 。而我們可以把多邊形 ACD 看成是一個三角形，既然是一個三角形，那麼此三角形任兩邊相加會怎樣？

學生：大於第三邊。

老師：很好，會大於第三邊。因此在 $\triangle ACD$ 中，我們可以得到 $\overline{AD} + \overline{DC} > \overline{AC}$ 。

老師：我們把所有的條件整理出來會得到 $\overline{AD} + \overline{DC} > \overline{AC} > \overline{BC}$ ，所以贏家會是誰呢？

學生：小豬。

老師：對，這題的答案是小豬。

單元九 平行線與截角性質

Parallel Line and the Properties of Truncation

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

我們先利用矩形特性來帶入平行線概念並使學生知其定義，之後我們在介紹截線與截角，並由已知一截角利用三角形內角和、平角與對頂角求出各截角的方式，同時歸納平行線的同位角相等內錯角相等同側內角互補等性質。

■ 詞彙 Vocabulary

單字	中文	單字	中文
parallel line	平行線	corresponding angle	同位角
truncation	截角	alternate interior angle	內錯角
intercept	截線	interior angles on the same side	同側內角

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

① _____ are intercepted by _____, _____.

例句：Two lines **are intercepted by** a straight line L and a set of corresponding angle is 80° and 75° respectively, what is the acute angle of intersection of these two lines?

兩直線被一直線 L 所截，且其中一組同位角分別為 80° 及 75° ，則此兩直線相交而成的銳角是幾度？

■ 問題講解 Explanation of Problems

說明

Students will become familiar with the properties of parallel lines and understand how to use it 使學生更熟悉平行線的性質並實際應用他。

運算問題的講解

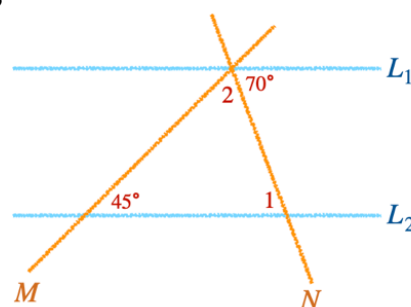
例題一

說明：用平行線的性質求出角度。

Find out an angle using the properties of parallel lines

(英文) As shown, $L_1 \parallel L_2$, M , and N are all intercepts of L_1 and L_2 . The intersection point is on L_1 , what are the actual angles of $\angle 1$ and $\angle 2$?

(中文) 如圖， $L_1 \parallel L_2$ ， M 及 N 都是 L_1 、 L_2 的截線，且交點在 L_1 上，求 $\angle 1$ 、 $\angle 2$ 。



(翰林版課本)

Teacher: Let's find $\angle 1$ first. We already know that $L_1 \parallel L_2$. Yet the alternate interior angles are equal, we know that $\angle 1 = 70^\circ$.

Teacher: Let's check $\angle 2$ next. The angle on the left of $\angle 2$ is named $\angle 3$. In the same way, as we found $\angle 1$, what is $\angle 3$?

Student: $\angle 3 = 45^\circ$.

Teacher: That's right. $\angle 3 = 45^\circ$. Since the same-side interior angles are supplementary, $\angle 2 + \angle 3$ and $\angle 1$ are supplementary. In other words, $\angle 2 + \angle 3 + \angle 1 = 180^\circ$. So $\angle 2 + 45^\circ + 70^\circ = 180^\circ$, what is the actual angle of $\angle 2$?

Student: $\angle 2 = 65^\circ$.

Teacher: Right answer. So, the answers for this question are: $\angle 1 = 70^\circ$ and $\angle 2 = 65^\circ$.

老師：我們先來求 $\angle 1$ 。首先我們知道 $L_1 \parallel L_2$ ，又因為內錯角相等，所以 $\angle 1 = 70^\circ$ 。

老師：我們再來看 $\angle 2$ ，我們將 $\angle 2$ 左邊的角稱作 $\angle 3$ 。如同求 $\angle 1$ 的方法， $\angle 3$ 等於多少？

學生： $\angle 3 = 45^\circ$ 。

老師： 沒錯， $\angle 3 = 45^\circ$ 。接著又因為同側內角互補，因此 $\angle 2 + \angle 3$ 跟 $\angle 1$ 會是互補的。
換句話說， $\angle 2 + \angle 3 + \angle 1 = 180^\circ$ 。故 $\angle 2 + 45^\circ + 70^\circ = 180^\circ$ ，那麼 $\angle 2$ 等於多少？

學生： $\angle 2 = 65^\circ$ 。

老師： 答對了，所以這題的答案： $\angle 1 = 70^\circ$ 、 $\angle 2 = 65^\circ$ 。

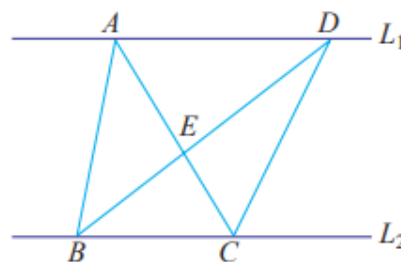
例題二

說明：利用平行線作固定面積的圖形變化。

Using parallel lines to create different figures with same area

(英文) As shown, $L_1 \parallel L_2$, the area of $\triangle ADE$ is 12, the area of $\triangle ABE$ is 9, and the area of $\triangle BCE$ is 6, what is the area of the quadrilateral $ABCD$?

(中文) 如圖， $L_1 \parallel L_2$ ， $\triangle ADE$ 的面積是 12， $\triangle ABE$ 的面積是 9， $\triangle BCE$ 的面積是 6，求四邊形 $ABCD$ 的面積。



(翰林版課本)

Teacher: We are now asked for the area of the quadrilateral $ABCD$. From the picture, we can tell that the area of quadrilateral $ABCD$ equals

$$\triangle ABE + \triangle BCE + \triangle ADE + \triangle CED.$$

Since we already know the areas of $\triangle ABE$, $\triangle BCE$, and $\triangle ADE$, we just need to find out the area of $\triangle CED$ for the answer.

Teacher: Let's find the area of $\triangle CED$ now. We know that $\triangle ABC$ and $\triangle BCD$ share the same bottom side \overline{BC} from the picture. Yet, because $L_1 \parallel L_2$, $\triangle ABC$ and $\triangle BCD$ have the same height.

Therefore, we learn that the areas of $\triangle ABC$ and $\triangle BCD$ are equal.

Teacher: Besides, $\triangle ABC$ can be written as $\triangle ABE$ plus $\triangle BCE$, and $\triangle BCD$ as $\triangle BCE$ plus $\triangle CED$. So, $\triangle ABE + \triangle BCE = \triangle BCE + \triangle CED$. Use elimination method, and we will get that the area of $\triangle ABE$ equals the area of $\triangle CED$.

Then what is the area of $\triangle CED$?

Student: It is 9.

Teacher: Yes, since we know that the area of $\triangle ABE$ is 9, the area of $\triangle CED$ is also 9. Now, we know the area of $\triangle CED$, we just add up all the areas of $\triangle ABE$, $\triangle BCE$, $\triangle ADE$, and $\triangle CED$ for the answer. What would it be?

Student: It would be 36.

Teacher: That's great. The area of the quadrilateral $ABCD$ is exactly 36 square unit.

老師：現在題目要我們求出四邊形 $ABCD$ 的面積，而從圖中四邊形 $ABCD$ ，等於 $\triangle ABE + \triangle BCE + \triangle ADE + \triangle CED$ 。其中， $\triangle ABE$ 、 $\triangle BCE$ 、 $\triangle ADE$ 的面積題目已經告訴我們了，也就是說，我們只要再求出 $\triangle CED$ 的面積就可以算出答案了。

老師：那麼我們現在來求 $\triangle CED$ 的面積。從圖中我們可以知道 $\triangle ABC$ 與 $\triangle BCD$ 有相同的底 \overline{BC} ，又因為 $L_1 \parallel L_2$ ，故 $\triangle ABC$ 與 $\triangle BCD$ 的高會等長，因此 $\triangle ABC$ 的面積等於 $\triangle BCD$ 的面積。

老師：而我們又可以把 $\triangle ABC$ 寫成 $\triangle ABE$ 加上 $\triangle BCE$ ， $\triangle BCD$ 寫成 $\triangle BCE$ 加上 $\triangle CED$ 。故 $\triangle ABE$ 的面積 + $\triangle BCE$ 的面積等於 $\triangle BCE$ 的面積 + $\triangle CED$ 的面積。再做整理一下，就變成 $\triangle ABE$ 的面積等於 $\triangle CED$ 的面積。所以 $\triangle CED$ 的面積等於多少？

學生：9。

老師：對，因為題目告訴我們 $\triangle ABE$ 的面積是 9，所以 $\triangle CED$ 的面積也是 9。現在我們知道 $\triangle CED$ 的面積了，接下來我們只要把 $\triangle ABE$ 、 $\triangle BCE$ 、 $\triangle ADE$ 、 $\triangle CED$ 的面積全部相加起來就好了。請問相加後答案是多少？

學生：36。

老師：非常棒，四邊形 $ABCD$ 的面積就是 36 平方單位。



單元十 平行四邊形

Parallelogram

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

這一節我們利用「平行線的截角性質」和「三角形全等性質」來推導出平行四邊形的性質及四種的判別方法（兩組對邊等長或兩組對角相等的四邊形、兩對角線互相平分的四邊形、一組對邊平行且等長的四邊形）。

■ 詞彙 Vocabulary

單字	中文	單字	中文
parallelogram	平行四邊形		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

① _____ bisect each other.

例句：Two diagonals of a parallelogram **bisect each other**.

平行四邊形的兩條對角線互相平分。

② _____ divided _____ in quarters.

例句：The two diagonals of a parallelogram **divided** their area **in quarters**.

平行四邊形的兩條對角線將其面積四等分。

■ 問題講解 Explanation of Problems

說明

Students will be able to solve complicated geometric questions by utilizing the properties of parallelogram.

利用平行四邊形的性質使學生能解決個複雜的幾何問題。

運算問題的講解

例題一

說明：利用平行四邊形的性質求出角度。

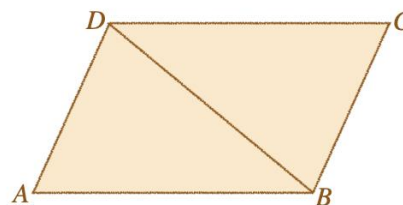
Find an actual angle using the properties of parallelogram.

(英文) As shown, in the parallelogram $ABCD$, $\angle A = 80^\circ$. If $\angle ABD:\angle DBC = 3:2$, what is the actual angle of $\angle DBC$?

(中文) 如圖，平行四邊形 $ABCD$ 中， $\angle A = 80^\circ$ 。

若 $\angle ABD:\angle DBC = 3:2$ ，

則 $\angle DBC$ 的度數為何？



(109 年國中會考第 5 題)

Teacher: From this question, we know that this is a parallelogram. On the basis of the properties of parallelogram that diagonals are equal, that is $\angle A = \angle C = 80^\circ$ and $\angle ABC = \angle ADC$. Also, the interior angles on the same side are supplementary, so $\angle ABC = 180^\circ - 80^\circ = 100^\circ$.

Student: Next, it says that $\angle ABD:\angle DBC = 3:2$. In other words, how much does $\angle DBC$ account for of $\angle ABC$?

Student: It accounts for $\frac{2}{5}$.

Teacher: Great, $\frac{2}{5}$ exactly. So, $\angle DBC$ equals $\angle ABC \times \frac{2}{5}$. And we've found that $\angle ABC = 100^\circ$, what is the actual angle of $\angle DBC$?

Student: It is 40° .

Teacher: Correct, $100^\circ \times \frac{2}{5}$ is 40° . So, we know that $\angle DBC = 40^\circ$.

老師：根據題目我們知道這是一個平行四邊形，而根據平行四邊形的性質-兩組對角相等，因此 $\angle A = \angle C = 80^\circ$ 、 $\angle ABC = \angle ADC$ 。又因兩平行線的同側內角互補，故 $\angle ABC = 180^\circ - 80^\circ = 100^\circ$ 。

學生：接著，題目告訴我們 $\angle ABD : \angle DBC = 3 : 2$ 。那麼換句話說 $\angle DBC$ 占 $\angle ABC$ 的幾分之幾？

老師： $\frac{2}{5}$ 。

學生：很好，就是 $\frac{2}{5}$ 。所以 $\angle DBC$ 會等於 $\angle ABC \times \frac{2}{5}$ ，而我們在前面算過 $\angle ABC = 100^\circ$ ，故 $\angle DBC$ 是多少？

老師： 40° 。

老師：沒錯， $100^\circ \times \frac{2}{5}$ 就是等於 40° 。故 $\angle DBC = 40^\circ$ 。

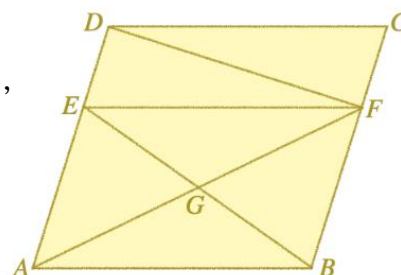
例題二

說明：利用平行四邊形性質計算面積。

Find an area using the properties of parallelogram.

(英文) As shown, the quadrilateral $ABCD$ is a parallelogram, and $\overline{EF} \parallel \overline{AB}$. If the area of the quadrilateral $ABFE$ is 32 and the area of the quadrilateral $CDEF$ is 14, what is the area of the quadrilateral $DEGF$?

(中文) 如圖，四邊形 $ABCD$ 為平行四邊形， $\overline{EF} \parallel \overline{AB}$ ，如果四邊形 $ABFE$ 的面積為 32，四邊形 $CDEF$ 的面積為 14，求四邊形 $DEGF$ 的面積。



(翰林版課本)

Teacher: We are asked for the area of the quadrilateral $DEGF$. From the picture, we can tell that the quadrilateral $DEGF$ equals $\triangle EGF$ plus $\triangle DEF$, so we just need to find and add up the areas of $\triangle EGF$ and $\triangle DEF$.

Teacher: The quadrilateral $ABCD$ is a parallelogram, and $\overline{EF} \parallel \overline{AB}$. From this, we learn that quadrilateral $ABFE$ and $CDEF$ are parallelograms, too. Because of the properties of parallelogram, the two diagonals divide the parallelogram into four equal areas, we

know that the area of quadrilateral $ABFE$, which is 32, divided by 4 equals the area of $\triangle EGF$. Then what is the answer?

Student: The answer is 8.

Teacher: That's right. Now we know the area of $\triangle EGF$ is 8, let's move on to the area of $\triangle DEF$. With the properties of parallelogram that any diagonal divides a parallelogram into two congruent triangles, and that \overline{DF} is a diagonal of the parallelogram $CDEF$, the area of quadrilateral $CDEF$, 14, divided by 2 would be the area of $\triangle DEF$. Everyone, what is the answer?

Student: It is 7.

Teacher: Yes, it is 7. Now we have the areas of $\triangle EGF$ and $\triangle DEF$. We just need to add them up and we'll get the area of the quadrilateral $DEGF$. Therefore, the area of the quadrilateral $DEGF$ would be $8 + 7 = 15$ square unit.

老師：現在題目要我們求四邊形 $DEGF$ 的面積，而我們可以從圖看到邊形 $DEGF$ 可以寫成 $\triangle EGF$ 加上 $\triangle DEF$ ，所以我們只要求出 $\triangle EGF$ 和 $\triangle DEF$ 的面積，之後再把他們相加就行了。

老師：因為四邊形 $ABCD$ 為平行四邊形而 $\overline{EF} \parallel \overline{AB}$ ，所以四邊形 $ABFE$ 和四邊形 $CDEF$ 也是平行四邊形。因為平行四邊形的性質-兩條對角線將其面積四等分，故 $\triangle EGF$ 的面積會等於四邊形 $ABFE$ 的面積 32 除以 4。答案是多少？

學生：8。

老師：沒錯。現在我們知道 $\triangle EGF$ 的面積等於 8 了，接下來我們換求 $\triangle DEF$ 的面積。因為 \overline{DF} 為平行四邊形 $CDEF$ 的對角線和平行四邊形的性質-任一條對角線均可將它分成兩個全等的三角形，所以我們可以得知 $\triangle DEF$ 的面積等於四邊形 $CDEF$ 的面積 14 除以 2。請同學算算答案是多少？

學生：7。

老師：答對了。現在我們有 $\triangle EGF$ 和 $\triangle DEF$ 的面積了，接下來我們把他們相加便是四邊形 $DEGF$ 的面積，故四邊形 $DEGF$ 的面積等於 $8 + 7 = 15$ 平方單位。

單元十一 特殊四邊形與梯形

Irregular Quadrilateral and Trapezoid

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

在此章節我們先利用「三角形全等性質」和「平行線截角性質」來介紹等腰梯形底角相等且對角線等長的特性，之後再介紹梯形兩腰中點連線段的性質，並利用對角線的等長、平分、垂直等關係來介紹特殊四邊形對角線的性質，最後引導學生利用對角線來判別特殊四邊形的方法。

■ 詞彙 Vocabulary

單字	中文	單字	中文
kite	箏形	lower line	下底
rhombus	菱形	side	腰
rectangle	長方形（矩形）	midsegment	中位線
square	正方形	base angle	底角
trapezoid	梯形	isosceles trapezoid	等腰梯形
upper line	上底		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

① _____ in which _____ perpendicular and bisects _____.

例句：A quadrilateral **in which** one diagonal is **perpendicular and bisects** the other diagonal is a kite.

一條對角線垂直平分另一條對角線的四邊形是箏形。

■ 問題講解 Explanation of Problems

說明

Students will find the angle or the length faster by utilizing the properties of certain quadrilaterals and trapezoid.

利用特殊四邊形和梯形的性質使學生可以更快的計算出角度或長度。

運算問題的講解

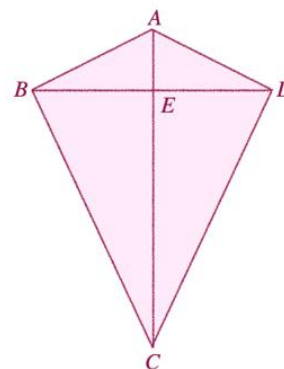
例題一

說明：利用特殊四邊形對角線的性質進行計算以求長度。

Find the length using the properties of diagonals of certain quadrilaterals.

(英文) As shown, in the kite $ABCD$, $\overline{AB} = \overline{AD} = 5$, $\overline{BC} = \overline{CD} = 12$, and $\angle ABC = \angle ADC = 90^\circ$, please find \overline{AC} and \overline{BD} .

(中文) 如圖，箏形 $ABCD$ 中， $\overline{AB} = \overline{AD} = 5$ ， $\overline{BC} = \overline{CD} = 12$ ， $\angle ABC = \angle ADC = 90^\circ$ ，求 \overline{AC} 及 \overline{BD} 。



(翰林版課本)

Teacher: Let's start with \overline{AC} . The question says that $\angle ABC = \angle ADC = 90^\circ$. In other words, $\triangle ABC$ and $\triangle ADC$ are right triangles. Thus, we can use Pythagorean

theorem we've learned before, one square leg plus another square leg equals square hypotenuse, to get the length of \overline{AC} . How much is \overline{AC} , please?

Student: $\overline{AC} = 13$.

Teacher: Exactly. $\overline{AC} = \sqrt{5^2+12^2} = 13$. Let's do \overline{BD} now. From the textbook, we learned that the area of a kite equals one half of the product of the two diagonals. So, once we figure out the area of the kite, we get \overline{BD} . Except for this way, what else can we do to find the area of the kite?

Student: Find and combine the areas of $\triangle ABC$ and $\triangle ADC$.

Teacher: Yes, find and combine the areas of $\triangle ABC$ and $\triangle ADC$, and we can get the area of the kite, which is half of the product of the two diagonals. Also, $\triangle ABC$ and $\triangle ADC$ are two congruent right triangles, the equation would be $(5 \times \frac{12}{2}) \times 2 = \frac{13 \times \overline{BD}}{2}$. What will this become if we simplify it?

Student: $\overline{BD} = \frac{120}{13}$.

Teacher: Good for you. So, the answers for this question are $\overline{AC} = 13$ and $\overline{BD} = \frac{120}{13}$.

老師：我們先來看 \overline{AC} ，因為題目告訴我們 $\angle ABC = \angle ADC = 90^\circ$ ，換句話說， $\triangle ABC$ 和 $\triangle ADC$ 為直角三角形，所以我們可以利用之前學過的畢氏定理，兩股平方和相加等於斜邊平方可以得到 \overline{AC} 的長度。現在請同學幫我算一下 \overline{AC} 等於多少？

學生： $\overline{AC} = 13$ 。

老師：沒錯， $\overline{AC} = \sqrt{5^2+12^2} = 13$ 。算完 \overline{AC} 我們現在來算 \overline{BD} 。根據課本裡面提到箏形的面積會等於兩條對角線長乘積的二分之一，所以我們只要算出箏形的面積就可以算出 \overline{BD} 。那麼這個箏形的面積除了兩條對角線相乘再除二分之一外還能怎麼算？

學生：算出 $\triangle ABC$ 和 $\triangle ADC$ 的面積再相加。

老師：沒錯，算出 $\triangle ABC$ 和 $\triangle ADC$ 的面積再相加就可以得到箏形的面積也就是等於兩條對角線長乘積的二分之一。而又因為 $\triangle ABC$ 與 $\triangle ADC$ 是兩個全等的直角三角形，所以我們可以寫成 $(5 \times \frac{12}{2}) \times 2 = \frac{13 \times \overline{BD}}{2}$ 。整理一下會得到什麼呢？

學生： $\overline{BD} = \frac{120}{13}$ 。

老師：非常棒，所以這題的答案就是 $\overline{AC} = 13$ 、 $\overline{BD} = \frac{120}{13}$ 。

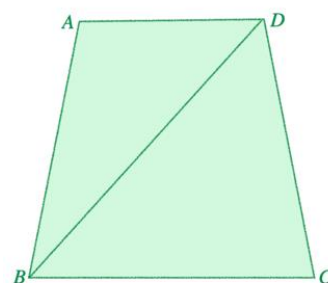
例題二

說明：利用等腰梯形的性質以及平行線截角性質求解。

Solve questions by utilizing the properties of isosceles trapezoid and parallel line truncations

(英文) In the isosceles trapezoid $ABCD$, $\overline{AD} \parallel \overline{BC}$. If $\angle A = 100^\circ$, $\angle ABD = 30^\circ$, and $\overline{AB} = 15$, what are the actual angle of $\angle BDC$ and the length of \overline{BC} ?

(中文) 等腰梯形 $ABCD$ 中， $\overline{AD} \parallel \overline{BC}$ ，若 $\angle A = 100^\circ$ ， $\angle ABD = 30^\circ$ ， $\overline{AB} = 15$ ，求 $\angle BDC$ 以及 \overline{BC} 的長。



(翰林版課本)

Teacher: Let's start with $\angle BDC$. The question says $\overline{AD} \parallel \overline{BC}$, and based on the properties that alternate interior angles of two parallel lines are equivalent and that interior angles on the same side are supplementary, we can get that $\angle ADB = \angle DBC = 180^\circ - 100^\circ - 30^\circ = 50^\circ$. We also know that the quadrilateral $ABCD$ is an isosceles trapezoid, what are the properties of an isosceles trapezoid?

Student: Having two diagonals with equal length.

Teacher: Good, what else?

Student: Having two same base angles.

Teacher: That's right! An isosceles trapezoid has two same base angles. We know that $\angle A = 100^\circ$, so, $\angle A = \angle D = 100^\circ$.

Thus, $\angle BDC = \angle ADC - \angle ADB = 100^\circ - 50^\circ = 50^\circ$.

Teacher: Next, let's do \overline{BC} . Do you have any idea why an isosceles trapezoid is called so?

Student: Because the two legs of it are equal.

Teacher: Right, that is why it is called an isosceles trapezoid. It means that $\overline{AB} = \overline{CD}$. Besides, because $\angle DBC = 50^\circ$ and $\angle BDC = 50^\circ$, $\triangle CBD$ is an isosceles triangle, then we know $\overline{CD} = \overline{BC}$. Therefore, $\overline{AB} = \overline{CD} = \overline{BC}$, and $\overline{AB} = 15$.

What is \overline{BC} then?

Student: $\overline{BC} = 15$.

Teacher: That's right. So, the answers for the question are $\angle BDC = 50^\circ$ and $\overline{BC} = 15$.

老師：我們先來算 $\angle BDC$ 。

因為題目說 $\overline{AD} \parallel \overline{BC}$ ，而我們根據平行線內錯角相等和平行線段的同側內角互補的性質，可以得到 $\angle ADB = \angle DBC = 180^\circ - 100^\circ - 30^\circ = 50^\circ$ 。現在題目還告訴我們四邊形 $ABCD$ 為等腰梯形，各位同學等腰梯形有哪些性質呢？

學生：兩條對角線等長。

老師：很好，還有呢？

學生：兩組底角分別相等。

老師：沒錯！因為等腰梯形的兩組底角分別相等而 $\angle A = 100^\circ$ ，所以我們可以得到 $\angle A = \angle D = 100^\circ$ 。因此 $\angle BDC = \angle ADC - \angle ADB = 100^\circ - 50^\circ = 50^\circ$ 。

老師：接著我們來算 \overline{BC} ，大家知道等腰梯形為什麼叫等腰梯形嗎？

學生：因為他兩腰等長。

老師：對，就是因為他的兩腰等長，所以被稱作等腰梯形，也就是說 $\overline{AB} = \overline{CD}$ 。又因為 $\angle DBC = 50^\circ$ 、 $\angle BDC = 50^\circ$ ，所以 $\triangle CBD$ 為等腰三角形，故 $\overline{CD} = \overline{BC}$ 。因此 $\overline{AB} = \overline{CD} = \overline{BC}$ ，而 $\overline{AB} = 15$ 。所以 \overline{BC} 等於多少？

學生： $\overline{BC} = 15$ 。

老師：答對了。因此這題的答案便是 $\angle BDC = 50^\circ$ 、 $\overline{BC} = 15$ 。

國內外參考資源 More to Explore

國家教育研究院樂詞網	
查詢學科詞彙 https://terms.naer.edu.tw/search/	
教育雲：教育媒體影音	
為教育部委辦計畫雙語教學影片 https://video.cloud.edu.tw/video/co_search.php?s=%E9%9B%99%E8%AA%9E	
Oak Teacher Hub	
國外教學及影音資源，除了數學領域還有其他科目 https://teachers.thenational.academy/	
CK-12	
國外教學及影音資源，除了數學領域還有自然領域 https://www.ck12.org/student/	
Twinkl	
國外教學及影音資源，除了數學領域還有其他科目，多為小學及學齡前內容 https://www.twinkl.com.tw/	

Khan Academy	
<p>可汗學院，有分年級數學教學影片及問題的討論。</p> <p>https://www.khanacademy.org/</p>	
Open Textbook (Math)	
<p>國外數學開放式教學資源</p> <p>http://content.nroc.org/DevelopmentalMath.HTML5/Common/toc/toc_en.html</p>	
MATH is FUN	
<p>國外教學資源，還有數學相關的小遊戲</p> <p>https://www.mathsisfun.com/index.htm</p>	
PhET: Interactive Simulations	
<p>國外教學資源，互動式電腦模擬。除了數學領域，還有自然科。</p> <p>https://phet.colorado.edu/</p>	
Eddie Woo YouTube Channel	
<p>國外數學教學影音</p> <p>https://www.youtube.com/c/misterwootube</p>	

國立臺灣師範大學數學系陳界山教授網站	
國高中數學雙語教學相關教材 https://math.ntnu.edu.tw/~jschen/index.php?menu=TeachingWorksheets	
2023 年第四屆科學與科普專業英文(ESP)能力大賽	
科學專業英文相關教材，除了數學領域，還有其他領域。 https://sites.google.com/view/ntseccompetition/%E5%B0%88%E6%A5%AD%E8%8B%B1%E6%96%87%E5%AD%B8%E7%BF%92%E8%B3%87%E6%BA%90/%E7%9B%B8%E9%97%9C%E6%95%99%E6%9D%90?authuser=0	



國中數學領域雙語教學資源手冊：英語授課用語

[八年級下學期]

A Reference Handbook for Junior High School Bilingual Teachers in
the Domain of Mathematics: Instructional Language in English

[8th grade 2nd semester]

- 研編單位：國立彰化師範大學雙語教學研究中心
- 指導單位：教育部師資培育及藝術教育司
- 撰稿：蔡庭宜、李珮慈
- 學科諮詢：張淑珠
- 綜合規劃：曾松德
- 編輯排版：吳依靜
- 封面封底：JUPE Design



發行單位 國立彰化師範大學雙語教學研究中心

NCUE BILINGUAL EDUCATION RESEARCH CENTER

指導單位 教育部師資培育及藝術教育司

MOE DEPARTMENT OF TEACHER AND ART EDUCATION