國中數學領域

雙語教學資源手冊 英語授課用語

A Reference Handbook for Junior High School Bilingual Teachers in the Domain of Mathematics: Instructional Language in English

〔八年級上學期〕

國立彰化師範大學雙語教學研究中心 發行

目次 Table of Contents

單元一 乘法公式
Multiplication Formula

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

這單元學生由觀察圖形面積的變化，再配合分配律推展完全平方公式以及平方差公式，並透過觀察數字間的關聯利用公式更快速地計算出結果。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
formula for the perfect square	完全平方公式	formula for the difference of two squares	平方差公式
square of the sum of two numbers	和的平方公式	square of the difference of two numbers	差的平方公式

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Using _____, calculate _____.

例句：**Using** the square of the sum of two numbers, **calculate** the value of $\left(150\frac{3}{4}\right)^2$.

利用和的平方公式，計算$\left(150\frac{3}{4}\right)^2$的值。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

Using the formula for the perfect square and the formula for the difference of two squares to do calculations.

我們利用完全平方公式和平方差公式來幫助計算。

❧ 運算問題的講解 ☙

例題一

說明：本題是讓學生熟悉完全平方公式的計算。

Students can get familiar with the formula for the perfect square.

（英文）Use the formula for the perfect square to simplify $(a+90)^2+(a-10)^2=?$

（中文）利用完全平方公式化簡：$(a+90)^2+(a-10)^2=?$

Teacher: Let's read the question first: "use the formula for the perfect square to simplify $(a+90)^2+(a-10)^2$." $(a+90)^2$ is the square of the sum of two numbers. Do you remember the expansion formula of $(a+b)^2$?

Student: $a^2+2ab+b^2$.

Teacher: Yes, it is $a^2+2ab+b^2$. So, the first binominal squared $(a+90)^2$will be $a^2+180a+8100$. Great, then what's the later part of the question?

Student: The square of the difference of two numbers.

Teacher: That's right, the square of the difference of two numbers. Then what is the expansion formula of $(a-10)^2$?

Student: $a^2-20a+100$.

Teacher: Exactly, it is $a^2-20a+100$. So, let's get back to the question, $(a+90)^2+(a-10)^2$ equals $a^2+180a+8100+a^2-20a+100$. The result will be $2a^2+160a+8200$ after we organize the formula.

老師： 我們先來看一下題目，題目說「利用完全平方公式化簡 $(a+90)^2+(a-10)^2$」。那麼我們先來做前半部分$(a+90)^2$，這是個和的平方公式。那麼$(a+b)^2$展開後是什麼？

學生： $a^2+2ab+b^2$。

老師： 對，$a^2+2ab+b^2$。所以我們套到題目裡面就是 $a^2+180a+8100$。
很好，那麼題目的後半部分是什麼？

學生： 差的平方公式。

老師： 對，是差的平方公式，那麼 $(a-10)^2$ 展開後又是什麼呢？

學生： $a^2-20a+100$。

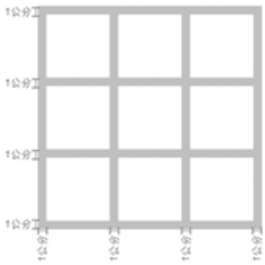
老師： 沒錯，$a^2-20a+100$。所以回到題目來，$(a+90)^2+(a-10)^2$就會等於 $a^2+180a+8100+a^2-20a+100$，之後我們在整理一下就會得到 $2a^2+160a+8200$。

例題二

說明：使學生可以從應用題中利用平方差公式更快地計算出結果。
Students can solve word problems more quickly by using the formula for the difference of two squares.

（英文）Cheryl would like to build a 9-square-grid garden (see diagram) by using bricks 1cm wide. Each square is made equal-sized, 5cm on each side. What is the minimum area of bricks that Cheryl needs to use for her garden?

（中文）雪菡想要用 1 公分寬的磚塊圍出一個九宮格花圃（如下圖），已知九宮格內的每一小格的邊長都是 5 公分，請問雪菡最少用了多少面積的磚塊？



Teacher: Let's read the question. It says what is the minimum area of bricks used for the garden. How do we find the minimum area?

Student: Subtract the cut-out area from the larger square.

Teacher: Exactly! The outer square without the cut-out area. We have already known the side lengths for the outer square are $5\times3+1\times4$ that gives 19. The area for this larger square is 19^2. How about the white squares?

Student: Find the area of one square then times 9.

Teacher: It is not wrong, but do we have another expression? What shape of these white squares can form?

Student: 5 times 3 then square the result.

Teacher: Excellent! The area of that white region gets 15^2, so the minimum area is actually $19^2 - 15^2$. Apply the formula for the difference of two squares can give us $19^2 - 15^2 = (19 - 15)(19 + 15) = 136$. Remember the number is in cm^2!

老師： 我們先來看一下題目，最少用了多少面積的磚塊，最少怎麼算？

學生： 最大的正方形減掉挖空的部分。

老師： 對！最外面的正方形剪掉裡面挖空的部分。最外面的正方形邊長為 $5 \times 3 + 1 \times 4$ 也就是 19，所以這個正方形的面積是 19^2。那白色的部分怎麼算？

學生： 先算出其中一塊的面積再乘以 9。

老師： 你這麼說也沒錯，但還有沒有其他種算法，可以先觀察一下這幾個白色的格子可以組成什麼形狀。

學生： 把 5 乘以 3 再平方。

老師： 很好！因此白色的面積就是 15^2，所以這一題最少用了多少面積其實就是 $19^2 - 15^2$，我們再利用平方差公式 $19^2 - 15^2 = (19 - 15)(19 + 15) = 136$，記得單位是平方公分喔！

單元二 多項式的加減
Addition and Subtraction of Polynomials

國立彰化師範大學數學系　蔡庭宜、李珮慈

■ 前言 Introduction

學生在一年級學到的「式子的加減運算」是多項式加減的特例。而其運算的方法都是多項式加減時必備的知識。因此，此節的重點在於讓學生認識多項式，並知道同類項正式的定義以及建立直式的加減運算。

■ 詞彙 Vocabulary

單字	中文	單字	中文
polynomial	多項式	constant polynomial	常數多項式
monomial	單項式	polynomial of degree zero	零次多項式
term	項	polynomial of zero	零多項式
constant term	常數項	arrangement in descending power	降冪排列
coefficient	係數	arrangement in ascending power	升冪排列
degree	次數	the term with the highest degree	最高次項
like term	同類項		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ _____ is called _____.

例句：The degree order from lowest to highest **is called** arrangement in ascending power.
次數由低到高排列稱為升冪排列。

■ 問題講解 Explanation of Problems

ᔓ 說明 ᔕ

We guide students to know how to add or subtract polynomials.
我們使學生了解多項式的加減是如何運算的。

ᔕ 運算問題的講解 ᔓ

例題一

說明：使學生了解升冪與降冪。
Students can know what is descending and ascending power.

（英文）Arrange this polynomial $2x^3 + 9 - 3x + 9x^2$ in descending and ascending powers respectively.
（中文）將多項式 $2x^3 + 9 - 3x + 9x^2$ 依降冪與升冪分別排列。

Teacher: The question asks us to arrange this polynomial $2x^3 + 9 - 3x + 9x^2$ in descending and ascending powers respectively. Please make sure which order the question asks before answering it. Let's do the descending power first. Do you know what it is?

Student: The power of a term decreases from the term with the highest degree to the lowest.

Teacher: Excellent! The power of a term decreases from the term with the highest degree to the lowest. So, the degree order for this question in descending power starts from the third-degree term, second-degree term, first-degree term, to the constant term, and that gives us $2x^3 + 9x^2 - 3x + 9$. Next is the ascending power. What is it?

Student: Reverse the degree order of the descending power.

Teacher: Fantastic! Ascending power is to arrange the polynomial from the term with the lowest degree to the highest, so the answer is $9 - 3x + 9x^2 + 2x^3$.

老師： 將多項式 $2x^3 + 9 - 3x + 9x^2$ 依降冪與升冪分別排列。在解這種類型的題目一定要先看清楚是降冪還是升冪，那我們先來算降冪排列。那你們知道什麼是降冪嗎？

學生： 次方由高排到低。

老師： 對！次方由高排到低。所以這題的次方由高到低就是 3 次、2 次、1 次、常數，因此答案就是 $2x^3 + 9x^2 - 3x + 9$。接著我們再來看升冪！那升冪又是什麼呢？

學生： 降冪倒過來。

老師： 非常好！升冪就是次方由低排到高，所以升冪的答案就是 $9 - 3x + 9x^2 + 2x^3$。

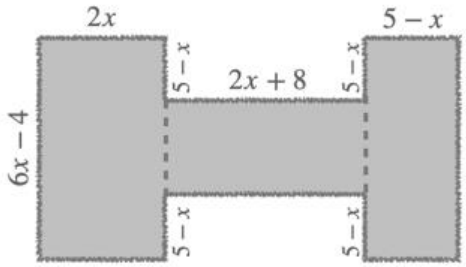
例題二

說明：本題使學生從應用題中更了解多項式的運算規則。

Students can know more how to solve polynomial equations by practicing the word problem below.

（英文）The school teacher in today's art class asks the class to use colored paper to fold a dumbbell (as illustrated). Find the perimeter of this dumbbell.

（中文）今天學校老師上美術課，請同學用色紙摺出一個運動器材。因為慕柏最近有在舉啞鈴，所以他就用色紙摺出一個啞鈴的圖形（如下圖）。請問此啞鈴的周長為多少？



Teacher: Okay, for this question, we need to find the perimeter. That is easy because we just need to add up all the side lengths around the figure.

That gives us $2(6x - 4) + 2 \times 2x + 2(2x + 8) + 6 \times (5 - x)$, and then we can see each term does have 2. So, we extract 2 from this expression and here we have

$2[(6x-4)+2x+(2x+8)+3\times(5-x)]$. Now, do you know how to simplify each term in these parentheses here?

Student: Add constant terms altogether. Add one-degree terms altogether.

Teacher: Exactly! Add constant terms altogether. Add one-degree terms altogether. More specifically, we add or subtract like terms. After simplifying these terms in the parentheses, we can get $2(7x+19)$. Finally, we follow the distributive law to multiply the 2 to each term inside the parenthesis and the answer is $14x+38$.

老師： 來，我們這一題要算的是周長，那就很簡單了！只要把圖形周圍的長度全部加起來就可以了。

因此我們可以得到 $2(6x-4)+2\times 2x+2(2x+8)+6\times(5-x)$，之後我們可以發現每個式子都有 2，所以我們把 2 提出來，便會得到 $2[(6x-4)+2x+(2x+8)+3\times(5-x)]$。那接下你們知道要怎化簡中括號裡的式子嗎？

學生： 常數項加常數項，一次項加一次項。

老師： 沒錯，就是常數項加常數項，一次項加一次項，正確來說是相同次方的數相加減。化簡完中括號內的式子後，我們會得到 $2(7x+19)$。最後我們再利用分配律將 2 乘進去便會得到答案 $14x+38$。

單元三 多項式的乘除
Multiplication and Division of Polynomials

國立彰化師範大學數學系　蔡庭宜、李珮慈

■ 前言 Introduction

這單元由發展$(a+b)(c+d)=ac+ad+bc+bd$的經驗，推展至多項式的乘法，並建立直式的乘法運算。因考量學生學習的現況，課本中僅教授多項式除法的運算過程，若教學中有需要額外補充，請教師自行至教師課本查閱。

■ 詞彙 Vocabulary

單字	中文	單字	中文
dividend	被除式	remainder	餘式
divisor	除式	divide	整除
quotient	商式		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ _____ is multiplied by _____.

例句：When six x **is multiplied by** three x, six is multiplied by three and x is multiplied by x.
$6x$ 乘以 $3x$ 時，6 與 3 相乘，x 與 x 相乘。

❷ Find _____ of _____.

例句：**Find** the quotient and the remainder **of** $(3x^2 + 4) \div 2x$.

求 $(3x^2 + 4) \div 2x$ 的商式及餘式。

❸ We know that _____ when we divide it by _____.

例句：**We know that** the quotient of the polynomial A is $x + 5$ **when we divide it by** $3x - 7$, and the remainder is 13. Please find the polynomial A.

已知多項式 A 除以 $x + 5$ 得商式為 $3x - 7$，餘式為 13，求多項式 A。

■ 問題講解 Explanation of Problems

說明

Students can know how the multiplication and division of polynomials work.

我們使學生了解多項式的乘除是如何運算的。

運算問題的講解

例題一

說明：使學生更了解多項式的除法並從商式、餘式和被除式去反推得出除式，同時瞭解怎麼用直列式做除法。

Students can understand more about polynomial division and how to use the quotient, the remainder, the dividend to blow back the divisor, as well as how to do the long division at the same time.

（英文）If a polynomial $2x^3 - 10x^2 + 20x + 20$ divided by $ax + b$ equals $x^2 + 10$ with a remainder of 120. What is the value of $a \times b$?

（中文）若多項式 $2x^3 - 10x^2 + 20x + 20$ 除以 $ax + b$，得商式為 $x^2 + 10$，餘式為 120，則 $a \times b$之值為何？

Teacher: We know that the dividend is equal to the divisor multiplied by the quotient plus the remainder, so we can express our question as

$2x^3 - 10x^2 + 20x + 20 = (ax + b)(x^2 + 10) + 120$. But since we are asked to find the divisor, we need to distribute $(x^2 + 10)$ and 120 to the dividend, and that gives us $[2x^3 - 10x^2 + 20x + 20 - 120] \div (x^2 + 10) = ax + b$. Now, please tell me the value of the polynomial in the square brackets.

Student: $2x^3 - 10x^2 + 20x - 100$.

Teacher: Excellent! $2x^3 - 10x^2 + 20x - 100$.
We can evaluate $(2x^3 - 10x^2 + 20x - 100) \div (x^2 + 10)$ using long division to get the quotient, and then we will know the value of a and b.

Teacher: In polynomial long division, we stop the division when the remainder is either a lower degree than the divisor or 0. Also, if there is a term missing, we need to fill in the blank with a placeholder 0. Now, let's get on to $(2x^3 - 10x^2 + 20x - 100) \div (x^2 + 0x + 10)$. What times x^2 can give us $2x^3$?

Student: $2x$.

Teacher: That is correct. So, $x^2 + 0x + 10$ times $2x$ that is equal to $2x^3 + 0 + 20x$. We add 0 in the place of second-degree term and then we subtract $2x^3 + 0 + 20x$ from $2x^3 - 10x^2 + 20x - 100$. That gives us $-10x^2 - 100$. The first term of this new dividend has the same degree of x^2, so we need to continue the process.

Teacher: Now what times x^2 will give us $-10x^2$?

Student: -10.

Teacher: Good job! So, $x^2 + 0x + 10$ times -10 that will come to $-10x^2 + 0 - 100$. Here we still need to fill the missing term with 0.
Again, we subtract $-10x^2 - 100$ from $-10x^2 - 100$ and get 0. The remainder is 0 and we are clear to stop. $2x - 10$ is our quotient. a is 2, and b is -10.
The value of $a \times b$ is -20.

老師： 我們知道被除式會等於除式乘上商再加上餘式，所以這一題我們可以寫成 $2x^3 - 10x^2 + 20x + 20 = (ax + b)(x^2 + 10) + 120$。但這一題問除式是多少？所以我們要把 $(x^2 + 10)$ 和 120 移項到被除式那邊就會變成 $[2x^3 - 10x^2 + 20x + 20 - 120] \div (x^2 + 10) = ax + b$。那同學可以先告訴我中括號內的式子會等於多少嗎？

學生： $2x^3 - 10x^2 + 20x - 100$？

老師： 對！$2x^3 - 10x^2 + 20x - 100$。

之後我們用列直式算出 $(2x^3-10x^2+20x-100)\div(x^2+10)$ 的商為何便可以知道 a 和 b 等於什麼了。

老師：在多項式的除法中我們需要算到餘式的次數比除式的次數低或餘式為 0 才能停止。如果有遇到缺項的情況同學要記得補 0。
那我們現在來算$(2x^3-10x^2+20x-100)\div(x^2+0x+10)$。那麼 x^2 要乘多少才會等於 $2x^3$？

學生：$2x$。

老師：對，所以我們 $x^2+0x+10$ 乘上 $2x$ 會得到 $2x^3+0+20x$。因為缺項需要補 0，所以我們在 x^3和 $20x$ 加上了 0。
之後我們在用 $2x^3-10x^2+20x-100$ 減掉$2x^3+0+20x$ 會得到 $-10x^2-100$。但 $-10x^2$ 的次方數等於 x^2 的次方數，因此我們要再繼續算下去。

老師：接下來那麼 x^2 要乘多少才會等於 $-10x^2$？

學生：-10。

老師：對，所以我們 $x^2+0x+10$ 乘上 -10 會得到 $-10x^2+0-100$。這裡一樣因為缺了一次項，所以我們需要再一次向的位置補 0。之後我們在用 $-10x^2-100$ 減掉 $-10x^2-100$ 會得到0。因為餘式為 0，我們可以停止計算了而除式是 $2x-10$。因此 a 是 2，b 是 -10，$a\times b$的值為 -20。

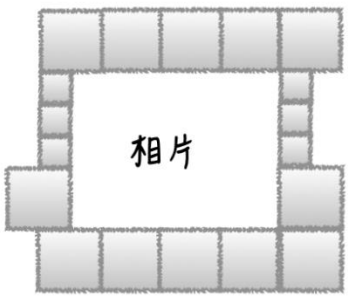
例題二

說明：本題使學生應用題中更加熟悉多項式的乘法。

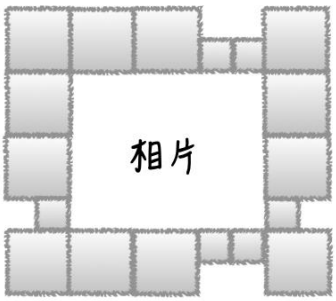
Students can get more familiar with polynomial long division by practicing the word problem below.

（英文）Today my sister bought two packs of square mosaic tiles. Each pack contains two different square sizes, 12 pieces with a side length of x cm and six pieces with a side length of 5 cm. She will use them to frame her two photos like the illustration below. Find the area of the first photo frame. If $x=6$, what is the difference of the total area in cm^2 between the first frame and the second one?

（中文）今天姊姊去買了兩包內含 12 個邊長為 x公分的正方形馬賽克拼貼磚和 6 個邊長為 5 公分的正方形馬賽克拼貼磚的組合包準備回家幫他的兩張相片各做一個相框。做完後的樣子如下圖，請問圖一的相片面積為多少平方公分？
如果 $x=6$ 那圖一和圖二的相片面積會相差多少平方公分？



(圖一)



(圖二)

Teacher: Let's look at the first question. What shape of the first photo would be?

Student: A rectangle?

Teacher: Do you know how to find the area of rectangles?

Student: The length times the width.

Teacher: Excellent! The length times the width. Let's look at the length of this rectangle first. Can we have it by adding up the lengths of the four squares, which is $4x$, and subtracting the 5cm square from it?

Student: Yes.

Teacher: So, the entire length is $4x - 5$ and then we need to figure out the width. The width here is composed of three $5cm$ squares and one $x$cm square. What is the value of the width?

Student: $x + 15$.

Teacher: Exactly! So, we just need to multiply the length by the width, which is $(4x - 5) \times (x + 15)$, and then simplify this binomial. That will be the answer to this question. Now, let's expand $(4x - 5) \times (x + 15)$.

Teacher: First, we multiply $4x$ by x and 15 and that give us $4x^2 + 60x$. Then, we do the same with -5 and we can have $-5x - 75$. The result will be

$4x^2 + 60x - 5x - 75$ and we combine like terms.

That gives us $4x^2 + 55x - 75$, so the answer is $4x^2 + 55x - 75$ in cm^2.

Teacher: Now the second question asks us to subtract out the first photo from the second photo to find a difference. First, we apply the same method to find the area of the second photo frame. Who can answer me what is the length and width of it?

Student: The length is $2x + 10$ and the width is $2x + 5$.

Teacher: That is correct. The length is $2x + 10$ and the width is $2x + 5$. How about the area?

Student: $4x^2 + 30x + 50$ in cm^2

Teacher: Great job! We have figured out the area of both the first and the second photo, and the only thing we need to do now is to subtract them, which will be

$(4x^2 + 55x - 75) - (4x^2 + 30x + 50)$. In the last chapter, we have learned and practiced how to add or subtract polynomials, so now we can simplify this expression. That gives us $25x - 125$, and the question tells us that x is 6.

We plug the value of x back in $25x - 125$ and we have the answer

$25 \times 6 - 125 = 25$ cm^2.

老師： 我們先看第一個問題，你們覺得圖一是什麼圖形？

學生： 長方形？

老師： 那你們知道長方形的面積要怎麼算嗎？

學生： 長乘以寬。

老師： 很好，就是長乘以寬。那麼我們先來看長，這個長方形的長是不是由這四個正方形的邊長和 $4x$ 再減掉這個邊長為 5 的正方形。

學生： 對。

老師： 所以這個長就是 $4x - 5$。我們再接著看寬，這裡的寬是由三個邊長為 5 的正方形和一個邊長為 x 的正方形所組成的。那麼寬會是多少？

學生： $x + 15$。

老師： 對，所以我們只要再把長和寬相乘起來也就是 $(4x - 5) \times (x + 15)$ 這個式子展開便是這題的答案了。那麼接下來我們就來展開 $(4x - 5) \times (x + 15)$這個式子。

老師： 首先我們先把 $4x$ 乘以 x 和 15 會等於 $4x^2 + 60x$，之後再將 -5 乘以 x 及 15 會等於 $-5x - 75$。所以展開後的式子便是 $4x^2 + 60x - 5x - 75$ 再將式子化簡一下就能到 $4x^2 + 55x - 75$。所以答案就是 $4x^2 + 55x - 75$ 平方公分。

老師： 那接下來看第二個問題，兩張相片面積差多少？首先我們用一樣的方法把圖二相框的面積算出來，有誰可以回答我圖二相框的長和寬是多少嗎？

學生： 長是 $2x + 10$，寬是 $2x + 5$。

老師： 沒錯，長是 $2x + 10$，寬是 $2x + 5$。那面積又是多少呢？

學生： $4x^2 + 30x + 50$ 平方公分。

老師： 對很好，所以我們現在知道了圖一相片的面積以及圖二相片的面積，那麼接下來我們只要把他們相減也就是$(4x^2 + 55x - 75) - (4x^2 + 30x + 50)$。因為我們在上一章已經學過並練習過多項式的加減運算了所以我們就直接把他化簡出來，結果便是 $25x - 125$。但因為題目說 x 等於 6 所以我們要再把 6 代入 $25x - 125$，所以答案會是 $25 \times 6 - 125 = 25$ 平方公分。

單元四 二次方根的意義
Definition of Square Root

國立彰化師範大學數學系　蔡庭宜、李珮慈

■ 前言 Introduction

這單元藉由用「正方形面積反求邊長」的方法，讓學生先知道無理數的存在（但無理數運算及明確的概念屬於高中範圍），再引進根號數的符號，之後再學習根號數的相反數，使學生知道正負根號數的區別，以作為平方根意義的先備知識。

■ 詞彙 Vocabulary

單字	中文	單字	中文
root	根號	rational number	有理數
perfect square	完全平方數	digit-by-digit calculation	十分逼近法
square root	平方根(二次方根)	radical	根式；根數

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Compare the sizes of _____.

例句：**Compare the sizes of** $\sqrt{34}$, 15, $\sqrt{121}$.

比較 $\sqrt{34}$、15、$\sqrt{121}$ 三數的大小。

❷ Approximate _____to _____decimal place by rounding.

例句：**Approximate** the value of $\sqrt{19}$ **to** the first **decimal place by rounding.**

以四捨五入法求 $\sqrt{19}$ 的近似值到小數點後第一位。

❸ Determine whether _____.

例句：**Determine whether** 23 is the square root of 529.

判別 23 是否為 529 的平方根。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

Students can familiarize themselves with the concept of the root by solving a question as a warm-up for the following chapter.

我們利用題目來熟悉根號的概念以為下章節做鋪墊。

❧ 運算問題的講解 ☙

例題一

說明：利用此題來整合學生在 2-1 這一單元中所學的所有觀念。

This question below summarizes all the concepts that students have learned in the chapter 2-1.

（英文）Which of the following description about $\sqrt{18}$ is wrong?

(A) We can find the coordinate of $\sqrt{18}$ on the number line

(B) $\sqrt{18} = 3\sqrt{2}$

(C) $\sqrt{18}$ is the positive square root of 16

(D) $\sqrt{18} > 4$

（中文）有關$\sqrt{18}$的敘述，下列何者<u>錯誤</u>？

(A) 在數線上可以找到坐標為 $\sqrt{18}$ 的點

(B) $\sqrt{18} = 3\sqrt{2}$

(C) $\sqrt{18}$是 16 的正平方根

(D) $\sqrt{18} > 4$

Teacher: Let's start with option A. It says that we can find $\sqrt{18}$ (Square root of 18) on the number line. What do you think?

Student: That is correct.

Teacher: How so?

Student: No idea.

Teacher: Because we can find $\sqrt{2}$ on the number line and because we can simplify $\sqrt{18}$ to $3\sqrt{2}$, which means three times $\sqrt{2}$, we can definitely find $3\sqrt{2}$ (3 times root 2) on the number line.

Option A is correct.

Teacher: Let's do option B. First, we should simplify $\sqrt{18}$. What is the prime factorization of 18?

Student: 2×3^2.

Teacher: Excellent, it is 2×3^2. So, $\sqrt{18}$ is the same thing as $3\sqrt{2}$. Option B is proved correct. Next is option C. How can we prove it?

Student: By calculating the positive and negative square root of 16.

Teacher: Exactly, by calculating the positive and negative square root of 16. But note here, we are asked to find the positive square root. We only need to solve the positive square root of 16, which is 4. So, option C is wrong. We should choose C.

Teacher: We've known the answer, but we still need to go through option D. The option C is proved to us that 4 is equal to $\sqrt{16}$. Then, which one is bigger, 18 or 16?

Student: 18.

Teacher: That's right. It's 18, so which one is bigger? $\sqrt{18}$ or $\sqrt{16}$?

Student: $\sqrt{18}$.

Teacher: Great job! We can conclude that $\sqrt{18}$ is bigger than 4, which means option D is also correct.

老師： 我們先來看 A 選項，A 選項說數線上可以找到 $\sqrt{18}$ 這個點，你們覺得是對的還是錯的。

學生：對的。

老師：那為什麼是對的呢？

學生：不知道。

老師：我們知道 $\sqrt{2}$ 可以在數線上畫得出來，而 $\sqrt{18}$ 可以化成 $3\sqrt{2}$，也就是三倍的$\sqrt{2}$，所以 $3\sqrt{2}$ 也可以在數線上找到。所以 A 選項是對的。

老師：我們再來看 B 選項，首先我們先將 $\sqrt{18}$ 化簡一下，你們知道的 18 質因數分解會等於多少嗎？

學生：2×3^2。

老師：對，就是 2×3^2，因此我們可以得到 $\sqrt{18} = 3\sqrt{2}$。所以 B 選項還是對的。接著 C 選項，16 的平方根怎麼算？

學生：正負 16 開根號。

老師：沒錯，就是正負 16 開根號。但要注意喔，題目這裡問的是正平方根，所以應該只有正 16 開根號，也就是 4，所以 C 選項是錯的，這題的答案選 C。

老師：雖然已經知道答案了，但我們還是要把 D 選項看完。我們在講解 C 選項時，便知道了 4 會等於 $\sqrt{16}$，那麼 18 跟 16 誰會比較大？

學生：18。

老師：對，是 18。所以 $\sqrt{18}$ 跟 $\sqrt{16}$ 哪一個比較大？

學生：$\sqrt{18}$。

老師：很好，所以結論就是 $\sqrt{18}$ 會大於 4，也就是說 D 選項也是對的。

例題二

說明：利用實際的範例幫助學生了解根式的意義。

A real-world example can aid students to understand more about the radical.

（英文）A Go board is marked with 324 small squares 1 cm wide. Find the side length of the largest square that all the small squares can form.

（中文）圍棋棋盤上有 324 個邊長為 1 公分的小方格，則盤面上所有小方格所構出的最大正方形的邊長為多少？

Teacher: The question asks for all the 324 1-by-1 squares to make up the largest square possible, which means the area of the largest square is 324.

Teacher: As we have known, the area of a square is equal to the length multiplied by the width, or the length of one side squared. That is equivalent to saying that the length is equal to the square root of the area. Then we can write it like this $\sqrt{324}$. Now, can someone tell me what the prime factorization of 324 is?

Student: 2 squared times 3 to the fourth power.

Teacher: Excellent! The prime factorization of 324 is $2^2 \times 3^4$, so $\sqrt{324}$ is the same thing as $\sqrt{2^2 \times 3^4}$ or we can express it like this $\sqrt{(2 \times 3^2)^2}$. Why we can write the square root like this expression is already covered in the seven-grade math.

Teacher: So, the length of this largest square is equal to $\sqrt{324} = \sqrt{(2 \times 3^2)^2} = 2 \times 3^2 = 18$ cm^2.

老師： 題目說有 324 個邊長為 1 的小方格要構成一個最大的正方形，那就表示這個最大的正方形面積為 324。

老師： 而我們都知正方形的面積等於邊長乘邊長，也就是邊長的二次方。
換句話說，邊長會等於面積再開根號。因此我們便可以知道邊長會等於$\sqrt{324}$。
那有人可以回答我 324 的質因數分解是多少嗎？

學生： 二的二次方乘上三的四次方。

老師： 很好，324 的質因數分解就是 $2^2 \times 3^4$，所以$\sqrt{324}$ 等於 $\sqrt{2^2 \times 3^4}$ 再整理一下就會得到$\sqrt{(2 \times 3^2)^2}$。至於開根號裡面為什麼可以變這樣是國一的內容喔。

老師： 因此我們的邊長就會等於 $\sqrt{324} = \sqrt{(2 \times 3^2)^2} = 2 \times 3^2 = 18$ 平方公分。

單元五 根式的運算
Radical Operation

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

在這單元中我們先教學生根式的乘除，之後再教根式的加減，這樣會使學生便於判別同類根式，以利根式的化簡，進而處理根式的四則運算。

■ 詞彙 Vocabulary

單字	中文	單字	中文
radical	根式	rationalizing denominator	有理化分母
simplest radical	最簡根式	similar radicals	同類根式

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Simplify _____ to _____.

例句：**Simplify** $\sqrt{198}$ **to** its simplest radical form.

將 $\sqrt{198}$ 化為最簡根式。

❷ Use _____ to expand _____, and simplify the result.

例句：**Use** $(a+b)^2=a^2+2ab+b^2$ **to expand** $\left(3\sqrt{5}+2\sqrt{7}\right)^2$**, and simplify the result.**

利用 $(a+b)^2=a^2+2ab+b^2$ 展開 $\left(3\sqrt{5}+2\sqrt{7}\right)^2$，並化簡其結果。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

Students can not only use similar radicals to simplify radical expressions faster and more effectively, but perform basic operations with radicals.
我們利用同類根式使學生可以更快、更好的簡化式子，並做根式四則運算。

❧ 運算問題的講解 ☙

例題一

說明：為了使學生更加了解同類根式，並簡化式子。
Students can know more about similar radicals and simplify radical expressions.

（英文）Simplify $3\sqrt{24}+\sqrt{96}+\sqrt{45}-\sqrt{125}$.
（中文）計算 $3\sqrt{24}+\sqrt{96}+\sqrt{45}-\sqrt{125}$，並將其結果簡化。

Teacher: For this type of question, always remember to simplify the given expression to its simplest radical form first and then combine similar radicals. So, our first step is to build factor trees for each radical.

Teacher: After we are done, we can come to $3\sqrt{2^3\times 3}+\sqrt{2^5\times 3}+\sqrt{3^2\times 5}-\sqrt{5^3}$.
Now, we can simplify that to its simplest radical form, and that give us $6\sqrt{6}+4\sqrt{6}+3\sqrt{5}-5\sqrt{5}$.

Teacher: Finally, we can rewrite it by combining like terms. Please note that only similar radicals can be combined. Who knows the answer?

Student: $10\sqrt{6}-2\sqrt{5}$.

Teacher: That is correct! The answer to this question is $10\sqrt{6}-2\sqrt{5}$.

老師：我們以後看到這種題目都要記得，要先把題目給的數字都化簡成最簡根式之後再來做運算。所以第一步我們先來把每個數字質因數分解。

老師：分解完後，我們會得到 $3\sqrt{2^3\times 3}+\sqrt{2^5\times 3}+\sqrt{3^2\times 5}-\sqrt{5^3}$。之後再把他們化成最簡根式，便會得到 $6\sqrt{6}+4\sqrt{6}+3\sqrt{5}-5\sqrt{5}$。

老師：然後我們就可以做最後的整理了！注意要是同類項才可以合併喔。
那麼有誰要來告訴我們答案嗎？

學生： $10\sqrt{6}-2\sqrt{5}$ 。

老師： 答對了。這題最後的答案便是 $10\sqrt{6}-2\sqrt{5}$。

例題二

說明：利用乘法公式將帶有根式的分數有理化。

Using the multiplication rule to rationalize a binomial denominator.

（英文）Simplify $\frac{\sqrt{6}}{\sqrt{2}-1}$ (Using the multiplication rule)

（中文）簡化 $\frac{\sqrt{6}}{\sqrt{2}-1}$。（請利用乘法公式）

Teacher: Generally, we can't simplify a radical expression without the multiplication rule. Now for this question, our first step is to multiply both the numerator and denominator by $\sqrt{2}+1$. We can get the difference of two squares in the denominator like this expression $\frac{\sqrt{6}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$.

Teacher: The next step is to expand the whole fraction. What will the denominator become? We learned this skill in the last chapter.

Student: 1.

Teacher: Exactly. We expand the denominator and can have the square root of 2 squared minus 1 squared. So, it's 1.

Teacher: And the numerator is going to become $\sqrt{6}\times\sqrt{2}+\sqrt{6}\times 1$, or $\sqrt{12}+\sqrt{6}$. We can simplify this further and come to $2\sqrt{3}+\sqrt{6}$.

Teacher: Because the denominator is 1, the answer to this question is $2\sqrt{3}+\sqrt{6}$.

老師： 我們一般在做根式的有理化都會利用到乘法公式。所以我們這題的第一步便是先上下同乘以 $\sqrt{2}+1$，使分母變成平方差的形式，所以我們寫出來的樣子便是 $\frac{\sqrt{6}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$。

老師： 我們第二步要做的就是將這個式子展開。那麼分母展開後的結果會是什麼？這是上一單元才剛教過喔。

學生： 1。

老師： 沒錯，分母展開後就會變成根號 2 的平方減掉 1 的平方也就是 1。

老師： 接著我們再把分子展開，展開後我們會得到$\sqrt{6}\times\sqrt{2}+\sqrt{6}\times 1$，也就是$\sqrt{12}+\sqrt{6}$。最後再化簡一下就可以得到$2\sqrt{3}+\sqrt{6}$。

老師： 因為分母為1，所以這題的答案便是$2\sqrt{3}+\sqrt{6}$。

單元六 畢氏定理
Pythagorean Theorem

國立彰化師範大學數學系 蔡庭宜、李珮慈

■ 前言 Introduction

這單元先利用<<周髀算經>>內畢氏定理證明使學生認識何謂畢氏定理。之後再利用畢氏定理及七年級所學過的 $\overline{AB} = |a - b|$ 推導出在笛卡爾坐標系平面上的兩點距離的公式。

■ 詞彙 Vocabulary

單字	中文	單字	中文
hypotenuse	斜邊	Pythagorean theorem	畢達哥拉斯定理
leg	股邊	distance formula	距離公式

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ We know _____ as follows, find _____.

例句：**We know** the lengths of the two legs of each of the right triangles **as follows, find** the length of the hypotenuse.

已知下列各直角三角形的兩股長，求斜邊的長。

❷ **In the figure, _____ are right-angled, _____.**

例句：**In the figure,** the slide and the stairs **are right-angled,** and the three sides of the right triangle are 5, 13, $\sqrt{194}$. Find the height h of the slide.

如圖，滑道和樓梯成直角，直角三角形的三邊長分別為 5, 13, $\sqrt{194}$，求滑梯的高 h。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

Students can apply the Pythagorean theorem to do the calculation.

我們利用畢氏定理來幫助計算。

❧ 運算問題的講解 ☙

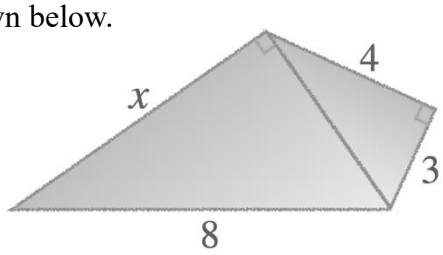
例題一

說明：為了使學生熟悉如何用畢氏定理求邊長。

Students can know how to figure out a side length using the Pythagorean theorem.

（英文）Find the value of x in the triangle shown below.

（中文）求下圖中 x 的值。



Teacher: This question asks for the value of x. Generally, when we need to find a side length of a right triangle, the Pythagorean theorem is our go-to formula. But, do you notice that this given triangle seems to miss one side length?

Student: Yes.

Teacher: So, the first thing we need to do is to find the length of the hypotenuse to the right triangle with the leg measurements of 3 and 4. Do you still remember what the Pythagorean theorem tells us?

Student: The length of a leg squared plus the length of the other leg squared is equal to the length of the hypotenuse squared.

Teacher: Exactly! First, let's call this unknown hypotenuse as y, and the Pythagorean theorem tells us that $3^2 + 4^2 = y^2$.

We expand this product and can come to $9 + 16 = y^2$. $9 + 16 = 25$, so $y^2 = 25$. What is the value of y?

Student: 5.

Teacher: Great job! Now we know the other side length is 5. We can apply the Pythagorean theorem to solve for x, so all we do now is to simplify this expression

$5^2 + x^2 = 8^2$. That is equal to $x^2 = 64 - 25 = 39$. Do you know the answer?

Student: $\sqrt{39}$.

Teacher: Excellent! Since the length must be positive, the answer is just $\sqrt{39}$.

老師： 這一題是要我們求出 x 的值。大多數我們在求直角三角形邊長的值時，會選擇用畢氏定理去做計算。可是我們看到上面這張圖，我們可以看到我們要求的三角形邊長是不是還少了一個。

學生： 是。

老師： 所以我們需要先把兩股長為 3 跟 4 的三角形的斜邊長算出來。那還記得畢氏定理的公式是什麼呢？

學生： 兩股平方相加會等於斜邊的平方。

老師： 沒錯。所以首先我們假設斜邊長為 y，之後在根據畢氏定理寫出 $3^2 + 4^2 = y^2$，把他乘開來後我們就可以得到 $9 + 16 = y^2$ 而 $9 + 16 = 25$，所以 $y^2 = 25$，那麼 y 會是多少？

學生： 5。

老師： 非常好！現在我們已經知道另一個邊的長為 5 了，那接下來我們一樣用畢氏定理就可以求出解了。把他寫成式子就是 $5^2 + x^2 = 8^2$，再整理一下就會得到 $x^2 = 64 - 25 = 39$，那同學們答案是多少呢？

學生： $\sqrt{39}$。

老師： 很好！因為長度一定大於 0，所以答案就只有$\sqrt{39}$。

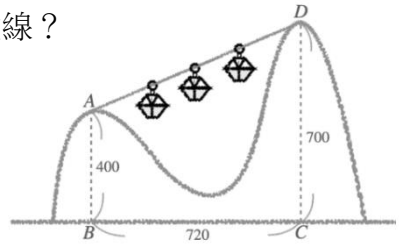
例題二

說明：本題利用簡單的應用題使學生更加明白畢氏定理的使用。

Students can have a better understanding of how to apply the Pythagorean theorem by solving an easy word problem.

（英文）Today Xiao Hua plans to build a gondola lift model. He sketches out his idea on the computer (as illustrated) for the necessary material needed. If $\overline{AB} = 400$ mm (line AB is four hundred millimeters), $\overline{DC} = 700$ mm, and $\overline{BC} = 720$ mm, how about the cable $\overline{AD} = ?$ How many *centimeters* of the cable should Xiao Hua prepare for his model?

（中文）今天簫譁打算做一個空中纜車模型，於是他先用電腦繪製一張平面圖(如下圖)思考要準備多少材料，若 $\overline{AB} = 400$ 毫米、$\overline{DC} = 700$ 毫米、$\overline{BC} = 720$ 毫米而纜線 $\overline{AD} = ?$，請問簫譁需要準備多少公分的纜線？



Teacher: Let's look at the drawing. What shape do you think it is?

Student: A trapezoid.

Teacher: That's right, so we need to draw a line that crosses the trapezoid from A and touches the side $\overline{DC}$ marked as P. This line is parallel to $\overline{BC}$. Also, you will notice that line breaks the trapezoid into a rectangle and a triangle. Is $\overline{AP}$ perpendicular to $\overline{DC}$?

Student: Yes.

Teacher: Excellent! Since $\overline{AP}$ is perpendicular to $\overline{DC}$, $\triangle APD$ is a right triangle. What strategy can we apply to solve this problem?

Student: The Pythagorean theorem.

Teacher: Yes, the Pythagorean theorem is a correct answer. Because $ABCP$ is a rectangle, that tell us that $\overline{AP}$ is equal to $\overline{BC}$. The length of $\overline{BC}$ is 720. Who knows what the length of $\overline{PD}$ is?

Student: 300.

Teacher: Why?

Student: Because $700 - 400 = 300$.

Teacher: Exactly! Just like what we did previously, because $ABCP$ is a rectangle, the length

of $\overline{PC}$ will be equal to 400. $\overline{PD}$ is equal to $700-400$, which is 300.

Teacher: So, we can say $300^2+720^2=\overline{PD}^2$. This is just the Pythagorean theorem. Now, we just need to simplify this expression. What is it then?

Student: 780.

Teacher: Yes, but 780 is not our answer, because the unit the question asks for is cm. We are dealing with mm now, so we need to convert it into *cm*. Can anyone tell me 780 mm is equal to what cm?

Student: 78.

Teacher: Excellent, so the correct answer to this question is 78 cm.

老師： 我們先來看這一題的圖，你們覺得他是個什麼形狀。

學生： 梯形。

老師： 對，所以我們需要在 A 點的地方作一條通過它且 $\overline{BC}$ 平行的線，把這個梯形分成一個長方形和一個三角形，之後我們在與 $\overline{DC}$ 相交的那個點上記作P點。那 $\overline{AP}$ 跟 $\overline{DC}$ 這兩條線有沒有相互垂直？

學生： 有。

老師： 很好！既然它們有垂直，那麼 $\triangle APD$ 就是一個直角三角形。所以我們可以用什麼方法算呢？

學生： 畢氏定理。

老師： 對，就是畢氏定理。因為 $ABCP$ 是一個長方形，所以我們可以知道 $\overline{AP}$ 會等於 $\overline{BC}$，因此 $\overline{BC}$ 的長度等於 720，那誰知道 $\overline{PD}$ 的長為多少？

學生： 300。

老師： 為什麼？

學生： 因為 $700-400=300$。

老師： 沒錯！跟剛剛一樣。因為 $ABCP$ 是一個長方形，所以 $\overline{PC}$ 的長度也會等於 400。因此 $\overline{PD}$ 會等於$700-400$，也就是 300。

老師： 所以我們把它列式，就會得到 $300^2+720^2=\overline{PD}^2$。我們只要在把 $\overline{PD}$ 算出來就好了。那算出來是多少？

學生： 780。

老師： 對，算出來會是 780，但這不是答案喔。這題要的單位公分，但我們算出來的單位是毫米，所以我們要把他做單位換算。780 毫米等於幾公分？

學生： 78。

老師： 對，所以這題正確的答案是 78 公分。

單元七 提公因式與乘法公式作因式分解
Grouping by Common Factors and Factorization by Multiplication Formula

國立彰化師範大學數學系　蔡庭宜、李珮慈

■ 前言 Introduction

這一節先由七年級所學過的「因數、倍數」來引入「因式」的概念，再利用多項式除法是否整除，來判別一個多項式是否為另一多項式的因式。在這個章節我們總共會介紹兩種因式分解的方法，第一種，是利用分配律來導出「提公因式」；第二種，是我們在第一章學的乘法公式，但在利用乘法公式進行因式分解時，一定要先檢查多項式是否為乘法公式的形式。

■ 詞彙 Vocabulary

單字	中文	單字	中文
factor	因式	Factorization	因式分解
multiple	倍式	Grouping by common factor	提公因式法

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Factorize _____.

例句：**Factorize** the following equations.

因式分解下列各式。

❷ **_____is the common factor of _____ and_____.**

例句：If $x-4$ **is the common factor of** $5x^2-21x+a$ **and** x^2+bx-4, find $a-b$.
如果 $x-4$ 是 $5x^2-21x+a$ 和 x^2+bx-4 的公因式，求 $a-b$。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

Students will become familiar with the concept of root and get prepared for the next chapter by doing the following questions.
我們利用題目來熟悉根號的概念以為下章節做鋪墊。

❧ 運算問題的講解 ☙

例題一

說明：利用直式多項式除法加強學生對因式的概念。
strengthen the concept of factor using polynomial long division

（英文）Are $(3x-5)$ and $(3x+1)$ both factors of $(3x^2-2x-5)$?
（中文）$3x-5$ 與 $3x+1$ 都是 $3x^2-2x-5$ 的因式嗎？

Teacher: Before we start, let's review "factor" first. Factor is used to identify if a polynomial can be divided by another. If yes, this polynomial is then called the factor of another one.

Teacher: Now read the question: Determine whether $3x-5$ and $3x+1$ are factors of the polynomial $3x^2-2x-5$ or not. This is quite simple. All we need is polynomial division to know if they can divide $3x^2-2x-5$. And then we will find the answer. Now do the problem on your own first, please.

Teacher: Have you finished?

Student: Yes.

Teacher: Can $3x^2-2x-5$ be divisible by $3x-5$?

Student: Yes, it can.

Teacher: Correct! $3x^2-2x-5$ is divisible by $3x-5$, so $3x-5$ is a factor of $3x^2-2x-5$. In addition, the quotient of the formula $3x^2-2x-5$ divided by

$3x-5$ is $x+1, x+1$ is also a factor of $3x^2-2x-5$!

Teacher: How about $3x+1$? Can it divide $3x^2-2x-5$?

Student: No, it can't.

Teacher: Great, since $3x+1$ can't divide $3x^2-2x-5$, it is not a factor of the polynomial $3x^2-2x-5$.

老師：在做這題以前我們要先來複習什麼是因式，其實就是判斷一個多項式是否可以整除另一個多項式，如果能整除我們則稱這個多項式是另一個多項式的因式。

老師：現在我們再看回題目，題目叫我們判斷 $3x-5$ 與 $3x+1$ 是否為 $3x^2-2x-5$ 的因式。那其實很簡單，我們只需用第一章學過的多項式除法來看看他們是否可以整除 $3x^2-2x-5$ 便知道這題的答案了。現在先請同學算算看。

老師：各位同學算好了嗎？

學生：算好了。

老師：那算出來 $3x-5$ 可以整除 $3x^2-2x-5$ 嗎？

學生：可以。

老師：沒錯！因為 $3x-5$ 可以整除 $3x^2-2x-5$，所以 $3x-5$ 是 $3x^2-2x-5$ 的因式。順帶一提，因為 $3x^2-2x-5$ 除以 $3x-5$ 的商是 $x+1$，所以 $x+1$ 也是 $3x^2-2x-5$ 的因式喔！

老師：那換 $3x+1$ 可以整除 $3x^2-2x-5$ 嗎？

學生：不行。

老師：很好，因為 $3x+1$ 不可以整除 $3x^2-2x-5$ ，所以 $3x+1$ 不是 $3x^2-2x-5$ 的因式喔。

例題二

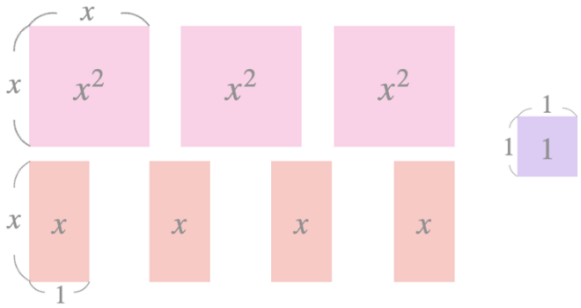
說明：利用簡單的圖形組合使學生實際操作因式分解。

let students practice factorization with simple figures

（英文）The following figures are given: 3 big squares with an area of x^2, 4 rectangles with an area of x, and a small square with an area of 1. If a student can combine these 8 figures as a big rectangle which has a width $x + 1$ long, then how long will the length be?

（中文）今天老師給學生們 3 個面積為 x^2 的大正方形、4 個面積為 x 的長方形、1 個面積為 1 的小正方形，如下圖，希望在不重疊的情況下，學生可以把這 8 個拼成一個寬為 $x + 1$ 的大長方形。

請問大長方形的長為何？



Teacher: Let's read the question first: 3 big squares with an area of x^2, 4 rectangles with an area of x, and a small square with an area of 1 are given for a big rectangle which has a width $x + 1$ long, and the length is asked for. Since we already know the width, we can know the length by factorizing the total area.

Teacher: So, first of all, we are going to find out the total area of the big rectangle. We can get the answer by adding up the areas of all these figures because each figure will be used. Then, how much is the total area?

Student: $3x^2 + 4x + 1$.

Teacher: Great, it is exactly $3x^2 + 4x + 1$. Then we factorize $3x^2 + 4x + 1$ to find another factor besides $x + 1$. What should we do?

Student: Divide the polynomial using division.

Teacher: That's right. Get the quotient by calculating the polynomial division. What is the quotient of $3x^2 + 4x + 1$ divided by $x + 1$?

Student: It is $3x + 1$.

Teacher: Good for you! So $3x^2 + 4x + 1$ equals $(3x + 1)(x + 1)$. The answer for this question is $3x + 1$ then.

老師： 我們先來看題目，題目說現在有 3 個面積為 x^2 的大正方形、4 個面積為 x 的長方形、1 個面積為 1 的小正方形要拼出寬為 $x+1$ 的大長方形，問我們這個長方形的長為多少。既然題目給了我們寬，那我們只要把面積算出來再做因式分解就可以知道長為多少了。

老師： 所以我們現在先把圖形總面積算出來，因為他每塊都有用到，所以我們只要把每塊的面積相加起來便是答案，那麼面積是多少？

學生： $3x^2+4x+1$。

老師： 很好，就是 $3x^2+4x+1$。之後我們再將 $3x^2+4x+1$ 因式分解，找出除了$x+1$以外的另一個因式，那我們該怎麼做？

學生： 用除法把他們相除。

老師： 沒錯，就是用多項式的除法把商算出來。那算出來 $3x^2+4x+1$ 除以 $x+1$ 的商是多少呢？

學生： $3x+1$。

老師： 好棒！所以 $3x^2+4x+1$ 會等於 $(3x+1)(x+1)$。因此這題的答案便是 $3x+1$。

單元八 利用十字交乘法因式分解
Factorization by Cross-method

國立彰化師範大學數學系　蔡庭宜、李珮慈

■ 前言 Introduction

十字交乘法是分解二次多項式最常用的方法，也是解二元一次方程式的方法之一。本節由觀察$(x+p)(x+q)$的展開式，及一次項、常數項與 p、q 的關係，進而發展出十字交乘法的記法。

■ 詞彙 Vocabulary

單字	中文	單字	中文
cross method	十字交乘法		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ _____ using the cross method.

例句：Factorize $-6x^2-11x+10$ **using the cross method.**

利用十字交乘法，因式分解 $-6x^2-11x+10$。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

The questions will be utilized to get familiar with the concept of root and get prepared for the next chapter.
我們利用題目來熟悉根號的概念以為下章節做鋪墊。

❧ 運算問題的講解 ☙

例題一

說明：利用計算題使學生練習十字交乘。

Practice cross method through calculation questions.

（英文）Factorize $-4x^2 - 38x - 48$ using the cross method.
（中文）利用十字交乘法，因式分解 $-4x^2 - 38x - 48$。

Teacher:	Let's read the question. We are asked for doing this using cross method. But we can simplify the question. Find out the common factor, and then use the cross method. Let's start from the common factor. Do you notice that each coefficient is a multiple of 2?
Student:	Yes.
Teacher:	It would be easier to do if the first term is positive, so I factor out -2. What would the polynomial look like after factoring out the negative?
Student:	$-2(2x^2 + 19x + 24)$.
Teacher:	Correct! Next, we just need to factorize $2x^2 + 19x + 24$ using the cross method. Let's start from the first term, $2x^2$. It is easier because it can only be $2x \times x$. Next, let's move to the constant term, 24. It can be factorized into many sets. What sets can they be?
Student:	(1, 24), (2, 12), (3, 8), and (4, 6).
Teacher:	That's right, they are the very four sets of the numbers,(1, 24), (2, 12), (3, 8), and (4, 6). We can now try these numbers to multiple $2x$ and x, to see if the result is added to $19x$. In fact, we don't have to try them all. Take (1, 24), 24 is over 19 so it must be more than 19 after adding up.
Teacher:	Let's try (4, 6). $2x$ times 6 plus x times 4 is equal to $2x$, not $19x$.

Then we switch. $2x$ times 4 plus x times 6 is equal to $14x$, not $19x$, either.

So, neither $(2x + 4)(x + 6)$ nor $(2x + 6)(x + 4)$ is the answer.

Teacher: Now, please try (2, 12) and (3, 8) using the same method to see which set of number is the answer.

Teacher: What is your answer?

Student: $(2x + 3)(x + 8)$.

Teacher: Let's see if it is the answer. $2x$ times 8 plus x times 3 is equal to $19x$. But don't forget the -2 we factored out earlier.

The correct answer is $-2\ (2x + 3)(x + 8)$.

老師： 我們來看這題，題目指定要我們用十字交乘法來做，但其實我們可以先把題目簡化一下，先把公因數提出來，再來用十字交乘法。那我們先來找它的公因數，大家有沒有發現它的每一個係數都是 2 的倍數？

學生： 有。

老師： 因為我想要首項是正的，這樣會比較好算，所以我先把-2 提出來。提出來後的式子會長什麼樣子呢？

學生： $-2(2x^2 + 19x + 24)$。

老師： 沒錯！那接下來我們只要用十字交乘法來因式分解 $2x^2 + 19x + 24$ 就行了。

我們先來分解首項 $2x^2$。他比較簡單，因為他只可以分解 $2x \times x$。

接著我們來拆常數項 24，他跟首項不一樣他可以被拆成很多組，那有哪些組呢？

學生： (1, 24)、(2, 12)、(3, 8)和(4, 6)。

老師： 對，就是(1, 24)、(2, 12)、(3, 8)和(4, 6)這四組。那麼我們現在可以來試試看，看你一組和 $2x$、x 相乘再相加後的和等於 $19x$。其實我們也不用每組都試，像 (1, 24) 這一組 24 本身就超過 19 所以相加後一定也會大於 19。

老師： 那我們先來試試 (4, 6)。將 $2x$ 乘上 6 再加上 x 乘上 4 會等於$16x$ 不等於 $19x$，之後我們再交換一下，換成 $2x$ 乘上 4 再加上 x 乘上 6 會等於 $14x$也一樣不等於 $19x$，所以 $(2x + 4)(x + 6)$ 和 $(2x + 6)(x + 4)$ 都不是答案。

老師： 那請同學用相同的方法再試試看 (2, 12)、(3, 8) 這兩組哪個是答案吧。

老師： 你們算出來答案是多少呢？

學生： $(2x + 3)(x + 8)$。

老師： 答案是不是那個呢？我們現在來看看。將 $2x$ 乘上 8 再加上 x 乘上 3 會等於$19x$。

但別忘了還有我們一開始提出來的 -2，所以正確的答案是$-2(2x+3)(x+8)$。

例題二

說明：利用已知條件回推出解答，使學生活用第三章所學習的內容。

Students will be able to find an answer with given clues and practically use what they learn in chapter 3

（英文）If $77x^2+2x-15$ can be factorized as $(ax-3)(bx+c)$, and we know that a, b, and c are integers, what will $a+b\times c$ be?

（中文）若 $77x^2+2x-15$ 可因式分解成 $(ax-3)(bx+c)$，其中 a、b、c 均為整數，求 $a+b\times c$ 。

Teacher: We already know that $77x^2+2x-15$ can be factorized as $(ax-3)(bx+c)$, so if we think backwards to expand $(ax-3)(bx+c)$, we will get the result $abx^2+acx-3bx-3c$.

Teacher: In the end, we will find that $77x^2+2x-15$ equals $abx^2+(ac-3b)x-3c$ which means $ab=77$, $ac-3b=2$, and $3c=15$. What answer will we know first from these three formulas?

Student: $c=5$.

Teacher: Correct, it is $c=5$. In other words, $a\times b=77$ and $5a-3b=2$. All we need to do next is to solve the simultaneous equations. However, we are studying the cross method, so we are going to do it using the cross method.

Let's start from 77. 77 can be factorized as 7×11 and 1×77. But 1×77 is obviously wrong, so we just need to find out a and b, which is 11 and which is 7.

Teacher: The difference between 11×5 and 7×3 is way more than we expected, how about the difference between 5×7 minus 11×3?

Student: The difference is 2.

Teacher: That's right. So we learn that $a=7$, $b=11$, and $c=5$. What is the result if we substitute $7+11\times 5$ for $a+b\times$ c?

Student: It is 62.

Teacher: Exactly, $7+11\times 5$ equals 62. This is the final answer.

老師： 題目是說 $77x^2+2x-15$ 可以分解成 $(ax-3)(bx+c)$，那我們可以反過來看，將 $(ax-3)(bx+c)$ 展開就會是 $abx^2+acx-3bx-3c$。

老師： 之後再整理一下，就會得到 $77x^2 + 2x - 15$ 會等於 $abx^2 + (ac - 3b)x - 3c$也就是 $ab = 77$、$ac - 3b = 2$、$3c = 15$。從這三個式子裡我們最先得到的解是什麼？

學生： $c = 5$。

老師： 對，就是 c 等於 5。換句話說就是 $a \times b = 77$ 且 $5a - 3b = 2$。

接下來我們其實只解聯立方程式就行了，但是我們現在是在教十字交乘法，所以我們要用十字交乘法來做。

首先我們先來拆解 77，77 可以被拆成 7×11 和 1×77 ，但 1 跟 77 的這個組合一看就知道一定是不對的，所以我們只剩 11 跟 7 需要代入找出誰是 a，誰是 b 。

老師： 如果是 11 乘 5 而 7 乘 3 得出來的答案差距會有比我們要的還要大，所以我們將 5 乘上 7 減掉 3 乘上 11 算出來會是多少？

學生： 2。

老師： 對，所以我們得到 a 是 7、 b 是 11 然後 c 是 5。之後再把 a、b、c 代入到我們最後要算的式子 $a + b \times c$。那麼值求出來會是多少。

學生： 62。

老師： 沒錯， $7 + 11 \times 5$ 會等於 62 這便是答案。

單元九 因式分解法解一元二次方程式
Solving the Quadratic Equation by Factorization

國立彰化師範大學數學系　蔡庭宜、李珮慈

■ 前言 Introduction

本章節主要是讓學生認識一元二次方程式並利用第三章所學的因式分解來求一元二次方程式的解。

■ 詞彙 Vocabulary

單字	中文	單字	中文
quadratic equation	一元二次方程式	double root	重根
Solving the quadratic equation	解一元二次方程式	square root	平方根

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ **If _____ is a solution of _____, find_____.**

例句：**If** $x = 2$ **is a solution of** $x^2 + 3x + m = 0$, **find** the value of m.

若 $x = 2$ 為 $x^2 + 3x + m = 0$ 的一個解，求 m 的值。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

To solve quadratic equations using factorization we learned in chapter 3.
我們利用第三章所學的因式分解來求一元二次方程式的解。

❧ 運算問題的講解 ☙

例題一

說明：讓學生了解如何用因式分解求一元二次方程式的解。
Students can understand how to solve quadratic equations using factorization.

（英文）Solve $4x(3x-2)=3(3x-2)$.
（中文）解 $4x(3x-2)=3(3x-2)$？

Teacher: Let's start from moving all the terms to the left of the equal sign. By doing so, we can get $4x(3x-2)-3(3x-2)=0$. Then take a look at the equation, what do you notice?
Student: There are (3*x*-2) in both terms.
Teacher: Exactly, $(3x-2)$ in both terms. So now, if we factor out $(3x-2)$, we get $(4x-3)(3x-2)=0$. Would it possibly be 0 if we multiply two numbers which are not 0?
Student: No, it wouldn't.
Teacher: Good. So, it means that either $(4x-3)$ or $(3x-2)$ equals 0. Then we can write down the equations: $4x-3=0$ or $3x-2=0$.
Teacher: By calculation, we can get the solution that $x=\frac{3}{4}$ or $x=\frac{2}{3}$.

老師：首先，我們先將方程式都移到等號的左邊，這樣會得到 $4x(3x-2)-3(3x-2)=0$。接下來我們來觀察一下方程式，你們有發現什麼嗎？
學生：都有 $3x-2$。
老師：沒錯，就是都有 $(3x-2)$，所以我們現在把它提出來就可以得到 $(4x-3)(3x-2)=0$。那麼兩個不為 0 的數相乘出來的結果有可能會是 0 嗎？

學生： 不會。

老師： 很好，所以這就意味著 $4x-3$ 和 $3x-2$ 其中一個會等於 0。我們把它寫成式子便是 $4x-3=0$ 或者 $3x-2=0$。

老師： 計算一下，我們就可以得到 $x=\frac{3}{4}$ 或 $x=\frac{2}{3}$。

單元十 配方法與公式解
Completing the Square and Quadratic Formula

國立彰化師範大學數學系　蔡庭宜、李珮慈

■ 前言 Introduction

在本節中，學生還會學會另外兩種解一元二次方程式的方法，第一種是配方法，第二種是公式解。老師可以先舉一些無法利用因式分解來求解的一元二次方程式，來吸引學生使學生有學習新方法的動機，之後再來教學生這兩種方法。

在教配方法前，可先利用平方根的概念來求解，並讓學生觀察左邊平方式的展開與逆回，引入配方法的學習。在熟悉配方法後，便可進行公式解的學習，因國中不討論虛數，所以判別式 $b^2-4ac<0$ 時，以「負數無平方根，所以沒有解」的方式處理，因此有關公式解的題目通常會以檢查判別式入手。若學生沒有檢查判別式，而是直接代公式也沒有關係，因為根號內若出現負數，學生也能知道此方程式是沒有解的。

■ 詞彙 Vocabulary

單字	中文	單字	中文
completing the square	配方法	perfect square expression	完全平方式
quadratic formula	公式解	distinct roots	相異根
discriminant	判別式	no solution	無解

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ If _____ can be matched in _____.

例句：**If** $x^2 + mx + 1$ **can be matched in** a perfect square expression, then $m = \pm 2$.

若 $x^2 + mx + 1$ 可配成完全平方式，則 $m = \pm 2$ (plus or minus 2)。

❷ The equation of the form _____, plus _____.

例句：The equation of the form $x^2 \pm mx$, plus $\left(\frac{m}{2}\right)^2$, can be matched in a perfect square expression $\left(x \pm \frac{m}{2}\right)^2$. (Parentheses x plus or minus m over (or by) two and then square.)

形如 $x^2 \pm mx$ 的式子，加上 $\left(\frac{m}{2}\right)^2$ 後，可配成完全平方式 $\left(x \pm \frac{m}{2}\right)^2$。

■ 問題講解 Explanation of Problems

ଓ 說明 ଓ

To become familiar with finding the solution of a quadratic equation in one variable by completing the square and quadratic formula.

我們利用題目來熟悉如何用配方法及公式解求一元二次方程式的解。

ଓ 運算問題的講解 ଓ

例題一

說明：使學生學會如何用配方法求一元二次方程式的解。

Students will learn how to find the solution of a quadratic equation in one variable by completing the square.

（英文）Find the solution of $x^2 + 4x - 3 = 0$ by completing the square.

（中文）利用配方法求 $x^2 + 4x - 3 = 0$ 的解。

Teacher: Let's read the question. We are asked to find the solution of this equation by

completing the square. First of all, we move the constant term to the right of the equal sign. It is then written as $x^2 + 4x = 3$. Then look at the left. What is the coefficient of x?

Student: It is 4.

Teacher: That's right. Do you remember what the square of the sum of two numbers is?

Student: $a^2 + 2ab + b^2$.

Teacher: Correct. $(a + b)^2$ equals $a^2 + 2ab + b^2$. If we apply this formula into the question, we will learn that $a = x$ and $2ab = 4x$.

Therefore, we know that b equals 2.

Teacher: However, if we want to make the equation in the form of $(x + 2)^2$, we still need 4 for the constant term. As a result, we need to plus 2^2 on both sides of the equation, which will be written as $x^2 + 4x + 2^2 = 3 + 2^2$. Then simplify the equation, we will get $(x + 2)^2 = 7$. Finally, we are going to use what we learned in chapter 3 to find the square roots on both sides at the same time. What is the result?

Student: $x + 2 = \pm\sqrt{7}$.

Teacher: That's right. The solution is $x = \pm\sqrt{7} - 2$.

老師：我們先來看一下題目，這題是要求我們利用配方法來求出這個方程式的解。首先，我們把常數放到等式的右邊變成 $x^2 + 4x = 3$。然後我們再來看等式的左邊，x 的係數是多少？

學生：4。

老師：對，那和的完全平方公式是什麼？

學生：$a^2 + 2ab + b^2$。

老師：沒錯，$(a + b)^2$ 會等於 $a^2 + 2ab + b^2$。所以我們代進去題目裡看可以得到 $a = x$ 而 $2ab = 4x$，也就是說 b 會等於 2。

老師：但如果我們想要寫成 $(x + 2)^2$，那常數還差了一個 4。因此我們要將等式的左右兩邊加上 2^2，也就是這樣：$x^2 + 4x + 2^2 = 3 + 2^2$。

再整理一下會得到 $(x + 2)^2 = 7$。之後我們利用第三章學的將兩邊同時開根號便是答案，那開完根號會是什麼？

學生：$x + 2 = \pm\sqrt{7}$。

老師：對，因此答案就是 $x = \pm\sqrt{7} - 2$。

例題二

說明：利用公式解來求一元二次方程式的解。

Find the solution of a quadratic equation in one variable using quadratic formula.

（英文）Find the solution of $-x^2+4x+12=0$ using quadratic formula.

（中文）利用公式解求 $-x^2+4x+12+0$ 的解。

Teacher: As usual, let's read the question first. It asks us to find the solution using quadratic formula.

Teacher: In an equation of the form $ax^2+bx+c=0$, if $b^2-4ac\geq 0$, we can apply the quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Otherwise, if b^2-4ac is less than 0, there will be no solution. In this question, $a=-1$, $b=4$, and $c=12$. Is there any solution for this question or not? Why?

Student: Yes, there is a solution because $b^2-4ac>0$.

Teacher: Great. We just need to substitute them for the quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, and we will get $x=\frac{-4\pm\sqrt{4^2-4\times(-1)\times(12)}}{2\times(-1)}$. Then we organize the equation, it will become $x=\frac{-4\pm\sqrt{16+48}}{-2}=\frac{-4\pm\sqrt{64}}{-2}=\frac{-4\pm 8}{-2}$.

Teacher: Finally, we will find out the result by calculating how much $\frac{-4+8}{-2}$ and $\frac{-4-8}{-2}$ are. What is the result?

Student: $x=6$ or $x=-2$.

Teacher: That's great!

老師：我們一樣先來看一下題目，這題是要求我們利用公式解來求解。

學生：一個 $ax^2+bx+c=0$ 的方程式如果 $b^2-4ac\geq 0$，則有公式解 $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$。反之，若小於 0，則無解。

在這一題中 $a=-1$、$b=4$、$c=12$，請問這題有沒有解？為什麼？

老師：因為 $b^2-4ac>0$，所以有解。

老師：很好，所以我們只要把他代入公式解 $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ 就好了。

代進去後我們會得到 $x=\frac{-4\pm\sqrt{4^2-4\times(-1)\times(12)}}{2\times(-1)}$ 把他整理一下會得到

$x = \frac{-4\pm\sqrt{16+48}}{-2} = \frac{-4\pm\sqrt{64}}{-2} = \frac{-4\pm8}{-2}$。

老師： 最後，我們只要算出 $\frac{-4+8}{-2}$ 和 $\frac{-4-8}{-2}$ 等於多少便是答案。所以答案是少？

學生： $x = 6$ 或 $x = -2$。

老師： 太棒了！

單元十一 一元二次方程式應用問題
Application Problem

國立彰化師範大學數學系　蔡庭宜、李珮慈

■ 前言 Introduction

本節由「假設、列式、解方程式至檢查解是否符合題意」來進行應用問題的教學，一貫於七年級解應用問題的概念，惟因一元二次方程式常有兩個相異的解，所以特別強調最後的檢查。

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Find _____ according to _____.

例句：**Find** the unknown number **according to** the listed equations.
依據所列的方程式求出未知數。

■ 問題講解 Explanation of Problems

ଓ 說明 ଓ

To solve the problems in our lives using quadratic equation in one variable.
我們利用一元二次方程式來解決生活中的問題。

∞ 運算問題的講解 ∞

例題一

說明：利用一元二次方程式來處理分隊問題。

Solve a grouping problem using a quadratic equation in one variable.

（英文）School had a fair today. Students in class one of third grade were divided into groups. The number of the students in each group was 3 more than the number of the groups. Two groups were in charge of decorating and cleaning up, the rest groups were in charge of taking care of the stall. It is known that there were 36 students taking care of the stall, how many students were for decorating and cleaning up?

（中文）今天學校舉辦園遊會，三年一班的導師將學生們分成若干小組，每小組人數比小組數目多 3，其中兩小組負責場佈及最後的收拾，其餘的負責顧攤。已知顧攤的共有 36 人，求場佈及最後的收拾的人數。

Teacher: The question says that they were grouping. The number of the students in each group was 3 more than the number of the groups. Two groups were in charge of decorating and cleaning up, and the rest groups were in charge of taking care of the stall. We know that there were 36 students taking care of the stall, and we are asked for the number of the students who were in charge of decorating and cleaning up.

Teacher: The very first step of solving a word problem of quadratic equation in one variable is to make an assumption. Let's assume that the students were divided into x groups. Two groups were in charge of decorating and cleaning up, so there were $(x-2)$ groups in charge of taking care of the stall. Since the number of the students in each group was 3 more than the number of the groups, the number of the students in each group was $(x+3)$.

Teacher: The second step is to set up an equation. 36 students who took care of the stall were in $(x-2)$ groups.

So, according to the question, we can make the equation $(x-2)(x+3)=36$.

Teacher: We will get $x^2+x-6=36$ by expanding the equation. Then we organize the equation, it will become $x^2+x-42=0$.

Teacher: Next is the third step, solve for the unknown number. We have learned many ways to solve an equation before. What if I want to factorize it, what would the equation be?

Student: $(x+7)(x-6)=0$.

Teacher: Exactly, $(x+7)(x-6)=0$. So x is equal to 6 or -7.

Teacher: And we are moving to the fourth step, see if the result conforms to the question and answer. The number of the students cannot be negative, so x is 6. Then we substitute 6 for the equation, we will get that the number of the students who were in charge of decorating and cleaning up is $2(x+3)=2\times(6+3)=18$

老師： 現在題目說學生要分組，每小組人數比小組數多 3，兩小組負責場佈及最後的收拾，其餘的負責顧攤。已知顧攤的共有 36 人，問場佈及最後的收拾的人數。

老師： 我們在做一元二次方程式應用題的第一步便是假設。所以我們先假設一共分成 x 小組，其中有兩組去負責場佈及最後的收拾，所以負責顧攤的有$(x-2)$ 組。而因為每小組人數會比小組數多 3，所以每小組人數為$(x+3)$ 人。

老師： 做完假設後，就是第二步——列式。已知顧攤共 36 人。而負責顧攤的有$(x-2)$ 組，依題意我們可列出方程式 $(x-2)(x+3)=36$。

老師： 把方程式展開便會得 $x^2+x-6=36$，再整理一下就變成 $x^2+x-42=0$。

老師： 再來就是第三步——求未知數。我們在前面有學過許多解方程式的方法，那麼如果我想用因式分解來做，則式子可被分解成什麼樣子？

學生： $(x+7)(x-6)=0$。

老師： 沒錯， $(x+7)(x-6)=0$。所以 x 便會等於 6 或 -7。

老師： 接著我們來到第四步——是否符合題意並寫答。因為人數不可以是負數，所以 x 等於 6。再把 6 代回去便會得到負責場佈及最後的收拾的人數有 $2(x+3)=2\times(6+3)=18$ (人)。

例題二

說明：利用一元二次方程式來處理經費問題。

Use a quadratic equation in one variable to solve a budget problem.

（英文）A class leader asked a bookseller how much an original textbook for their course is today. The bookseller said that they must order at least 32 books for a shipment, otherwise they would have to purchase the books on their own. It was 4,000 dollars per book, but if they order over 32 books, they will get all books 100 dollars off for an extra book. It is known that the class leader collected 129,600 dollars, how many books they ordered in total?

（中文）今天班代詢問書商某科上課所需的原文書需要多少錢。書商告知訂購數目要達 32 本才會出貨不然要自行去買，而一本 4,000 元，但訂購數目若超過 32 本，每增加 1 本，則每本可折扣 100 元，已知班代共代收到 129,600 元，則總共訂購了幾本？

Teacher: It says that they have order at least 32 books. Each book is 4,000 dollars. If they order over 32 books, they will get all books 100 dollars off for an extra book.

Teacher: Let's assume that there were x extra books, so they ordered $(32+x)$ books. As a result, the price for each book would become $(4000-100x)$ dollars. Also, we know that the class leader collected 129,600 dollars in total. Therefore, we can set up an equation according to the question: $(32+x)(4000-100x)=129600$.

Teacher: Let's simplify the equation first. We factor out the common factor of 4000 and $100x$ and get that $100(32+x)(40-x)=129600$. Then both sides of the equation divided by 100, the equation will become $(32+x)(40-x)=1296$. After that, we expand it and move the terms to the left of the equal sign. What will the equation be after being organized?

Student: $x^2-8x+16=0$

Teacher: Yes, it is $x^2-8x+16=0$. Don't you think it looks familiar? Has anyone noticed what the formula is used?

Student: Square of the difference of two numbers.

Teacher: That's right! It is the square of the difference of two numbers. The equation can be shortened as $(x-4)^2=0$. The solution will be $x=4$. Please be aware that the solution is repeated root!

Teacher: Finally, we take the solution of x back for the amount of the order.
$32+x=32+4=36$(本). They ordered 36 books in total.

老師：題目說訂購數目要達 32 本，每本收費 4,000 元，如果超過 32 本，則每增加 1 本，每本可折扣 100 元。

老師：我們先假設增加 x 本，則總共訂購 $(32+x)$ 本，因此每本的費用會變成 $(4000-100x)$ 元。又因為班代共代收到 129,600 元，所以我們根據題意可列出方程式 $(32+x)(4000-100x)=129600$。

老師：我們先來整理一下，首先我們先把 4000 和 $100x$ 的公因數提出來，變成 $100(32+x)(40-x)=129600$，再把等式的兩邊同除 100 就會得到 $(32+x)(40-x)=1296$。之後我們再展開整理到等號的左邊，請問整理完後的式子會是什麼？

學生：$x^2-8x+16=0$。

老師：對，$x^2-8x+16=0$。那麼同學有沒有覺得很眼熟？有誰發現這是什麼公式了嗎？

學生：差的平方公式。

老師：沒錯，這是差的平方公式！所以我們直接寫成 $(x-4)^2=0$，因此答案是 $x=4$，注意是重根喔。

老師：最後我們再利用求出來的 x 去算題目要求的訂購數，則共訂購了 $32+x=32+4=36$ (本)。

例題三

說明：利用一元二次方程式及計算機來認識黃金比例。

Use a quadratic equation in one variable and a calculator to know golden ratio.

（英文）The so-called golden ration body shape means the ratio of one's height over his lower body is equal to the ratio of his lower body over his upper body. A belly button divides a body into upper and lower parts. If a 180-cm-tall model's body meets the standard of golden ratio, how long is his lower body? (A calculator is available, round it to the nearest tenth.)

（中文）一般所謂的黃金比例身材，是指全身身長與下半身長的比等於下半身長與上半身長的比，我們是以肚臍的位置來作為上下半身的分界。今天由一位身高為 180 公分的模特兒，擁有黃金比例的身材，則他的下半身長為多少公分？（可用計算機計算，四捨五入到小數點後第一位）

Teacher: Golden ration body shape means the ratio of one's height over his lower body is equal to the ratio of his lower body over his upper body. Assume that his lower body is x cm, and his upper body is $180 - x$ cm.

Teacher: Next, set up an equation according to the question: $180 : x = x : (180 - x)$. We have learned that inner terms multiplying equals outside terms multiplying. For this question, it would be $x \times x = 180 \times (180 - x)$. Then we expand the equation and move all the terms to the left to the equal sign, it will become $x^2 + 180x - 32400 = 0$.

Teacher: 32400 is a large number, so we are going to use the quadratic formula. We can get $x = \frac{-180 \pm \sqrt{180^2 - 4 \times 1 \times (-32400)}}{2} = \frac{-180 \pm \sqrt{16200}}{2}$ using the quadratic formula. Is $\sqrt{16200}$ an integer?

Student: No, it is not.

Teacher: We can just use a calculator, or, of course, you can use digit-by-digit calculation. You can check the book if you are not sure about how to operate a calculator. What is the number shown on you calculator?

Student: 111.246118 and -291.246118

Teacher: Great! We are asked to round the number to the nearest tenth. Since a height cannot be negative, his lower body is about 111.2-cm long.

老師： 黃金比例身材就是指全身身長與下半身長的比等於下半身長與上半身長的比。因此我們假設下半身長為 x 公分，則上半身長為 $180 - x$ 公分。

老師： 之後再依照題意列式，$180 : x = x : (180 - x)$。

我們在一下比例那章學過，內項乘以內項會等於外項乘以外項，依這題來說就是 $x \times x = 180 \times (180 - x)$。

再把他展開整理到等號的左邊，$x^2 + 180x - 32400 = 0$。

老師： 因為 32400 很大，所以我們選擇直接用公式解來做。

我們利用公式解可以得到 $x = \frac{-180 \pm \sqrt{180^2 - 4 \times 1 \times (-32400)}}{2} = \frac{-180 \pm \sqrt{16200}}{2}$。

$\sqrt{16200}$ 算出來會是整數嗎？

學生： 不會。

老師： 所以我們直接利用計算機來做計算，當然你們若是想要用十分逼近法來做計算也是可以的。若是不知道要怎麼按可以看課本，課本有寫。你們用計算機算出來是多少？

學生： 111.246118 和 −291.246118。

老師： 很好，題目叫我們去到小數點後第一位再加上身高不會是負數，所以下半身長約為 111.2 公分。

單元十二 統計資料處理
Statistical Data Processing

國立彰化師範大學數學系　蔡庭宜、李珮慈

■ 前言 Introduction

本節在一開始先複習七下第五章學過的統計圖表，接著再教學生相對次數及累積次數，並將其製成相對次數分配表和累積次數分配表，之後再利用累積次數分配表繪製累積次數分配折線圖，最後再介紹累積相對次數分配表，並將其繪製成累積相對次數分配折線圖。在折線圖的部分，我們配合資訊融入可利用電腦軟體繪製。整節中，從相對次數分配、累積次數到累積相對次數都是用相同的例題與數字，在教學上可以減少計算。

■ 詞彙 Vocabulary

單字	中文	單字	中文
relative frequency	相對次數	Relative frequency distribution	相對次數分配
cumulative frequency	累積次數	cumulative frequency distribution	累積次數分配
line graph	折線圖	cumulative relative frequency	累積相對次數

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Using _____, complete _____ and draw _____.

例句：**Using** a calculator, **complete** a table of relative frequency distribution of students' weight in Pei Ru's class **and draw** a line chart of relative frequency distribution.

利用計算機，完成珮茹班上同學體重相對次數分配表，並繪製相對次數分配折線圖。

❷ The table on the ____ is _____, and produces _____.

例句：**The table on the** right **is** a statistical table of the school cooperative's operating income for the first half of the year, **and produces** a table of the cumulative frequency distribution of income.

右表是學校合作社上半年營業收入統計表，製作收入的累積次數分配表。

❸ What is the percentage of _____.

例句：**What is the percentage of** students who are 160~170 centimeters in height in the class?

身高 160~170 公分的人數占全班的百分比是多少？

■ 問題講解 Explanation of Problems

ଓ 說明 ୬

Students can know more about understand the cumulative number distribution table on the statistical chart.

我們利用題目是學生更加的了解統計圖表上累積次數分配表。

∞ 運算問題的講解 ∞

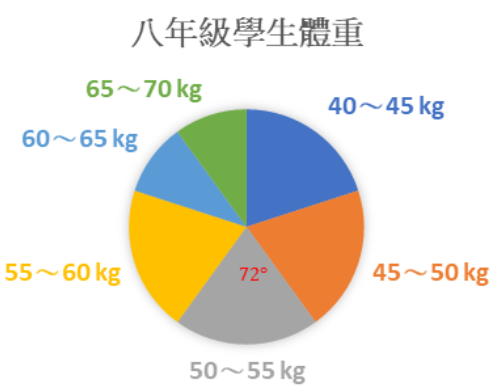
例題一

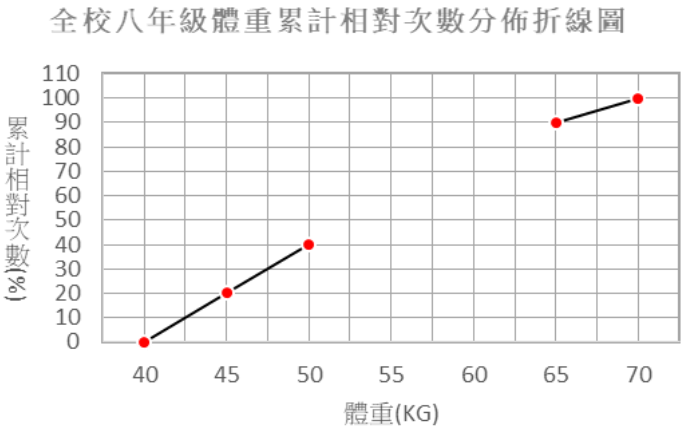
說明：利用實際範例使學生理解並應用累積次數分配折線圖。

A real-life example can help students to understand a cumulative frequency distribution graph and know how to use it.

（英文）The health service center of the school looked into a pie graph and a cumulative frequency distribution graph for all the eighth-grades, but some parts of the graphs went missing. We know that the student weighed 55 ~ 60 kgs is 10% more than ones in the category of 60 ~ 65 kgs in the school. What percentage is the student weighed 55 to 60 kgs comprised of the total eighth-grade population? If there are 120 students in this category, how many eighth-grade students in total does the school has?

（中文）健康中心調查全校八年級同學體重的圓形圖與累積相對次數分配折線圖，不小心漏掉部分圖形，已知 55 ~ 60 公斤比 60 ~ 65 公斤多 10% ，請問 55 ~ 60 公斤的人占全校的百分比是多少？若 55 ~ 60 公斤的同學有 120 人，那麼全校總共有多少人？





（翰林二上課本第 208 頁，109 年出版）

Teacher: Let's read the first question. It asks what percentage the student weighed 55 to 60 kgs is comprised of the total eighth-grade population. We knew this category has 10% more students than the other group weighed 60 ~ 65 kgs. Let's just say the percentage for the heavier weight of 60 ~ 65 kgs is $(x - 10)\%$ and for the group of 55 ~ 60 kgs is x %.

Teacher: The pie graph also tells us that the angle for 50 ~ 55 kgs is measured 72°, so we could say the percentage for students in this range is equal to 72° divided by 360 and times 100%. It is 20%.
Who knows where we can find the percentage of 40 ~50 kgs?

Student: From the cumulative relative distribution graph.

Teacher: Excellent! Now you know where to find the answer. What percentage does this category account for?

Student: 40%.

Teacher: That is right. From the line graph, we can see that 50 makes up 40% of the total population, and that is the percentage for the student weighed between 40 and 50kgs. We can even say that the percentage of 40 ~ 55 kgs is $40 + 20 = 60(\%)$. Now, what percentage does the 40 ~ 65 kgs account for?

Student: 90%.

Teacher: Very nice! Now we know the percentage for the students weighed 40 to 55 kgs and 40 to 65 kgs are 60% and 90% respectively. Can anyone tell me how to solve for the percentage of 55 ~ 65 kgs?

Student: The percentage of 40 ~ 65 kgs minus that of 40 ~ 55 kgs.

Teacher: Exactly! That is 90% of 40 ~ 65 kgs minus 60% of 40 ~ 55 kgs and now we can have the percentage of 55 ~ 65kgs. Who knows the answer?

Student: 30%.

Teacher: That is correct. Now, we know that the percentage of 55 ~ 65 kgs is 30%. In other words, x% of 55 ~ 65 kgs plus $x - 10\%$ is equal to 30% of 55 ~ 65kgs. That gives us $x + x - 10 = 30$ and $x = 20$. So, the student who weighed 55 to 65kgs makes up 20% of the total school population combined.

Teacher: Let's look at the second question. It asks "if there are 120 students in the category of 55 ~ 60 kgs, how many eighth-grade students in total the school has." We can just call the entire eighth grade as x and we already knew the student weighed 55 to 60 kgs makes up 20% of the entire eighth-grade population. That is equivalent to say that x times 20% of the student weighed 55 to 60 kgs is equal to 120.
We can write it down like this expression, $x \times 20\% = 120$, and that gives us

$x = \frac{120}{0.2} = 600$. So, the school has 600 eighth-grade students in total.

老師： 我們先來看第一個問號，題目現在問 55 ~ 60 公斤的人占全校的多少百分比，而我們已知 55 ~ 60 公斤比 60 ~ 65 公斤多 10% ，因此我們先假設 60 ~ 65 公斤

的百分比為 $(x-10)\%$，所以 55～60 公斤的百分比為 $x\%$ 。

老師：我們又可以從圓餅圖得知 50～55 公斤所占的角度是 72°，因此 50～55 公斤的人所占的百分比會等於所占的角度 72 除上 360 再乘上 100% 也就是 20%。那麼有人知道 40 ~50 公斤的百分比要從哪裡得知嗎？

學生：累積相對次數分配折線圖。

老師：很好，現在你們知道要從哪裡看了。那麼 40 ~50 公斤的百分比是多少呢？

學生：40%。

老師：沒錯，在折線圖上我們可以看到 50 對應到的的值 40%，那麼 40 ~50 公斤的百分比就是 40%。所以 40～55 公斤的百分比為 $40+20=60(\%)$。那麼 40～65 公斤占多少百分比呢？

學生：90%。

老師：不錯，現在我們知道 40 ~ 55 公斤的百分比是 60% 和 40 ~ 65 公斤的百分比是 90%。請問同學 55～65 公斤的百分比怎麼求？

學生：40～65 公斤的百分比減掉 40～55 公斤的百分比。

老師：沒錯，就是用 40 ~ 65 公斤的百分比 90% 減掉 40 ~ 55 公斤的百分比 60% 這樣就算出 55～65 公斤的百分比了。那算出來 55～65 公斤的百分比是多少？

學生：30%。

老師：答對了，現在我們知道了 55～65 公斤的百分比是 30%，換句話說就是 55～60 公斤的百分比 x 加上 60～65 公斤的百分比 $x-10$ 會等於 55～65 公斤的百分比 30%，我們整理出來就是 $x+x-10=30$，在計算一下便能得到 $x=20$。

故 55～60公斤的人占全校的百分比為 20% 。

老師：我們接著看第二個問號，問「如果 55 ~ 60 公斤的同學有 120 人，那全校總共有多少人」。我們先全校八年級有 x 人，而從第一個問題可以得出 55～60 公斤的同學 20% ，也就是說全校八年級的人數 x 乘上 55 ~ 60 公斤的同學所占的百分比 20% 會等於 55 ~ 60 公斤的同學人數 120。把他寫成式子就是 $x\times 20\%=120$，整理一下 $x=\frac{120}{0.2}=600$。

故全校八年級共有 600 人。

例題二

說明：利用基測題目使學生熟悉相對次數分配表之使用方法。

Students can get familiar with how to use a relative frequency distribution table by solving a word math problem drawn from the Basic Competence Test (BCT).

（英文）The following is a frequency distribution table. The frequency values with the age groups of 36 ~ 42 and 50 ~ 56 appear illegible because of smears or overuse. If the relative frequency of staff aged 36 ~ 42 and 50 ~ 56 are $a\%$ and $b\%$ respectively. Find the value of $a + b$. (the twentieth question from 2012 BCT math exam)

Age (years)	22 ~ 28	29 ~ 35	36 ~ 42	43 ~ 49	50 ~ 56	57 ~ 63
Frequency (persons)	6	40	[illegible]	42	[illegible]	2

（中文）下表為某公司 200 名職員年齡的次數分配表，其中 36 ~ 42 歲及 50 ~ 56 歲的次數因汙損而無法看出。若 36 ~ 42 歲及 50 ~ 56 歲職員人數的相對次數分別為 $a\%$、$b\%$，則 $a + b$ 之值為何？

年齡（歲）	22 ~ 28	29 ~ 35	36 ~ 42	43 ~ 49	50 ~ 56	57 ~ 63
次數（人）	6	40	[illegible]	42	[illegible]	2

（101 年基測第 20 題）

Teacher: We know relative frequency is equal to class frequency divided by the total frequency combined. The question tells us that the relative frequency of the employees aged 36 ~ 42 and 50 ~ 56 are $a\%$ and $b\%$ respectively. Now, we need to find the value of $a + b$.

Teacher: The total population of the company is 200 people. We also know the total number of each age group except the ones of 36 ~ 42 and 50 ~ 56. How many people aged 36 ~ 42 and 50 ~ 56 has the company employed?

Student: 110 people.

Teacher: Very nice! 110 is the sum of people aged 36 ~ 42 and 50 ~ 56. In other words, the frequency of these two age groups is $a + b = 110$. The total frequency combined is 200. So, what we need to do now is simply to divide 110 by 200 and then we can solve for $a + b$. That is $110 \div 200 = \frac{110}{200} = \frac{55}{100} = 55\%$.

The answer is $a + b = 55$.

老師：我們知道相對次數會等於組次數除上總次數。現在題目告訴我們 36 ~ 42 歲及 50 ~ 56 歲職員人數的相對次數分別為 a%、b%，問「$a + b$ 的值是多少？」

老師：現在公司的總人數是 200 人，而除了 36 ~ 42 歲及 50 ~ 56 歲其他的人數我們都知道了，那麼 36 ~ 42 歲和 50 ~ 56 歲總共有多少人？

學生：110 人。

老師：很好！36 ~ 42 歲加上 50 ~ 56 歲的人數為 110 人。換句話說 $a + b$ 的組次數就是 110，而總次數是 200。因此我們只要用 36 ~ 42 歲加上 50 ~ 56 歲的人數 110 人除以公司總人數 200 就知道 $a + b$ 的值了。

把他寫成式子：$110 \div 200 = \frac{110}{200} = \frac{55}{100} = 55\%$，故答案就是 $a + b = 55$。

國內外參考資源 More to Explore

國家教育研究院樂詞網 查詢學科詞彙 https://terms.naer.edu.tw/search/	
教育雲：教育媒體影音 為教育部委辦計畫雙語教學影片 https://video.cloud.edu.tw/video/co_search.php?s=%E9%9B%99%E8%AA%9E	
Oak Teacher Hub 國外教學及影音資源，除了數學領域還有其他科目 https://teachers.thenational.academy/	
CK-12 國外教學及影音資源，除了數學領域還有自然領域 https://www.ck12.org/student/	
Twinkl 國外教學及影音資源，除了數學領域還有其他科目，多為小學及學齡前內容 https://www.twinkl.com.tw/	

Khan Academy 可汗學院，有分年級數學教學影片及問題的討論。 https://www.khanacademy.org/	
Open Textbook (Math) 國外數學開放式教學資源 http://content.nroc.org/DevelopmentalMath.HTML5/Common/toc/toc_en.html	
MATH is FUN 國外教學資源，還有數學相關的小遊戲 https://www.mathsisfun.com/index.htm	
PhET: Interactive Simulations 國外教學資源，互動式電腦模擬。除了數學領域，還有自然科。 https://phet.colorado.edu/	
Eddie Woo YouTube Channel 國外數學教學影音 https://www.youtube.com/c/misterwootube	

國立臺灣師範大學數學系陳界山教授網站	
國高中數學雙語教學相關教材 https://math.ntnu.edu.tw/~jschen/index.php?menu=Teaching_Worksheets	
2023 年第四屆科學與科普專業英文(ESP)能力大賽	
科學專業英文相關教材，除了數學領域，還有其他領域。 https://sites.google.com/view/ntseccompetition/%E5%B0%88%E6%A5%AD%E8%8B%B1%E6%96%87%E5%AD%B8%E7%BF%92%E8%B3%87%E6%BA%90/%E7%9B%B8%E9%97%9C%E6%95%99%E6%9D%90?authuser=0	

國中數學領域雙語教學資源手冊：英語授課用語

[八年級上學期]

A Reference Handbook for Junior High School Bilingual Teachers in the Domain of Mathematics: Instructional Language in English

[8th grade 1st semester]

- 研編單位：國立彰化師範大學雙語教學研究中心
- 指導單位：教育部師資培育及藝術教育司
- 撰稿：蔡庭宜、李珮慈
- 學科諮詢：張淑珠
- 綜合規劃：曾松德
- 編輯排版：吳依靜
- 封面封底：JUPE Design

發行單位　國立彰化師範大學雙語教學研究中心
NCUE BILINGUAL EDUCATION RESEARCH CENTER

指導單位　教育部師資培育及藝術教育司
MOE DEPARTMENT OF TEACHER AND ART EDUCATION