國中數學領域

雙語教學資源手冊 英語授課用語

A Reference Handbook for Junior High School Bilingual Teachers in the Domain of Mathematics: Instructional Language in English

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單元一 整數的加法與減法
Addition and Subtraction of Integers

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■ 前言 Introduction

學生會開始計算整數的加法及減法，並利用已學會的技巧去處理生活中會面對到的問題。老師們在這一節裡會接觸到如何完整的用英文表達一個式子，主要是用加法(plus)及減法(minus)去連接數字。並請注意加法及正數的念法要區分出來，這樣學生會更容易了解您在表達的是動作（做加或減），或是在描述該數的性質為正數或負數。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
integer	整數	**positive integer**	**正整數**
addition	**加法**	**negative integer**	**負整數**
subtraction	**減法**	number line	數線
operation	運算	whole number	非負整數
positive	正	**opposite number**	**相反數**
negative	負	commutative property	交換律
origin	原點	associative property	結合律
sum	和	commutative property of addition	加法交換律
addend	加數	associative property of addition	加法結合律
sign	同號異號裡的號		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Show _____ on _____. Solve it.

例句：**Show** positive two plus positive three **on** the number line, then **solve it (the equation)**.
在數線上圖解正 2 加正 3，並求出結果。

❷ Add _____ with _____.

例句：**Add** the two numbers **with** the same sign.
同號數相加。

❸ Adding (Subtracting) _____ means _____.

例句：**Adding** a negative number **means** shifting the point to the left on the number line.
加一個負數代表用箭頭移動原來的數往左邊幾個單位。

❹ When _____, _____ is _____.

例句：**When** *a* is greater than *b*, negative *a* plus *b* **is** a negative number.
當 *a* 大於 *b* 時，負 *a* 加 *b* 是負整數。

❺ The sum of _____ is _____.

例句(1)：**The sum of** a number and its opposite number **is** zero.
一數與該數相反數的和為零。

例句(2)：**The sum of** two positive numbers **is** positive.
兩個正整數的和是正數。

❻ When _____, _____ is greater than/ equal to/ less than _____.

例句：**When** *b* is less than 0, *a* plus *b* **is less than** *a*.
如果 *b* 是負數，則 $a + b$ 小於 *a*。

❼ **If _____, then _____.**

例句：**If** a and b are integers, **then** $a + b = b + a$ (read as: a plus b equals b plus a).
如果 a 和 b 是整數，則 $a + b = b + a$。

■ 問題講解 Explanation of Problems

➿ 說明 ➿

We use the arrows on the number line to represent the addition of positive and negative numbers. We use an arrow pointing to the right to represent a positive number, and an arrow pointing to the left as a negative number.
我們用箭頭來表示正數及負數的加法，箭頭向右代表正數而箭頭向左代表負數。

We also let students know the commutative property and associative property of addition to help them simplify an expression in a faster way.
我們利用加法的交換律及結合律來幫助計算。

Subtracting a number is equivalent to adding its opposite number.
For example, $5 - 3$ equals $5 + (-3)$. (read as: 5 minus 3 equals 5 plus negative 3)
在做整數的減法時，我們會用引入減一個數等同於加上它的相反數來去做運算。

➿ 運算問題的講解 ➿

例題一

說明：介紹學習加法的交換律跟結合律有什麼好處？通常是為了方便運算，或是可以加快得到答案的速度。

（英文）Simplify the numerical expression, $1039 + (-178) + 78$.
（中文）計算 $1039 + (-178) + 78$。

Teacher: What is the advantage of knowing commutative property and associative property?
Student: I am not sure. Can it increase the speed of simplifying an equation.
Teacher: Correct. In the following question, it says: “simplify the numerical expression, $1039 + (-178) + 78$.” By using the associative property, you don’t need to follow

the usual order of operations, which is doing the calculations from left to right. You can do the addition of the last two terms first and some of you might be able to calculate the rest in your heads.

Student: Okay, I can do that. The result is negative 100. And then I can also use mental math to solve $1039 + (-100)$, which is 939.
That seems easier than if I do $1039 + (-178)$ first.

老師： 知道交換律跟結合律的好處嗎？

學生： 不確定。簡化算式可以變快嗎？

老師： 對。在下面的問題中，它說：計算 $1039 + (-178) + 78$。用結合律，就不用按照一般的左到右順序來計算。
可以先加後面兩項有些人就可能在腦中算出剩下的數字。

學生： 好，我會。答案是-100。然後我還可以使用心算來解決$1039 + (-100)$，即 939。

例題二

說明：本題是跟學生介紹去括號的規則，用來簡化運算。

（英文）Simplify $465 - (464 + 367) - (10 - 366)$ (read as: 465 minus the sum of 464 and 367, then minus the difference between 10 and 366.)
Simplify the numerical expression (Simplify the expression, or Simplify)
（中文）計算 $465 - (464 + 367) - (10 - 366)$

Teacher: In fact, we know the opposite number of $(a + b)$ is $-(a + b)$ and the sum of them is 0. How do you write that in an equation?

Student: Like this: $(a + b) - (a + b) = 0$.

Teacher: We also know that $a + b + (-a - b) = a + b - a - b = 0$. Now, compare those two equations and check the relation between $-(a + b)$ and $-a - b$.

Student: I think that they are the same.

Teacher: Correct. Furthermore, we also know that $-(a - b) = -a + b$. Now use those two rules to simplify the following question: $465 - (464 + 367) - (10 - 366)$. After deleting the parentheses, how should you rewrite it?

Student: It becomes: $465 - 464 - 367 - 10 + 366$.

Teacher: Right, so which part of the expression will you do first?

Student: I will do 465 minus 464 first. And then use commutative property to add 366.

Teacher: Good strategy. Then $366+1-367$ can be simplified to 0. Only -10 is left and -10 is the final answer.

老師：事實上，我們知道 $(a+b)$ 的相反數是$-(a+b)$，兩者的和為 0。怎麼用算式表示？

學生：像這樣：$(a+b)-(a+b)=0$。

老師：我們也知道 $a+b+(-a-b)=a+b-a-b=0$。比較一下這兩個算式，並觀察 $-(a+b)$ 和 $-a-b$ 之間的關係。

學生：我覺得它們一樣。

老師：正確。此外，我們也知道 $-(a-b)=-a+b$。現在，使用這兩個規則簡化以下問題：$465-(464+367)-(10-366)$。去括號會變成怎樣呢？

學生：變成：$465-464-367-10+366$。

老師：對，那要先處理算式的哪個部分？

學生：我會先做 $465-464$，然後使用交換律加上 366。

老師：解法不錯。然後 $366+1-367$ 可以簡化為 0。只剩下-10，答案就出來了。

∞ 應用問題 / 會考素養題 ∞

例題一

說明：這題是 105 年會考數學第二題裡的一部分。學生只要知道減一個數等於加上此數的相反數，就可以順利完成題目的<u>上半部分</u>。原題目為：$|-5-(-11)|\div\frac{3}{2}\times 4$

（英文）Simplify $-5-(-11)$ (negative 5 minus negative 11)
（中文）計算 $-5-(-11)$

Teacher: Let's look at the next question. Simplify negative 5 minus negative 11. In the first step, draw a number line.

Student: Done.

Teacher: Can you locate negative 5 on the number line?

Student: Yes, it is five units to the left of 0.

Teacher: Let's review the rule we just learned in this lesson. What should you do if you

subtract an integer?

Student: Convert subtracting an integer to adding its opposite.

Teacher: Excellent! Now, back to the question. What is the opposite of negative 11.

Student: Positive 11.

Teacher: Great! What should you do if you see “minus negative 11” in the question?

Student: Rewrite it as plus the opposite of negative 11, which is 11.

Teacher: Good. Now rewrite the original question as negative 5 plus 11. What does the result look like on the number line?

Student: There’s an arrow from negative 5 pointing to the right by 11 units. Then it stops at 6, so the answer is 6.

Teacher: Correct!

老師： 讓我們看看下一道問題。計算−5 − (−11)。先畫一條數線。

學生： 畫好了。

老師： 你數線上找得到 −5 嗎？

學生： 可以，它在 0 的左邊五個單位。

老師： 讓我們回顧一下本課中剛學的規則。如果你減去一個整數，應該怎麼做？

學生： 把減去一個整數轉換成加上它的相反數。

老師： 非常好！現在回到這個問題。−11 的相反數是什麼？

學生： 11。

老師： 很好！如果問題中出現 −(−11)，你應該怎麼做？

學生： 重寫成加上−11 的相反數，也就是 11。

老師： 很好。現在把原問題重寫成 −5 加上 11。在數線上，這個結果是什麼樣子？

學生： 有一個從 −5 指向右邊 11 單位的箭頭，然後停在 6，所以答案是 6。

老師： 正確！

例題二

說明：利用股票價值起伏，請學生判斷投資是否賺錢或賠錢。

（英文）Matt Chen bought some stock in DEF Audio company. The stock went up $2 per share in the first week, went down $4 in the second week, and went up $3 in the third week. If Matt paid $30 per share for the stock,

a. Express the value of the stock after three weeks with only addition.

b. Did he make money or lose money?

（中文）小明花了 30 元買了 DEF 這間公司的股票，這些股票在第一週漲了 2 元，在第二週跌了 4 元，而在第三週又再漲了 3 元。

a. 用加法來表示三週後的股價。

b. 請問小明是賺錢還是賠錢呢？

Teacher: After reading the word problem, you can use the following steps to solve it. Firstly, what is the question asking?

Student: The question is asking if Matt made money or lost money.

Teacher: Right. Secondly, what information is given?

Student: Matt paid $30 per share. The price went up $2, went down $4, and then went up $3.

Teacher: Okay, you have all the information you need. Now express the given information as a sum of positive and negative integers.

Student: I will write it as $30 + 2 + (-4) + 3$.
(Thirty plus 2, plus negative 4, plus positive 3)

Teacher: Simplify $30 + 2 + (-4) + 3$ by drawing arrows on the number line.

Student: Start with number 30 on the number line. Draw an arrow to two units to the right (because you see plus 2) that stops at 32. Draw an arrow to 4 units to the left from 32 which stops at 28. Draw the last arrow to 3 units to the right from 28. The arrow ends at 31. Because the new price is greater than the old price, it means that Matt made money.

老師： 讀完這題後，就能按照下列步驟解題。首先，問題在問什麼？

學生： 問題是問小明賺還是虧。

老師： 對。然後題目還給了什麼資訊？

學生： 小明每股付了 30 元。股價漲了 2 元，跌了 4 元，然後再漲了 3 元。

老師： 好的，需要的資訊都有了。現在前面的文字表達為正整數和負整數之和。

學生： 我會將它寫成30 + 2 + (−4) + 3。
老師： 在數線上畫箭頭，對30 + 2 + (−4) + 3做簡化。
學生： 從數線上的數字 30 開始。向右畫一個兩個單位的箭頭（因為看到加 2），停在 32。從 32 向左畫一個四單位的箭頭，停在 28。最後向 28 的右側畫一個三單位的箭頭。箭頭停在 31。新價格比舊價格高，所以小明有賺錢。

例題三

說明：利用生活上會遇到的問題，像是溫度的變化再搭配上零下溫度的情境，可以讓學生練習做整數的加法及減法。

（英文）Solve each problem by first expressing the given data as a sum of positive and negative numbers. Then compute the sum of the numbers and answer the questions.
The temperature in Seoul at 7:00 a.m. was −7°C. By 12:00 noon, the temperature had increased by 13°C, but it then decreased by 3°C between noon and 5:00 p.m. What was the temperature reading at 5:00 p.m.?
（中文）先將題目的數據表示為正負數相減的式子，然後再計算這些數的總和。
某日首爾上午七點的氣溫為−7°C，到中午十二點，氣溫上升 13°C；中午到下午五點的時段下降 3°C。請問下午五點的溫度是多少？

Teacher: First use "−7" (negative 7) to represent the temperature at 7:00 a.m. What symbol do you use to represent "increased by?"
Student: The addition sign "+".
Teacher: And what symbol do you use to represent "decreased by"?
Student: The subtraction sign "−".
Teacher: Now write an equation to express the changes in temperature.
Student: −7 + 13 − 3.
Teacher: Use a number line and arrows to represent the expression.
Student: Okay. The arrows pointing to the right represent addition and arrows pointing to the left represent subtraction.
Teacher: Where does the arrow end?
Student: It ends at 3. Therefore, the answer is 3°C at 5:00 p.m.

老師： 今天我們要學習用數學算式來表示溫度的變化。首先，用「−7」來表示早上

7 點的溫度。同學們知道怎樣表示「增加」嗎？

學生： 要用加號「＋」。

老師： 對，很好。那如果是「減少」呢？

學生： 要用減號「－」。

老師： 對。現在請你們寫出算式來表達溫度變化。

學生： $-7+13-3$。

老師： 很好，接下來用數線和箭頭來表示這個算式。

學生： 好的。向右的箭頭表示加法，向左的箭頭表示減法。

老師： 那麼箭頭最後停在哪裡呢？

學生： 停在 3 這個位置。所以答案就是下午 5 時的溫度是 3℃。

單元二 整數的乘法與除法
Multiplication and Division of Integers

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■ 前言 Introduction

在這一節中，國內的教科書是用生活的例子來帶入正負數的乘法規則，而部分國外的教科書則是利用乘法對加法的分配律來帶出正負得負及負負得正的結論；在最後則是希望讓學生熟練正負數的四則運算。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
multiplication	乘法	**factor**	**因數**
division	除法	calculator	計算機
product	**乘積**	commutative property of multiplication	乘法交換律
quotient	**商**	associative property of multiplication	乘法結合律
parentheses	括號()	distributive property	分配律
brackets	括號[]	multiplicand	被乘數
braces	括號{}	multiplier	乘數

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ The product of _____and _____ is a _____.

例句：**The product of** a positive number **and** a negative number **is a** negative number.
(正數)×(負數)得到負數。

❷ Multiplying _____ by _____ means the sum of _____ _____.

例句：**Multiplying** 5 **by**−3 **means the sum of** five −3. Or you add −3 five times.
5 乘−3 可視作連加−3 這個數 5 次。

❸ The product of −1 and _____ equals the opposite of _____.

例句：**The product of −1 and** 5 **equals the opposite of** 5, which is −5.
−1 乘上 5 會變成 5 的相反數，也就是−5。

❹ When _____, _____ is _____.

例句：**When** *a* and *b* are negative, the product of *a and b* **is** positive.
當 *a*、*b* 為負，兩數乘積為正數。

❺ For a product with no zero factors, if the number of negative factors is ___, the product is ____.

例句(1)：**If the number of negative factors is** 3, **the product is** negative.
3 個負數的積為負數。

例句(2)：**If the number of negative factors is** 4, **the product is** positive.
4 個負數的積為正數。

❻ The quotient of ___ and ___ (two positive or two negative number) is positive.

例句：**The quotient of** −10 **and** −2 **is positive.**
−10 除以−2 的結果為正數。

❼ The quotient of ___ (a positive number) and ___ (a negative number) is negative.

例句：**The quotient of** 15 **and** −3 **is negative.**
15 除以−3 的結果為負數。

■ 問題講解 Explanation of Problems

說明

We think of the product 3(−2) as the sum of three identical addends, −2. That is (−2)+(−2)+(−2) = −6. Thus 3(−2) = −6
3 乘以−2 可以視為−2 連加三次，結果為−6。因為 3 乘以−2 答案為−6。

What if the negative sign is in the front, such as −3 multiplied by 2. There are some different ways of explaining it. One way is by assuming the communicative property of multiplication for integers sustains and we write (−3)2=3(−2). Thus −3(2)= −6.
The other way is by using an example in daily life. For example, the water level of the reservoir decreased by 3 meters each day during the dry season. On the contrary, the water level of the reservoir increased by 3 meters each day during the monsoon season. The increase and decrease of the water level represents the positive and negative sign of the front factor (the multiplicand) respectively. The days to the future and the past compared today represent the positive and negative signs of the factor (multiplier) in the back.
在說明負數乘正數時，我們可以用生活實例如水庫水位升降對照今日的水位來示例，或者像是一些國外的教科書是直接在乘法交換律成立的情況下就得到負數乘以正數為負數的結果。

To illustrate the product of two negative integers is positive, we can use the reservoir as an example. Think of (−3)(−2) as finding water level 2 days ago during the dry season. Since the water level decreased by 3 meters each day, the water level should be (−3)(−2)=6, 6 meters

higher than today.
可再延續用水庫水位的例子來說明負數乘負數為正數的結果。

The product of 1 and any number equals that number.
1 與任何數的乘積都等於這個數本身。

The product of −1 and any number equals the opposite of that number.
−1 與任何數的乘積都等於這個數的相反數

The product of 0 and any number equals 0.
0 與任何數的乘積都是 0。

The product of even numbers of negative numbers is a positive number.
偶數個負數的積為正數。

The product of odd numbers of negative numbers is a negative number.
奇數個負數的積為負數。

For a product with many factors, if one of the number is zero, then the product is 0.
在連乘的算式中，如果有任一個數為 0，其乘積為 0。

For any two numbers, a and b, $a \times b = b \times a$ (read as a times b equals b times a)
乘法交換律 Commutative property of multiplication

For any three numbers, $(a \times b) \times c = a \times (b \times c)$ (read as The product of a and b times $c$equals a times the product of b and c.)
乘法結合律 Associative property of multiplication

The quotient of two positive or two negative numbers is positive.
任兩正數或負數的商為正數。

The quotient of a positive number and a negative number is negative.
一正數及一負數的商為負數。

If $a \neq 0$, then $0 \div a = 0$ (read as: If a is any number other than zero, then zero divides by a equals zero.)
零除以任何非零的數為 0。

For any three numbers, a, b and c, $(a + b) \times c = c \times a + c \times b$ (read as The sum of a and b times c equals c times a plus c times b.
乘法對加法的分配律成立。

運算問題的講解

例題一

說明：兩數相乘時，運用正負數的乘法規則（正正得正，正負得負，負正得正及負負得正）先判斷答案的正負號，再計算結果。

（英文）Find the product. (1) (−3)×6　(2) (−2)×(−5)×(−8)

（中文）計算下列各式的值：

(1) (−3)×6　(2) (−2)×(−5)×(−8)

Teacher: When doing multiplications of integers, it is quite similar to doing multiplication of two positive integers. We only need to do one more step which is telling what the sign is of the result.

Check this question. It says, “Find the product.”

(1) (−3) × 6

(2) (−2) × (−5) × (−8)

What is the sign of the result for question 1?

Student: It’s negative because the product of a negative number and a positive number is positive.

Teacher: Great. Then, what is the product of 3 times 6?

Student: 18. So, the result is −18.

Teacher: Excellent. Let’s check question 2. How many negative factors do you see in this question?

Student: Three. That is an odd number. Therefore, the result of the product is negative.

Teacher: Terrific. Now, find the product of 2 times 5 times 8.

Student: It is 80. The answer to this question is −80.

老師：做整數乘法時，跟一般兩個正整數的乘法非常相似，只要多一步，就能確定結果的正負號。請計算以下問題的積：

(1) (−3) × 6

(2) (−2) × (−5) × (−8)

第(1)小題的正負為何？

學生：負的，因為一個負數與一個正數相乘的結果是負數。

老師：太好了。那麼 3 乘以 6 等於多少？

學生：18。因此，結果是−18。

老師：太棒了。現在看問題(2)。你看到了幾個負數？

學生：三個。這是奇數，因此乘積為負。

老師： 很好。現在算出 2 乘以 5 乘以 8 的乘積。
學生： 80。所以答案是−80。

例題二

說明：本題是跟學生介紹利用乘法交換律或結合律可以用來加快運算速度。

（英文）Simplify $(-25) \times 13 \times (-4)$
Simplify the numerical expression (Simplify the expression, or Simplify)
（中文）計算 $(-25) \times 13 \times (-4)$

Teacher: We have used the communicative property and associative property of addition to increase the speed of calculation. Both properties also exist in the multiplication of integers. They are also served to help increase the speed of calculation. From the communicative property, we can switch 13 and −4. Now rewrite the question.
Student: So $(-25) \times 13 \times (-4) = (-25) \times (-4) \times 13$
Teacher: Right. If we don't consider the negative signs, which one seems more straightforward to calculate first, 25 times 4, or 4 times 13? The associative property of multiplication tells us that we will get the same result no matter which one we do first.
Student: I think 25 times 4 is more straightforward and I can calculate in my head. It is 100.
Teacher: We also know that the product of two negative numbers is positive. So −25 times −4 equals 100. Now, try to complete the rest of the question.
Student: 100 times 13 is 1300.
Teacher: Great. If you do the part of 4 times 13 first, you need to deal with 25 times 52 later. That does seem a little bit more complicated than our method.

老師： 我們使用加法的交換律和結合律來加快速度。這兩個特性在整數乘法中也成立，也有助於增加計算速度。根據加法交換律，我們可以對 13 和 −4 進行交換。現在重寫題目。
學生： 也就是說，$(-25) \times 13 \times (-4) = (-25) \times (-4) \times 13$。
老師： 對。如果不看負號，哪一個先算比較好，25 乘以 4 還是 4 乘以 13？乘法的結合律告訴我們，無論我們先做哪一個，都會得到相同的結果。
學生： 我認為 25 乘以 4 更好算，可以心算得出 100。
老師： 我們還知道兩個負數的乘積是正的。因此，−25 乘以−4 等於 100。現在，試著完成剩餘部分。

學生： $100 \times 13 = 1300$。

老師： 太棒了。如果先計算 4×13，後面還要處理 25×52，那可能會比我們的方法更複雜。

∞ 應用問題 / 會考素養題 ∞

例題一

（英文）Simplify $(-8)+(-2)\times(-3)$.

（中文）算式 $(-8)+(-2)\times(-3)$之值為何？

（110 年國中會考第 2 題）

Teacher: Let's see this question. Simplify negative 8 plus negative 2 times negative 3. According to the rule, we need to do the multiplication first, then do addition next. Let's find the result of negative 2 times negative 3.

Student: We know the product of two negative numbers is positive and 2 times 3 is 6. So the answer is 6, positive 6.

Teacher: Great. Then, simplify −8 plus 6. From the previous lesson, we know that the sum of two numbers with different signs equals the number with a larger absolute value minus the number with a smaller absolute value. In addition, place a negative sign if the one with a larger absolute value is negative or place a positive sign if the one with a larger absolute value is positive. Remember, when we simplify, we will not write out the positive sign. Just remind us that the answer is positive.
Start to simplify $-8+6$.

Student: $-8+6$
$= -(8-6)$
$= -2$
When simply −8 plus 6, we do 8 minus 6 which is 2. Since −8 is the one with a larger absolute value, the sum is negative. Therefore, the answer is negative 2.

Teacher: You are right.

老師： 來看這道題。化簡 $(-8)+(-2)\times(-3)$。根據運算規則，我們要先做乘法，再做加法。現在我們來算出$(-2)\times(-3)$。

學生： 我們知道兩個負數的乘積是正的，$2 \times 3 = 6$。所以答案是 6，正 6。

老師： 太棒了。然後，化簡−8 + 6。根據上一課，我們知道兩個符號不同的數字的和等於絕對值較大的數減去絕對值較小的數。此外，如果絕對值較大的數是負數，則加上負號；如果絕對值較大的數是正數，則加上正號。請記住，在簡化時，我們不會寫出正號，只會提醒我們答案是正的。

開始簡化 −8 + 6。

學生： −8+6

= −(8−6)

= −2

當化簡 −8+6 時，可以視為 8−6，得到 2。由於 −8 是絕對值較大的數，所以相加是負的。因此，答案是−2。

老師： 答對了。

例題二

說明：利用股票價值起伏，請學生計算賺錢或賠錢的金額。

（英文）Matt Chen owned 200 shares in GHI Audio Company. The stock went down $2 per share after the outbreak of COVID-19. How much money does Matt lose after the epidemic outbreaks?

（中文）小明持有 GHI 公司股票 200 股，在新冠肺炎爆發後，市值每股跌了 2 元，請問小明在疫情爆發後一共賠了多少錢？

Teacher: A lot of people lost their jobs after the outbreak of COVID-19. This virus really caused great damage to our economy. There is no surprise the stock price went down. “Go down” is key information in this question. It gives us a hint that we should use the negative sign to describe the drop of the price. Express “went down $2” in math expression.

Student: −2.

Teacher: Good. Since Matt owned 200 shares and each share went down 2 dollars. How do you write this in math expression?

Student: 200 × (−2).

Teacher: Let’s review the rule. What is the sign of the product of a positive number and a negative number?

Student: It is negative.

Teacher: Simplify the results.

Student: 200 × (−2)

= −(200×2) = −400

老師： 許多人在新冠肺炎爆發後失業，病毒重創經濟大環境。股價崩跌也不易外。「下跌」是這個問題中的關鍵，提示應該用負號來表達價格下降。
請用數學表達「下跌了 2 元」。

學生： -2。

老師： 很好。由於小明擁有 200 股，每股下跌了 2 元。請問如何用數學表達這個情況？

學生： $200 \times (-2)$。

老師： 讓我們回顧一下規則。一個正數和一個負數的乘積的符號是什麼？

學生： 是負的。

老師： 然後化簡。

學生： $200 \times (-2)$
$= -(200 \times 2)$
$= -400$

單元三 指數記法與科學記號
Exponents and Scientific Notation

國立新竹科學園區實驗高級中等學校　陳立業老師

■ 前言 Introduction

老師們在這一節會開始接觸到指數及底數的英文講法，介紹指數記法在現實生活中描述極小或極大的量有其方便性，最後讓學生了解科學記號是怎麼使用並認識一些長度單位的用法。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
exponent	**指數**	squared	2 次方
base	**底數**	cubed	3 次方
scientific notation	**科學記號**	**power**	**次方**
millimeter	毫米	nth power	*n* 次方
micrometer	微米	decimal	小數
nanometer	奈米	decimal point	小數點

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ a^b is read as a to the power of *b*.

例句：**We read 3^5 as three to the power of** five. (or three to the fifth power.)
3 的 5 次方的英文念法。

❷ Multiplying a number *a* **by itself** *n* **times can be written as a^n.**

例句：**Multiplying** 3 **by itself** 5 **times can be written as 3^5.**
$3 \times 3 \times 3 \times 3 \times 3$ 可寫成 3^5。

❸ The exponent of a^b is b and the base is *a*.

例句：**The exponent of 3^5 is** 5 **and the base is** 3**.**
3^5的指數為 5 而底數為 3。

❹ There are several ways of reading a^2. One way is to read as *a* squared, the square of *a*, or *a* to the second power.

例句：**There are several ways of reading 3^2. One way is to read as three squared, the square of 3, or 3 to the second power.**
3 的平方有以上幾種的念法。

❺ There are several ways of reading a^3. One way is to read as *a* cubed, the cube of *a*, or *a* to the third power.

例句：**There are several ways of reading 5^3. One way is to read as five cubed, the cube of 5, or 5 to the third power.**
5 的三次方有以上幾種的念法。

❻ There are several ways of reading a^n. One way is to read as a to the power of n or a to the nth power.

例句：**There are several ways of reading 5^n. One way is to read as five to the power of n or five to the nth power.**

5 的 n 次方有以上幾種的念法。

❼ Write 10^{-n} in decimal form.

例句：**Write 10^{-6} in decimal form.**

將 10^{-6} 寫成小數。

❽ Write a in scientific notation.

例句：**Write 35000 in scientific notation.**

將 35000 以科學記號表示。

■ 問題講解 Explanation of Problems

說明

為了讀寫的方便，我們可以將連續 n 個 a 相乘寫作 a^n，讀作「a 的 n 次方」

For the convenience of reading and writing, multiplying a by itself n times can be written as a^n and a^n is read as "a to the power of n."

如何將一個數以科學記號來表示，如果該數大於 1，則將小數點向左移直到小數點的左邊只剩一個非零的數，舉例來說將 3250.25 以科學記號表示時，會將小數點向左移三個位數後可寫作 3.25025×10^3，後面所乘上 10^3 中的指數 3 剛好就等於一開始小數點左移的位數。如果該數介於 0 跟 1 之間，則將小數點向右移直到小數點的左邊只剩一個非零的數，舉例來說將 0.000467 以科學記號表示時，會將小數點向右移四個位數後可寫作 4.67×10^{-4}，後面所乘上 10^{-4} 中的指數 -4 為小數點右移位數的相反數。記錄小數點移動的位數，若該數大於 1，則科學記號的指數即為該位數；若該數介於 0 跟 1，則科學記號的指數是該位數的相反數。

How do we write a number in scientific notation?

(1) If the number, x, is greater than 1, we move the decimal point to the left until there is only one nonzero digit to the left of the decimal point. We denote the value as a. Then we count how many of the places the decimal point is moved and denote it as n. We write x in scientific notation as $a \times 10^{n}$ where a is greater than or equal to 1 and less than 10. For example, if we want to write 3250.25 in scientific notation, we move the decimal point three places to the left, then 3250.25 is written as 3.25025×10^{3}. The power of 10 is 3 which is just the same number as the places we move.

(2) If the number, x, is between 0 and 1, we move the decimal point to the right until there is only one nonzero digit to the left of the decimal point. We denote the value as a. We also count how many of the places we have moved the decimal point and denote as n. We write x in scientific notation as $a \times 10^{-n}$. For example, if we want to write 0.000467 in scientific notation, we move the decimal point four places to the right, then 0.000467 is written as 4.67×10^{-4}. The power of 10 is -4 which is just the opposite number as the places we move.

If m is a positive integer, then $(0.1)^{m} = 10^{-m}$ (Read as: Zero point 1 to the power of m equals 10 to the power of negative m).
如果 m 為正整數，則 $(0.1)^{m} = 10^{-m}$

If a is a positive number greater than 1, then a^{n} increases as n increases
如果 a 是比 1 大的正數時，n 愈大，則 a^{n} 的值會愈大

If b is a positive number smaller than 1, then b^{n} decreases as n increases
如果 b 是比 1 小的正數時，n 愈大，則 b^{n} 的值會愈小

Given that n is a positive integer, if $a \times 10^{n}$ is the scientific notation of a number x, then there are $(n+1)$ digits in x. For example, if x equals 3.68×10^{5}, then there are five plus one which is 6 digits in x.
如果某數的科學記號表示法為 $a \times 10^{n}$，則該數的整數部分是 $(n+1)$ 位數。舉例來說，3.68×10^{5}此數的整數部分是 5+1 也就是 6 位數。

Given that n is a positive integer, if $a \times 10^{-n}$ is the scientific notation of a number x, then the first nonzero number appears in the nth place to the right of the decimal point of x. For example, if x equals 2.343×10^{-6}, then the first nonzero number appears in the 6^{th} place to the right of the decimal point of x.
如果某數的科學記號表示法為 $a \times 10^{-n}$，則該數從小數點後第 n 位開始出現不是 0 的數字。舉例來說，2.343×10^{-6}從小數點後第六位開始出現不是 0 的數字。

運算問題的講解

例題一

說明：讓學生練習將一數寫成指數記法。同樣地，當看到一個數寫成指數記法也能算出其值。

（英文）(1) Write 6×6×6×6×6×6×6 in exponential form.

(2) Write (−3)×(−3)×(−3)×(−3) in exponential form.

（中文）將下列各式寫成指數記法的形式：

(1) 6×6×6×6×6×6×6

(2) (−3)×(−3)×(−3)×(−3)

Teacher: When we see a number multiplied by itself many times, we will use exponential notation to express that number.
Check this question. It says, “Find the product. (1) Write 6×6×6×6×6×6×6 in exponential form.” Since 6 is multiplied by itself 7 times, we can write it as 6^7, read as six to the power of seven. What is the base and exponent in this exponential form?

Student: The base is 6, and the exponent is 7.

Teacher: Great. Then, you can do the second question now.

Student: It says, “Write (−3) × (−3) × (−3) × (−3) in exponential form.” The base is (−3), and it is multiplied by itself four times. I’d write it as $(-3)^4$.

Teacher: Excellent. If the base is negative, make sure the exponent is outside the parentheses. If you write -3^4 that will express 3 multiplied by itself 4 times with a negative sign in front of it.

Student: And the answer is negative 81, which is wrong.

Teacher: Terrific. Now, I believe you see the difference between the place of the exponent inside and outside of the parentheses.

老師： 當看到一個數字乘以自己很多次，我們會使用指數律來表達。
問題說：「求乘積。(1) 用指數形式寫出 6×6×6×6×6×6×6。」由於 6 乘以自己 7 次，我們可以寫成 6^7，讀作六的七次方。在這個指數形式中，底數和指數分別是什麼？

學生： 底數是 6，指數是 7。

老師： 太好了。那你現在可以做第(2)小題了。

學生： 它說：「用指數形式寫出 $(-3)\times(-3)\times(-3)\times(-3)$。」底數是$(-3)$，它被自己乘了四次。我會寫成 $(-3)^4$。

老師： 很好。如果底數是負數，請讓指數保持在括號外。如果你寫成 -3^4，那麼這表示 3 乘以自己 4 次再帶負號。

學生： 那答案變成負 81，這就錯了。

老師： 很棒。現在，我相信你看到了指數在括號內外的差異。

例題二

說明：本題是跟學生介紹如何利用科學記號表示一個數。

（英文）Write the number in scientific notation.

（中文）以科學記號表示法記錄下列各數。

(1) 13000　(2) 370　(3) 0.0000038

Teacher: Before we solve the question, let's review what a scientific notation for a number is. To write a number in scientific notation is to express it as $a \times 10^n$ if the number is greater than 1, or to express it as $a \times 10^{-n}$ if the number is between 0 and 1. Notice that a is greater or equal to 1 and less than 10. (or a is a number between 1 and 10) Now, check the first question. Write 13000 in scientific notation. To find the number n, we will record how many places we have moved the decimal point to the left until there is only one nonzero digitto the left of the decimal point. You can move the decimal point on your worksheet now.

Student: I will move the decimal point four places to the left, ending between one and three.

Teacher: Right. We can write out an equation to record the process as follows,

$13000 = 1.3\times10000$

Thirteen thousand equals one point three times ten thousand (There are four zeros after 1, which is the same as the number of places the decimal point is moved.)

$= 1.3 \times 10^4$

By the way, how many digits are there in 13000?

Student: There are five digits in 13000.

Teacher: You will see there is a relationship between the digits of a number and the value of n in its scientific notation. Now, write 370 in scientific notation.

Student: It is not difficult. We only need to move the decimal point two places to the left and

write it as $370 = 3.7 \times 10^2$

Teacher: Great. You might see there is a pattern now. In the scientific notation of 13000, the number of n is 4 and the digit of the number is 5. While in the scientific notation of 370, the number of n is 2 and 370 is a three-digit number.

Student: Aha, I see the pattern. The digit of the number is the number of *n* in its scientific notation plus one.

Teacher: Fantastic. We will do the last question. Write 0.0000038 in scientific notation. To find the number n, we will record how many places we have moved the decimal point to the right until there is only one nonzero digit. to the left of the decimal point. You can move the decimal point on your worksheet now.

Student: I will move the decimal point six places to the right, ending between three and eight.

Teacher: Right. We can write out an equation to record the process as follows,

$0.0000038 = 3.8 \times 0.000001$ (Read as: Three point eight times zero point zero zero zero zero zero 1)

$= 3.8 \times (0.1)^6$

$= 3.8 \times 10^{-6}$

By the way, which is the first nonzero number to the right of the decimal point in 0.0000038?

Student: 3 is the first nonzero number and it is the 6th place to the right of the decimal point. 6 is the opposite number of the exponent in scientific notation of the number.

Teacher: Excellent. You did find the pattern between the place of the first nonzero positive number less than one and the number of *n* in its scientific notation.

老師： 在解答問題之前，讓我們回顧一下科學記號的定義。若要以科學記號表示一個數字，如果某數大於 1 可以表示為 $a \times 10^n$，如果某數介於 0 與 1 之間，則可以將它表示為 $a \times 10^{-n}$。注意，a 大於等於 1 且小於 10（或者 a 是介於 1 和 10 之間的數字）。

現在，看第(1)小題，將 13000 表示為科學記號。為了找到 n，我們會記錄我們將小數點向左移幾位，直到小數點左邊只有一個非零數字。你現在可以在練習題本上移動小數點。

學生： 我會將小數點向左移動四個位數，停在 1 和 3 之間。

老師： 正確。我們可以撰寫一個方程式來記錄這個過程，如下所示：

13000=1.3×10000（1 後面有四個零，這與小數點移動的位數相同。）

$= 1.3 \times 10^4$

順便問一下，13000 有幾位數字？

學生： 13000 有五位數字。

老師： 你會發現數字的位數和其科學記號中 n 的值之間有一個關係。現在，把 370 表示為科學記號。

學生： 這並不困難。我們只需要將小數點向左移兩位，並將其寫成 $370 = 3.7 \times 10^2$。

老師： 太好了。你可能已經看到了一個模式。在 13000 的科學記號中，n 的值是 4，數字的位數是 5。而在 370 的科學記號中，n 的值是 2，370 是一個三位數。

學生： 啊哈，我看到規律了。數字的位數是其科學記號中 n 的值加 1

老師： 太棒了，我們來做最後一題。把 0.0000038 用科學記號寫出來。

為了找到指數 n，我們要把小數點往右移動幾位，直到小數點左邊只剩下一個非零數字為止。可以在你的練習題本上移動小數點。

學生： 我會把小數點往右移動六位，落在 3 和 8 之間。

老師： 對了。我們可以寫出一個方程式來記錄這個過程，如下所示，

$0.0000038 = 3.8 \times 0.000001$

$= 3.8 \times (0.1)^6$

$= 3.8 \times 10^{-6}$

順帶一提，在 0.0000038 中，小數點右邊第一個非零數字是多少？

學生： 3 是第一個非零數字，它是小數點右邊的第 6 位。6 是該數的科學記號指數的相反數。

老師： 太棒了，你發現了小於 1 的第一個非零正數的位數，和科學記號中的指數 n 的規律。

∞ 應用問題 / 會考素養題 ∞

例題一

（英文）How many times larger is 5^6 than 5^3?

（中文）5^6 是 5^3的多少倍？　（110 年國中會考第 5 題）

Teacher: This is a question from Comprehensive Assessment Program for Junior High School Students in 2021. It says, “How many times larger is 5^6 than 5^3.

This question tests whether you know the notation of a number in exponential notation… Please tell me what “ 5^6 ”and “ 5^3 ” means.

Student: 5^6 means 5 is multiplied by itself 6 times and 5^3 means 5 is multiplied by itself 3

times

Teacher: Great. We will rewrite the question as follows,

$$5^6 \div 5^3 = \frac{5^6}{5^3} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5}$$

(Pause for a while.)

How many "5s" can you cross out from the numerator and denominator?

Student: I can cross out three 5s. After we cross out three 5s, the answer is 125.

Teacher: You are right.

老師：這是 110 年國中會考題，它說：「5^6 是 5^3的多少倍？」這是在考你是否知道怎麼將一個數用指數符號表示。

請告訴我「5^6」和「5^3」分別代表什麼意思？

學生：5^6 表示 5 自己乘了 6 次，5^3表示 5 自己乘了 3 次。

老師：很好。我們將問題改寫為：

$$5^6 \div 5^3 = \frac{5^6}{5^3} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5}$$

（暫停一下。）

你可以從分子和分母中劃掉多少個「5」？

學生：我可以劃掉三個 5。當我們刪掉三個 5 後，答案是 125。

老師：答對了。

例題二

（英文）This size of severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) particles differ. The research shows that the diameter of the SARS-CoV-2 particles ranges from 50 nm and 140 nm. Given that a dust mite is 200 μm in diameter, if we take an 80 nm SARS-CoV-2 particle, how many times larger does a dust mite make than the SARS-CoV-2 particle?

（中文）新冠肺炎病毒的直徑介於 50 奈米至 140 奈米之間，那麼一個直徑為 200 微米的塵蟎是相對於一個直徑 80 奈米的新冠肺炎的病毒的幾倍？

Teacher: After the outbreaks of epidemics caused by Covid19-virus, our life was seriously impacted. In order to fight against the virus, one of the things we need to know is the size of the virus.

Student: Why does the size of the virus matter?

Teacher: Good question. The virus is airborne, and one way to prevent it is by wearing a mask. So we need to know whether the mask we wear is able to keep away the virus from getting inside our body. According to the research, the diameter of the SARS-CoV-2 particles range from 50 nm and 140 nm. What is 1 nm?

Student: 1 nm is 10^{-9} meter.

Teacher: Consider a mite with 200 μm in diameter, what is 1 μm?

Student: 1 μm is 10^{-6} meter.

Teacher: Let's compare their size. How much larger is a mite than a virus?

$$\frac{200\ \mu m}{80\ nm}$$
$$= \frac{200 \times 10^{-6}}{80 \times 10^{-9}}$$
$$= 2.5 \times 10^{3}$$
$$= 2500$$

Student: Wow, a mite is 2500 times larger than a virus!

老師： 在新冠肺炎病毒爆發之後，生活受到了嚴重影響。為了對抗病毒，需要知道病毒的大小。

學生： 病毒的大小為什麼重要？

老師： 這是個好問題。病毒用空氣傳播，戴口罩是預防病毒傳播的方法。因此，需要知道我們所戴的口罩能否夠防止病毒進入身體。根據研究，SARS-CoV-2 病毒的直徑範圍在 50 到 140 奈米之間。1 奈米是多少？

學生： 1 奈米等於 10^{-9}公尺。

老師： 考慮一個直徑為 200 微米的塵蟎，1 微米是多少？

學生： 1 微米等於10^{-6} 公尺。

老師： 比較它們的大小。塵蟎比病毒大幾倍？

$$\frac{200\ \mu m}{80\ nm}$$
$$= \frac{200 \times 10^{-6}}{80 \times 10^{-9}}$$
$$= 2.5 \times 10^{3}$$
$$= 2500$$

學生： 哇，塵蟎比病毒大 2500 倍！

單元四 質因數分解
Prime Factorization

國立新竹科學園區實驗高級中等學校　吳珮蓁老師

■ 前言 Introduction

學生學習質數、因數、質因數與質因數分解，並利用已學會的技巧去處理生活中會面對到的問題。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
divisible	被整除	factor	因數
multiple	倍數	composite number	合數
prime number	質數	**prime factorization**	**質因數分解**
prime factor	**質因數**	factor tree	樹狀圖
standard factorization	標準分解式	ladder diagram	階梯圖
short division	短除法	in ascending order	昇冪排列

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Tell whether A is divisible by B.

例句：**Tell whether** 62 **is divisible by** 3.
判斷 62 是否能被 3 整除。

❷ A, B are the factors of C.

例句：6 is divisible by 2 and 3. So 2 and 3 **are the factors of** 6.
6 可以被 2 和 3 整除，所以 2 和 3 是 6 的因數。

❸ Tell whether A is prime or composite.

例句：**Tell whether** 32 **is prime or composite.**
判斷 32 是質數還是合數。

❹ List all of the factors of A.

例句：**List all of the factors of** 24.
列出 24 所有的因數。

❺ Write the prime factorization of A.

例句：**Write the prime factorization of** 82.
求 82 的標準分解式。

❻ Write A as a product of two numbers.

例句：**Write** 36 **as a product of two numbers** in all possible ways.
將 36 寫成兩個數字的乘積，請列出所有可能答案。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

Prime number and composite number

Now, we are going to learn the prime number and composite number.

Let me take an example: 42 divided by 6 equals 7. We can also say that 42 is the product of 6 and 7, written as $42 = 6 \times 7$. “42” is the product, and “6” and “7” are both factors. The whole numbers that are multiplied to find the products are factors.

Besides 6 and 7, 42 can be written as a product of 2 and 21, 3 and 14, 1 and 42.

$42 = 6 \times 7 = 2 \times 21 = 3 \times 14 = 1 \times 42$

If we list all possible ways, we can find out all of the factors of 42. Therefore, the factors of 42 are 1, 2, 3, 6, 7, 14, 21, and 42.

A composite number is a whole number greater than 1 that has positive factors other than 1 and itself. For example, 42 has positive factors other than 1 and itself. And 42 is a composite number.

However, a prime number is a whole number greater than 1 whose positive factors are only 1 and itself. For example, 13 is the product of 1 and 13. There’s no way to write 13 as a product of other numbers. Therefore, the factors of 13 are 1 and itself. 13 is a prime number.

我們用例子講解質數與合數的定義。

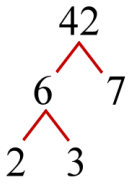
Prime factor and Prime factorization

Let me take 42 as an example again. We have listed the factors of 42, and they are 1, 2, 3, 6, 7, 14, 21, and 42. Among these factors, 2, 3 and 7 are prime numbers. They are **prime factors**.

The **prime factorization** of a number is the number written as the product of its prime factors. In elementary school, we have learned to use a factor tree and short division to factor a whole number. I will use 42 as an example.

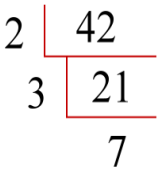
[Factor tree] We begin with any two factors of 42. Let’s start with 6 and 7. Then we keep finding the factors of each branch until we end at a prime number. For 6, we can have 2 and 3. Right now, we have 2, 3, and 7 at the end of each branch. They are all prime numbers, so the factor tree is complete.

The prime factorization of 42 is $2 \times 3 \times 7$.

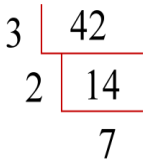


[Short division] (it is called Ladder diagram in some textbooks)

We choose a prime factor of 42 to begin, and let's start with 2. 42 divided by 2 equals 21, which is the quotient. Keep dividing the quotient by prime factors until the remaining is a prime number. 7 is a prime number, so that we can stop here. We can write the prime factorization of 42 as $2\times3\times7$.



We can start with different prime factors, and let's try 3 this time.



7 is a prime number, so we stop here. The prime factorization of 42 is $3\times2\times7$. Note that the prime factors may be written in a different order, but they are the same factors. We usually write the prime factors in ascending order, from the least to the greatest. This is the standard form of factorization (standard factorization).

When you finish the short division, the shape looks like a ladder. It is also named as "ladder diagram."

我們介紹質因數，並利用樹狀圖與短除法講解質因數分解，並寫成標準分解式。

∞ 運算問題的講解 ∞

例題一

說明：利用定義判別質數與合數。

（英文）Tell whether each number is prime or composite.

(1) 45　　(2) 17

（中文）判斷 45 和 17 是質數還是合數。

Teacher: Can you write 45 as a product of two numbers?

Student A: 5 and 9.

Student B: 3 and 15.

Student C: 1 and 45.

Teacher: You are all correct. We can write $45 = 1 \times 45 = 3 \times 15 = 5 \times 9$. All the positive whole numbers listed above are the factors of 45. In addition to 1 and 45. There are other positive factors. Hence, 45 is a composite number.

Can you write 17 as a product of two numbers?

Student A: 1 and 17.

Student B: 17 and 1.

Teacher: We can write 17=1×17, or 17=17×1. These two expressions are the same because of the commutative law (交換律). 1 and 17 are the only two factors of 17, so 17 is a prime number.

老師：你能用兩個數字的積來表示 45 嗎？

學生 1：5 和 9。

學生 2：3 和 15。

學生 3：1 和 45。

老師：你們都回答對了。我們可以寫成 $45 = 1 \times 45 = 3 \times 15 = 5 \times 9$。所有列出的正整數都是 45 的因數。除了 1 和 45 之外，還有其他正因數。因此，45 是一個合數。

你能用兩個數字的積來表示 17 嗎？

學生 1：1 和 17。

學生 2：17 和 1。

老師：我們可以寫成 $17 = 1 \times 17$，或 $17 = 17 \times 1$。由於交換律，這兩個算式相同。1 和 17 是 17 的唯一兩個因數，所以 17 是一個質數。

例題二

說明：本題練習質因數分解，並寫成標準分解式。

（英文）Write the prime factorization of 200.

（中文）求 200 的標準分解式。

Teacher: You can find the prime factorization of 200 with either method, factor tree or short division. Here I would like to demonstrate the short division again. First, choose a prime factor to start.

Student: 2.

Teacher: Alright. 200 divided by 2 is 100. Write the quotient, 100, below 200. Then keep dividing 100 by a prime factor.

Student: 5.

Teacher: Ok, 100 divided by 5 is 20. Write 20 below 100, and then keep dividing by a prime

factor.

Student: 2.

Teacher: Ok, everyone, please complete the process. (After 2 minutes) Would any of you like to show your **short division** on the board?

Student: Me.

Teacher: Yes, please.

Student:

$$\begin{array}{r|l} 2 & 200 \\ 5 & 100 \\ 2 & 20 \\ 5 & 10 \\ & 2 \end{array}$$

Teacher: Good job. According to the **short division**, we can write the prime factorization of 200 as $2 \times 5 \times 2 \times 5 \times 2$. When factors are repeated several times, we can use exponents to write prime factorization. An exponent shows how many times the base number is used as a factor. So, the prime factorization of 200 can be expressed as $2^3 \times 5^2$, which is also the standard form.

Student: Is there any other way? Do I have to start with 2 all the time?

Teacher: You don't have to start with 2 all the time. Let's try another way. Please find any two numbers whose product is 200.

Student: 4 and 50.

Teacher: We can write 200 as the product of two numbers, as $200 = 4 \times 50$.

We write 4 as the product of two numbers, and write 50 as the product of two numbers, as $200 = 2 \times 2 \times 5 \times 10$.

Keep doing this, write 10 as the product of two numbers, as

$200 = 2 \times 2 \times 5 \times 2 \times 5$.

Finally, the prime factorization of 200 can still be written as $2^3 \times 5^2$.

老師： 你可以用樹狀分解或短除法來找出 200 的質因數分解。這裡我想再次演示短除法的方法。首先，選擇一個質數因數開始。

學生： 2。

老師： 好的。200 除以 2 為 100。將商 100 寫在 200 下方。然後繼續用質因數去除 100。

學生： 5。

老師： 好的，100 除以 5 為 20。將商 20 寫在 100 下方，然後繼續用質因數去除。

學生： 2。

老師：好的，請大家一起完成。（2 分鐘後）有人想在黑板上寫出短除法嗎？

學生：我。

老師：好的，請上台。

學生：

$$\begin{array}{r|l} 2 & 200 \\ \hline 5 & 100 \\ \hline 2 & 20 \\ \hline 5 & 10 \\ \hline & 2 \end{array}$$

老師：很好！根據短除法，我們可以將 200 的質因數分解式寫為 $2 \times 5 \times 2 \times 5 \times 2$。當因數重複多次時，我們可以使用指數來表示質因數分解式。指數表示底數作為因數出現的次數。因此，200 的質因數分解式可以表示為 $2^3 \times 5^2$，這也是標準式。

學生：還有其他方法嗎？我每次都要從 2 開始嗎？

老師：不必每次都從 2 開始。讓我們試試其他方法。請找出任意兩個乘積為 200 的數字。

學生：4 和 50。

老師：我們可以將 200 寫成兩個數字的乘積，如 $200 = 4 \times 50$。將 4 寫成兩個數字的乘積，將 50 寫成兩個數字的乘積，如$200 = 2 \times 2 \times 5 \times 10$。繼續這樣做，將 10 寫成兩個數字的乘積，如$200 = 2 \times 2 \times 5 \times 2 \times 5$。最後，200 的質因數分解式一樣會寫作 $2^3 \times 5^2$。

∞ 應用問題 / 會考素養題 ∞

例題一

說明：本題練習質因數分解，並寫成標準分解式。

（英文）If a composite number has the first four prime numbers as factors, what is the smallest number it could be?

（中文）有一合數的因數包含最小的四個質數，請問該合數最小的可能值為何？

Teacher: What are the first four prime numbers?

Student: 1, 2, 3, and 5.

Teacher: A prime number is a whole number greater than 1 whose only positive factors are 1 and itself. According to the definition, 1 is not a prime number. Please revise your

answer.

Student: Ok. The first four prime numbers are 2, 3, 5, and 7.

Teacher: Correct. Let me ask you one more question. How many factors does this composite number have?

Student: 4. The question said so.

Teacher: Any other thoughts?

Student: At least 4! From the description, I only know that it has the first four prime numbers as factors, but I am not sure whether they are the ONLY factors or not. It might have other factors. This question didn't explain clearly.

Teacher: Aha, you found the tricky point. If this composite number has other factors, then there are infinite possible answers for this composite number. The prime factorization of this number is $2 \times 3 \times 5 \times 7 \times$ any number. However, have you noticed that the question asks for the smallest possible answer?

Student: If we multiply by 1, the answer should be the smallest. Am I right?

Teacher: Correct. In this case, the prime factorization is $2 \times 3 \times 5 \times 7 \times 1$. Therefore, the product is 210, which is the smallest composite number.

老師： 請問前四個質數是哪些數字？

學生： 1、2、3、5。

老師： 質數是指大於 1 且只有 1 和本身兩個正因數的整數。根據這個定義，1 不是質數，請更正你的答案。

學生： 好的，前四個質數是 2、3、5 和 7。

老師： 正確。讓我問你另外一個問題。這個合數有幾個因數？

學生： 4，題目是這樣說的。

老師： 還有其他想法嗎？

學生： 至少是 4 個！根據描述，我只知道它的因數包括前四個質數，但我不確定它是否有其他因數。這個問題沒有說清楚。

老師： 啊哈，你發現陷阱了。如果這個合數還有其他因數，那麼它就有無限可能的答案。這個數的質因數分解是 $2 \times 3 \times 5 \times 7 \times$任意數。不過，你有注意到這個問題要求最小的可能答案嗎？

學生： 如果乘以 1，答案就應該是最小的。對嗎？

老師： 正確。在這種情況下，質因數分解是 $2 \times 3 \times 5 \times 7 \times 1$。因此，乘積為 210，是最小的合數。

例題二

說明：利用因數分解找出所有可能解。

（英文）The area of a rectangular field is 120 cm^2. Find all possible whole number dimensions of the field. What is the smallest perimeter?

（中文）一塊長方形的面積為 120 cm^2，請問所有可能的長、寬（整數解）為何？最小的周長為何？

Teacher: The area of a rectangle is the product of its length and width. The dimension refers to its length and width. How do you solve this problem?

Student: We can find the factors of 120 first. Then pick two numbers whose product is 120.

Teacher: Can any of you provide an answer?

Student 1: 2 and 60.

Student 2: 10 and 12.

Student 3: 3 and 40.

Student 4: There are many possible answers. So how do I find all of them without missing any one?

Teacher: This is a good question. I suggest you try factors in ascending order. Let's start with the smallest factor, 1, and we will have 1× 120. Then try the next factor, 2, and we will have 2× 60. Then try the next factor, 3, and we will have 3× 40. With this pattern, you can find out all possible dimensions.

Listed below:

120 =

$1 \times 120 = 2 \times 60 = 3 \times 40 = 4 \times 30 = 5 \times 24 = 6 \times 20 = 8 \times 15 = 10 \times 12$

The perimeter is the boundary of a closed figure, and it is the sum of four sides.

Can you tell which dimension has the smallest perimeter?

Student: 10×12. The perimeter is $(10 + 12) \times 2 = 44$.

Teacher: Correct.

老師：矩形的面積是其長度和寬度的乘積，而大小指的是其長度和寬度。如何解決這個題目？

學生：我們可以先找出 120 的因數，然後挑選兩個乘積為 120 的數字。

老師：誰來給個答案？

學生 1：2 和 60。

學生 2： 10 和 12。

學生 3： 3 和 40。

學生 4： 有很多可能的答案。那麼，我該怎麼找到所有的答案，而不漏掉任何一個？

老師： 這是一個好問題。我建議你按升冪嘗試因數分解。讓我們從最小因數 1 開始，我們有 1× 120。然後嘗試下一個因數 2，我們有 2× 60。然後嘗試下一個因數 3，我們有 3× 40。有了這個模式，你可以找到所有可能的尺寸。

如下所示：

120 =

$1 \times 120 = 2 \times 60 = 3 \times 40 = 4 \times 30 = 5 \times 24 = 6 \times 20 = 8 \times 15 = 10 \times 12$

周長是封閉圖形的邊界，是四邊之和。你能告訴哪個尺寸的周長最小嗎？

學生： 10×12。周長為 $(10 + 12) \times 2 = 44$ 公分。

老師： 答對了。

例題三

說明：利用質因數分解解決生活中的問題。

（英文）Tony is going to use some digits from the prime factorization of 126 for his locker's code. The prime factorization of 126 is $a \times b \times 3^c$. He is going to use a, b, and c to create a 3-digit code. Please list all possible codes for Tony.

（中文）東尼用質因數分解法分解 126，將得到的某些數字做為他櫃子的密碼。126 的質因數分解可寫成 $a \times b \times 3^c$。他要使用 a、b 和 c 製作一組三位數的密碼，請列出所有可以使用的密碼組合。

Teacher: Let's work on the prime factorization of 126 first. You can use any method to figure this out.

Student A: $126 = 2 \times 7 \times 3^2$, so a is 2, b is 7, and c is 2.

Teacher: Well done.

Student B: But if I write in different order, the answers are different. $126 = 7 \times 2 \times 3^2$, so a is 7, b is 2, and c is 2

Teacher: Indeed. Let's write down the codes first, and then tell the difference. Please use a, b, and c to create 3-digit code, and remember that you can switch the orders for a, b, and c.

Student 1: 272, 227, and 722.

Student 2: 722, 272, and 227.

Student 3: The answers are the same!

Teacher: Yes, you are all right.

老師：讓我們先來找出 126 的質因數分解，任何方法都可以。

學生：$126 = 2 \times 7 \times 3^2$，所以 a 是 2，b 是 7，c 是 2。

老師：做得好。

學生：但是如果我以不同的順序寫，答案就會不同。$126 = 7 \times 2 \times 3^2$，所以 a 是 7，b 是 2，c 是 2。

老師：確實如此。讓我們先寫下密碼，然後分辨差異。請使用 a、b 和 c 建立三位數的密碼，請記住，a、b 和 c 的順序可以交換。

學生 1：272、227 和 722。

學生 2：722、272 和 227。

學生 3：答案都一樣！

老師：是的，你們都是對的。

單元五 最大公因數與最小公倍數
Greatest Common Factor & Least Common Multiple

國立新竹科學園區實驗高級中等學校　吳珮蓁老師

■ 前言 Introduction

學生學習最大公因數與最小公倍數的算法，並利用已學會的技巧去處理生活中會面對到的問題。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
common factor	公因數	**greatest common factor (GCF)**	最大公因數
common multiple	公倍數	**least common multiple (LCM)**	最小公倍數
relatively prime	互質	multiple	倍數

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Find _____ of _____.

例句：**Find** the greatest common factor **of** this set of numbers.
找出這組數字的最大公因數。

❷ What is the _____ of _____?

例句：**What is the** least common multiple **of** 6 and 12?
6 和 12 的最小公倍數為何？

■ 問題講解 Explanation of Problems

☙ 說明 ❧

[Greatest common factor]

We have learned to find the factors of one number. In this section, we will find the greatest common factor of two or more numbers. Please see the following example.

Factor of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factor of 18: 1, 2, 3, 6, 9, 18

Factors shared by two or more numbers are called common factors. 1, 2, 3, and 6 are the common factors of 24 and 18. Among these common factors, the greatest one, 6, is called the greatest common factor. We usually use GCF for abbreviation.

To find the GCF, we can list each number's factors, like what I did above. We can also use the prime factorization for GCF. We write the prime factorization of each number.

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
$18 = 2 \times 3 \times 3 = 2 \times 3^2$

Find the common prime factors: 2 and 3. Then we find the product of the common prime factors $2 \times 3 = 6$. This product of all the common prime factors is the greatest common factor. In addition to these two methods, we can use short division (ladder diagram) to find the GCF. We begin with a factor that divides into each number, which means finding a common factor of these numbers. Keep dividing until these numbers have no common factors except 1. Lastly, find the product of the numbers you divided by. $2 \times 3 = 6$. 6 is the GCF of 24 and 18.

2	24	18
3	12	9
	4	3

If the greatest common factor of whole numbers is 1, those numbers are said to be relatively prime. For example, 3 and 4 are relatively prime.

我們介紹最大公因數，和三種找到最大公因數的方法：列舉法、因數分解法、及短除法。

[Least common multiple]

A multiple of a number is the product of the number and any nonzero whole number. Let me exemplify multiples of some numbers.

The multiples of 9 are 9, 18, 27, 36, 45, 54, 63, …etc.

The multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, … etc.

The common numbers which exist above are 18, 36, and 54. These numbers are called "common multiple," the multiple shared by two or more numbers. The least of the common multiples, 18, is the least common multiple (LCM) of 9 and 6.

In addition to listing out the multiples to find the least common ones, we can use prime factorization. First, we write the prime factorization of each number:

$9 = 3^2$

$6 = 2 \times 3$

Then, we find the product of the highest power of each prime number in the prime factorizations. The highest power of the prime number 3 is 3^2, and the highest power of the prime number 2 is 2^1. The least common multiple is $3^2 \times 2^1$ is 18.

In addition to these two methods, we can use short division (ladder diagram) to find the LCM. We begin with a factor which divides into each number, and keep dividing until these numbers have no common factors, except 1. The least common multiple is the product of divisor and the quotients. In this case, divisor is 3, and the quotients are 3 and 2. The least common multiple is $3 \times 3 \times 2 = 18$.

3	9	6
	3	2

我們介紹最小公倍數，和三種找到最小公倍數的方法：列舉法、因數分解法、及短除法。

運算問題的講解

例題一

說明：本題是以因數分解法求出最大公因數。

（英文）Find the greatest common factor of 42 and 98, using prime factorization.
（中文）利用質因數分解法找出 42 和 98 的最大公因數。

Teacher: Please list the prime factorization of 42 and 98 respectively.
Student: $42 = 2 \times 3 \times 7$, $98 = 2 \times 7 \times 7$
Teacher: Then find the common factors now.
Student: 2 and 7.
Teacher: Find the product of the two numbers.
Student 1: 14.
Student 2: Why do we always find the greatest common factor, not the least common factor?
Teacher: Because the least common factor of two or more numbers is ALWAYS 1.

老師：請分別列出 42 和 98 的質因數分解。
學生：$42 = 2 \times 3 \times 7$，$98 = 2 \times 7 \times 7$
老師：現在找出它們的共同質因數。
學生：2 和 7。
老師：求此二數的積。
學生 1：14。
學生 2：為什麼總是找最大公因數，而非最小公因數？
老師：因為兩個或多個數字的最小公因數永遠都是 1。

例題二

說明：本題是以因數分解法求出最小公倍數。

（英文）Find the least common multiple of (1) 5 and 11 (2) 4, 5, and 13.
（中文）找最小公倍數 (1) 5 和 11 (2) 4、5 和 13

Teacher: Please list the prime factorization of 5 and 11 respectively.
Student: $5 = 5 \times 1$ and $11 = 11 \times 1$.

Teacher: We find the product of the highest power of each prime number in the prime factorizations for LCM. What is the LCM?

Student: The LCM is $5 \times 11 = 55$.

Teacher: Actually, when numbers are relatively prime, the LCM is the product of the prime numbers. Let's see the second question. Find the least common multiple of 4, 5, and 13.

Student: The common factor of 4, 5, and 13 is 1. They are relatively prime.

Teacher: Yes. How do you find the LCM?

Student: $4 \times 5 \times 13 = 260$.

Teacher: Correct.

老師： 請列出 5 和 11 的質因數分解式。

學生： $5 = 5 \times 1$，$11 = 11 \times 1$。

老師： 由質因數分解找出所有質數最高次的積，以此求出最小公倍數。最小公倍數是多少？

學生： 最小公倍數是 $5 \times 11 = 55$。

老師： 實際上，當數字是互質時，最小公倍數就是質數的乘積。現在來看第(2)小題。請找出 4、5 和 13 的最小公倍數。

學生： 4、5 和 13 的公因數為 1。因此為互質。

老師： 對，如何找到最小公倍數？

學生： $4 \times 5 \times 13 = 260$。

老師： 正確。

∞ 應用問題 / 會考素養題 ∞

例題一

說明：這題利用最大公因數解決日常生活問題。

（英文）Students are planning to hand out snack packs to orphanage children. The students have 48 pieces of handmade cookies and 360 candies. Every pack has the same contents and no leftover items. What is the greatest number of snack packs that can be made?

（中文）學生預計要發點心包給孤兒院的孩童，他們有 48 片手作餅乾和 360 顆糖果。每個點心包的內容物要相同，而且會發完沒有剩餘物。請問最多可發出幾份點心包？

Teacher: I will use some symbols to analyze the questions.

$$48 = \Delta \text{ (candies per pack)} \times \blacksquare \text{ (number of packs)}$$

$$360 = \nabla \text{ (cookies per pack)} \times \blacksquare \text{ (number of packs)}$$

■ is the number of packs; Δ is the number of candies per pack; ∇ is the number of cookies per pack. Please note that we use different symbols for candy and cookie. You can see that ■ is a common factor of 48 and 360. The question is asking for the "greatest number" of packs. Therefore, we need the greatest common factor of 48 and 360 for ■.

Now, please use prime factorization to find the GCF.

Student: $48 = 2^4 \times 3$; $360 = 2^3 \times 3^2 \times 5$

Teacher: What are the common prime factors?

Student: The common prime factors are 2^3 and 3.

Teacher: Can anyone tell me how to get the GCF?

Student: The product of all the common prime factor.

Teacher: Correct. The greatest common factor is the $2^3 \times 3 = 24$. The greatest number of snack packs is 24. Can you tell me the number of candies and cookies in each pack?

Student: $48 = 24 \times 2$; $360 = 24 \times 15$

Each pack will contain 2 pieces of handmade cookies and 15 candies.

Teacher: Well done.

老師：我會用一些符號來分析這一題。

$48 = \Delta$（每包糖果數量）$\times$ ■（包裝數量）

$360 = \nabla$（每包餅乾數量）$\times$ ■（包裝數量）

其中■為包裝數量，Δ為每包糖果數量，∇為每包餅乾數量。請注意我們使用不同的符號來表示糖果和餅乾。

你可以看到，■是 48 和 360 的公因數。問題要求「最多」可發出幾份點心包。因此，我們需要 48 和 360 的最大公因數來找到■。

現在，請使用質因數分解來找到最大公因數。

學生：$48 = 2^4 \times 3$；$360 = 2^3 \times 3^2 \times 5$

老師：哪些是共同的質因數？

學生：共同的質因數是 2^3 和 3。

老師：有人能告訴我如何得到最大公因數嗎？

學生：所有共同質因數的乘積。

老師：正確。最大公因數是 $2^3 \times 3 = 24$。最多能分 24 包。你能告訴我每包糖果和餅乾的數量嗎？

學生：$48 = 24 \times 2$；$360 = 24 \times 15$

每包有 2 塊手工餅乾和 15 顆糖果。

老師：沒錯，很好。

例題二

說明：這題利用最小公倍數解決日常生活問題。

（英文）You have a bag of pencils. The pencils in the bag are counted by threes, fives, and sevens. Please answer the following questions:

(1) If there is nothing left over each time, what is the fewest number of pencils that could be in the bag?

(2) If there are exactly two pencils left over each time, what is the fewest number of pencils that could be in the bag?

（中文）你有一袋筆，每三個一數，每五個一數，每七個一數。

(1) 每次數完都沒有剩餘的筆，袋內可能最少有幾支筆？

(2) 若每次數完都恰好剩餘兩支筆，袋內可能最少有幾支筆？

Teacher: When you count the pencils by threes, there's nothing left. It means that the number of pencils is the multiple of 3. When you count the pencils by fives and nothing left,

what does this mean to you?

Student: The number of pencils is the multiple of 5.

Teacher: Yes. When you count the pencils by sevens and nothing left, what does this mean?

Student: It is the multiple of 7.

Teacher: Correct. Based on the information, we know that the number of pencils is the multiple of 3; meanwhile, it is the multiple of 5 and 7. Therefore, we can say that the number is the "common multiple" of 3, 5, and 7.

Student: But there are many multiples, which one should we choose?

Teacher: Do you see that the question is asking for "the fewest" number of pencils? It means that the question is asking for the least common multiple.

Student: So… the least common multiple is $3 \times 5 \times 7 = 105$. The fewest number of pencils is 105.

Teacher: Now move on to the next question: If there are exactly two pencils left over each time, what is the fewest number of pencils.

Student: Why are there pencils left over?

Teacher: The reason is that the left-over pencils are not enough to be counted as a group of 3, 5, or 7. There are only two pencils left. Therefore, the fewest number of pencils is 105+2=107.

We just need to add the extra two pencils. You may doubt the answer. How about checking the answer?

Count 107 pencils by 3… How many groups of 3 would you have? How many pencils are left over? You can use division: $107 \div 3$

Student: $107 \div 3 = 35$ ... 2, 35 groups of 3, and 2 pencils left.

Teacher: OK. Please check $107 \div 5$ and $107 \div 7$.

Student: $107 \div 5 = 21$…2. 21 groups of 5, and 2 pencils left.

Student: $107 \div 7 = 15$…2. 15 groups of 7, and 2 pencils left.

Teacher: Correct. The fewest number of pencils is 107.

老師： 當你以三隻鉛筆為一組計算時，沒有多餘的鉛筆。這意味著鉛筆的數量是 3 的倍數。當你以五隻鉛筆為一組計算時，沒有多餘的鉛筆，這代表什麼呢？

學生： 鉛筆的數量是 5 的倍數。

老師： 是的。那以七隻鉛筆為一組計算時，沒有多餘的鉛筆，這代表什麼呢？

學生： 代表鉛筆的數量是 7 的倍數。

老師： 根據這些資訊，我們知道這支筆的數量是 3 的倍數，同時也是 5 和 7 的倍數，

因此我們可以說這個數字是 3、5 和 7 的「公倍數」。

學生： 但有很多個倍數，我們應該選擇哪一個？

老師： 你看到問題在問「最少」筆數，這表示問題正在詢問最小公倍數。

學生： 那麼... 最小公倍數是 $3 \times 5 \times 7 = 105$。最少的筆數是 105 支。

老師： 現在繼續下一個問題：如果每次剩下恰好兩支筆，最少有幾支筆？

學生： 為什麼會有剩下的筆？

老師： 原因是剩下的筆不足以被歸為 3、5 或 7 的一組。只剩下兩支筆。因此，最少的筆數是 $105 + 2 = 107$。

我們只需要加上額外的兩支筆。你可能會對答案表示懷疑，要怎麼樣檢查答案呢？

數 107 支筆，每 3 支計算一組...你會有多少組 3 支筆？還剩下多少支筆？你可以用除法：$107 \div 3$。

學生： $107 \div 5 = 21 \ldots 2$， 3 支筆一組的有 35 組，剩下 2 支筆。

老師： 好的，請驗算 $107 \div 5$ 和 $107 \div 7$。

學生： $107 \div 5 = 21 \ldots 2$，5 支筆一組的有 21 組，剩下 2 支筆。

學生： $107 \div 7 = 15 \ldots 2$，7 支筆一組的有 15 組，剩下 2 支筆。

老師： 正確。最少的筆數是 107 支。

例題三

說明：利用最小公倍數解決日常生活問題。

（英文）Serena helps in the recycling house for community service every 6 days, while Martin helps in the recycling house every 4 days. They start their community service on the same day. When will they both be in the recycling house for community service again?

（中文）Serena 每 6 天會去資源回收室做社區服務工作，Martin 每 4 天去資源回收室做社區服務工作。他們兩人在同一天開始作社區服務工作，下一次他們同時出現在資源回收室服務是甚麼時候？

Teacher: Serena goes to the recycling house every 6 days, and Martin goes to the recycling house every 4 days. So the days that they work together are the multiple of 4 and 6.

Student: 12, 24, 36, 48, etc.

Teacher: The question is asking for the next day that they work together. To determine the next day that Serena and Martin work together, find the least common multiple of 4 and 6.

Student: That would be 12.

Teacher: In 12 days, they will work together in the recycling house.

老師： Serena 每 6 天去一次資源回收室，而 Martin 每 4 天去一次。因此，他們一起工作的天數是 4 和 6 的公倍數。

學生： 12、24、36、48 等等。

老師： 問題是要求他們下一次一起工作的日期。要找出 Serena 和 Martin 一起工作的下一天，就要找到 4 和 6 的最小公倍數。

學生： 那就是 12。

老師： 是的，12 天後，他們同時會在資源回收室服務。

單元六 分數的加法與減法
Addition and Subtraction of Fractions

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■ 前言 Introduction

此單元教授分數的加法與減法，請老師注意分數的各種英文說法以及假分數、帶分數與小數點的轉換說法。進行分數的加法與減法時，會使用到通分的技巧，因此老師也要複習最小公倍數和最大公因數的概念。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
integer	整數	equivalent fractions	等值分數
fraction	**分數**	simplest form	最簡分數
decimal	小數	proper fraction	真分數
denominator	分母	improper fraction	假分數
numerator	分子	mixed number	帶分數
rational number	有理數	common factor	公因數
whole number	非負整數	common multiple	公倍數
half	一半	**greatest common factor, GCF**	**最大公因數**
third	第三；三分之一	**least common multiple, LCM**	**最大公倍數**
quarter	四分之一	fraction reduction	約分
fifth	第五；五分之一	fraction expansion	擴分

分數的講法

1. Half, third, fourth, fifth, sixth, seventh, eighth, ninth, tenth, …hundredth, thousandth

   $\frac{1}{2}$：1 over 2；one half

   $\frac{2}{3}$：2 over 3；two thirds

   $2\frac{3}{4}$：two and three fourths/quarters

   $1\frac{1}{2}$：one and a half

   $\frac{1}{10} = 0.1 = 10\%$：tenth

   $\frac{1}{100} = 0.01 = 1\%$：hundredth

2. **分子和分母的簡易講法**

   學生若不熟悉分子(numerator)與分母(denominator)的英文講法，教師可以分別用 the top number（上方）和 the bottom number（下端）來表示。

   例如：For two-thirds, the numerator (top number) is 2. The denominator (bottom number) is 3.

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ **Divide (break apart) _____ into _____.**
(_____ is divided into _____.)

例句(1)：The teacher **divided** our class **into** four groups.
老師把全班分成四組。

例句(2)：The rectangle **is divided into** four equal parts.
長方形被分成四等份。

❷ Multiply/Divide both the numerator and the denominator by a non-zero number.

例句(1)：To find the equivalent fractions of $\frac{3}{5}$ (read as: three-fifths), **multiply both the numerator and the denominator by a non-zero number.**

將五分之三的分子與分母同乘以一個非零的數字，可以得到等值分數。

例句(2)：For $\frac{3}{5}$, if we **multiply the numerator and the denominator by** negative two, we get $\frac{-6}{-10}$ (read as: negative six over negative ten). $\frac{-6}{-10}$ is equivalent to $\frac{3}{5}$.

將五分之三的分子與分母同乘以負二，可求得負十分之負六，是五分之三的等值分數。

例句(3)：If we **divide both the numerator and the denominator of** $\frac{20}{-15}$ **by** five, we get $\frac{4}{-3}$ (read as: four over negative three). This process is called fraction reduction.

將負十五分之二十的分子與分母同除以五，可求得負三分之四。這個過程稱為約分。

❸ Reduce/Simplify _____ (the fraction) to its simplest form.
Write_____ (the decimal) as a fraction in its simplest form.

例句(1)：**Reduce** $-\frac{14}{91}$ (read as: negative fourteen over ninety-one) **to its simplest form**.

It becomes $-\frac{2}{13}$ (read as: negative two over thirteen).

把負九十一分之十四化為最簡分數，可以得到負十三分之二。

例句(2)：**Write** 0.25 (read as: zero point two five or twenty-five hundredths) as a fraction **in its simplest form.** It is $\frac{1}{4}$ (read as: one quarter).

把零點二五化為最簡分數，是四分之一。

❹ For _____(a fraction), the numerator and denominator have a common factor _____.

例句(1)：**For** $\frac{22}{55}$ (twenty-two over fifty-five), **the numerator and denominator have a common factor of** 11 (eleven).

針對五十五分之二十二，分子和分母有一個公因數 11。

例句(2)：**For** $\frac{2}{5}$ (two fifths), **the numerator and denominator have no common factor** other than 1.

五分之二這個分數，分子和分母並沒有 1 以外的公因數。

❺ Convert _____(fractions) to a common denominator

例句(1)：When adding or subtracting fractions, we must first convert these fractions to a common denominator.

計算分數的加減法時，我們必須先把這幾個分數通分化成相同的分母。

例句(2)：To add two-thirds and one-fifth, you need to **convert these fractions to a common denominator** of 15。

計算三分之二加五分之一時，你必須把這兩個分數的分母化為 15。

❻ Order ____ (the fractions) from least to greatest (from greatest to least).

例句(1)：**Order** $-\frac{4}{7}$, $-\frac{5}{7}$ and $-\frac{6}{7}$ **from least to greatest**.

把七分之負四、七分之負五和七分之負六從小到大排列。

例句(2)：To **order** $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ **from greatest to least**, we have to convert these fractions to a common denominator.

為了比較二分之一、三分之二和四分之三的大小，我們必須先通分把這幾個分數的分母化為相同的正整數。

❼ ____ (a fraction) plus/minus ____(a fraction) equals _____(a fraction).

例句(1)：$\frac{2}{5}$ (two fifths) plus $\frac{1}{5}$ (one fifth) **equals** $\frac{3}{5}$ (three fifths).

五分之二加五分之一等於五分之三。

例句(2)：$\frac{2}{5}$ (two fifths) minus $\frac{1}{5}$ (one fifth) **equals** $\frac{1}{5}$ (one fifth).

五分之二減五分之一等於五分之一。

❽ _____ (A fraction) of _____is / are _____.

例句(1)：Three fifths **of** the students in this class **are** female.

班上五分之三的學生是女生。

例句(2)：A half **of** 10 **is** 5.

十的一半等於五。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

In this section, we will cover the addition and subtraction of fractions. The commutative and associative properties will also be applied in this section. Students have to familiarize themselves with how to convert fractions to a common denominator to add, subtract, and order fractions.

Remember that the order that we say a fraction in English is different from the order that we say a fraction in Chinese.

❧ 運算問題的講解 ☙

例題一

說明：此題為分數加法的基本運算，包含分數的擴分。

（英文）Evaluate $\frac{4}{21}+\frac{3}{7}$.

（中文）計算 $\frac{4}{21}+\frac{3}{7}$。

Teacher: To add these two fractions, can we add the numerators directly?

Student: No. The denominators are different.

Teacher: What is the least common multiple of 21 and 7?

Student: It is twenty-one.

Teacher: Great! Let's convert these two fractions to the same denominator. What will you get?

Student: Four over twenty-one plus nine over twenty-one. ($\frac{4}{21}+\frac{9}{21}$)

Teacher: Now, can we add these two numerators? Explain it.

Student: Yes, we can add the two numerators because they have the same denominator. It is equal to thirteen over twenty-one ($\frac{13}{21}$).

Teacher: Is it in the simplest form?

Student: Yes, because the numerator and denominator do not have any common factor other than one.

老師： 進行分數相加時，我們可以直接把分子相加嗎？

學生： 不行，因為分母不同。

老師： 21 和 7 的最小公倍數是多少？

學生： 是 21。

老師： 太好了！讓我們將這兩個分數換成相同的分母，會得到什麼？

學生： $\frac{4}{21}+\frac{9}{21}$。

老師： 現在可以把這兩個分子相加嗎？說看看。

學生： 可以，因為分母相同，相加等於 $\frac{13}{21}$。

老師： 是否為最簡分數？

學生： 是的，因為分子和分母除了 1 以外沒有其他公因數。

例題二

說明：此題仍為分數加法與減法的基本運算，包含交換律的應用。

（英文）Evaluate $\frac{9}{11}+[(-\frac{9}{17})+\frac{2}{11}]$.

（中文）計算 $\frac{9}{11}+[(-\frac{9}{17})+\frac{2}{11}]$。

Teacher: Commutative law of addition means that you can interchange the positions of two numbers when these two numbers are added.
That is, *a* plus *b* equals *b* plus *a* ($\boldsymbol{a+b=b+a}$).
Here, I would like to interchange the last two terms, $(-\frac{9}{17})+\frac{2}{11}=\frac{2}{11}+(-\frac{9}{17})$.
The new expression becomes $\frac{9}{11}+[\frac{2}{11}+(-\frac{9}{17})]$.
Can anyone tell me the purpose of interchanging these two terms?

Student: After interchanging these two terms, the first two terms have the same denominator. Then we can add the numerators of the first two terms directly.

Teacher: Correct. What is the new expression in this step?

Student: The expression becomes nine over eleven plus two over eleven minus nine over seventeen. $(\frac{9}{11}+\frac{2}{11}-\frac{9}{17})$

Teacher: What is the sum of the first two fractions?

Student: It is eleven over eleven, which is one.

Teacher: Yes, the expression becomes one minus nine over seventeen $(1-\frac{9}{17})$. How to do the subtraction in this case?

Student: Convert the integer one to a fraction as seventeen over seventeen $(\frac{17}{17})$, then the expression becomes $(\frac{17}{17}-\frac{9}{17})$. Therefore, the answer is eight over seventeen $(\frac{8}{17})$.

老師： 加法的交換律表示當兩個數字相加時，你可以交換這兩個數字的位置。也就是說，*a* 加上 *b* 等於 *b* 加上 *a*，$\boldsymbol{a+b=b+a}$。
這裡，我想交換後面兩項，$(-\frac{9}{17})+\frac{2}{11}=\frac{2}{11}+(-\frac{9}{17})$。

變成 $\frac{9}{11}+[\frac{2}{11}+(-\frac{9}{17})]$。

有人可以告訴我交換這兩項的目的嗎？

學生： 交換這兩項後，前兩項具有相同的分母。然後我們可以直接將前兩項的分子相加。

老師： 對的。這一步的表達式要怎麼列？

學生： 變成 $(\frac{9}{11}+\frac{2}{11}-\frac{9}{17})$。

老師： 前兩個分數的和是多少？

學生： $\frac{11}{11}$，也就是 1。

老師： 是的，表達式變成了$(1-\frac{9}{17})$。接下來要怎麼做減法？

學生： 將整數 1 轉換為$(\frac{17}{17})$，然後這個表達式變成了$(\frac{17}{17}-\frac{9}{17})$。答案是$(\frac{8}{17})$。

例題三

說明：此題為分數加法與減法的混合運算，加入了將帶分數轉換為假分數的計算，也加入了「負負得正」的概念。

（英文）Evaluate $\left[1\frac{2}{5}-(-\frac{3}{7})\right]+(\frac{2}{7})$.

（中文）計算 $\left[1\frac{2}{5}-(-\frac{3}{7})\right]+(\frac{2}{7})$。

Teacher: Write the mixed numbers as improper fractions. What do you get?

Student: It is five-fifths plus two-fifths $(\frac{5}{5}+\frac{2}{5})$. It turns out to be seven fifths $(\frac{7}{5})$.

Teacher: You are correct. The expression becomes $\left[\frac{7}{5}-(-\frac{3}{7})\right]+(\frac{2}{7})$.

Can anyone tell me the simplified form of negative three-sevenths $-\left(-\frac{3}{7}\right)$?

Student: Double negatives cancel each other, which makes a positive.

Therefore, it becomes positive three-sevenths $(\frac{3}{7})$.

Teacher: Associative law of addition means that you can add the terms in any order.

Here, the new expression is $(\frac{7}{5}+\frac{3}{7})+\frac{2}{7}$.

Which two terms can we add first?

Student: The second and the third term. (Three-sevenths and two-sevenths)

Teacher: After we add these two terms, what is the new expression?

Student: Seven-fifths plus five-sevenths $(\frac{7}{5}+\frac{5}{7})$.

Teacher: How to add these two terms?

Student: Convert the denominators to the same number, 35.

Teacher: When we have the denominator, 35, what is the new expression?

Student: Forty-nine over thirty-five plus twenty-five over thirty-five. $(\frac{49}{35}+\frac{25}{35})$

Teacher: Now, the answer is almost there. Anyone of you want to give it a try?

Student: Forty-nine plus twenty-five equals seventy-four.

Therefore, the answer is seventy-four over thirty-five. $(\frac{74}{35})$.

Teacher: Because seventy-four over thirty-five is in its simplest form, it is the answer to this problem.

老師： 先將帶分數轉換成假分數，你得到多少？

學生： $\frac{5}{5}+\frac{2}{5}$，相加等於 $\frac{7}{5}$。

老師： 對。算式變成 $[\frac{7}{5}-(-\frac{3}{7})]+(\frac{2}{7})$。有人能告訴我$-\left(-\frac{3}{7}\right)$簡化後會變什麼嗎？

學生： 負負得正，這使數值變成正的。因此，它變成了$\frac{3}{7}$。

老師： 加法結合律代表我們可以按任何順序相加。

在這裡，新的表達式是$(\frac{7}{5}+\frac{3}{7})+\frac{2}{7}$。哪兩項可以先相加？

學生： 第二和第三項 $(\frac{3}{7}+\frac{2}{7})$。

老師： 相加後，新的式子變成？

學生： $(\frac{7}{5}+\frac{5}{7})$。

老師： 這兩數如何相加？

學生： 將分母轉換為 5 和 7 的最小公倍數，35。

老師： 當分母是 35 時，新的表達式是什麼？

學生：$(\frac{49}{35}+\frac{25}{35})$。

老師：現在，答案快出來了。有人要想試試看嗎？

學生：$49+25=74$。因此，答案是 $\frac{74}{35}$

老師：$\frac{74}{35}$ 已經是最簡分數，所以這就是答案。

應用問題 / 會考素養題

例題一

說明：此題主要為利用分數加減換算距離的應用，最後化簡時需要使用假分數轉換為帶分數的概念。教師也可視學生程度加入絕對值的概念。

（英文）Find the distance between point A and point B given the coordinates of them are $-2\frac{3}{4}$ and $1\frac{3}{10}$ respectively.

（中文）求數線上A$(-2\frac{3}{4})$, B$(1\frac{3}{10})$ 兩點的距離。

Teacher: Try to draw a line and plot the two points on this line.
Which point is on the left, and which point is on the right? Explain your reasoning.

Student: Negative two and three-fourths $(-2\frac{3}{4})$ is on the left.

One and three-tenths $(1\frac{3}{10})$ is on the right because $1\frac{3}{10}$ is greater than $-2\frac{3}{4}$.

Teacher: Since $1\frac{3}{10}$ is greater than $-2\frac{3}{4}$, $1\frac{3}{10}-(-2\frac{3}{4})$ would be the distance between the two points. How to evaluate the difference?

Student: Because double negatives make a positive, we have to evaluate $1\frac{3}{10}+2\frac{3}{4}$.

Teacher: It is noted that the two fractions have different denominators. Therefore, we have to convert the two fractions to the same denominator. What is the least common multiple of 10 and 4?

Student: Because $10=5\times2$ and $4=2\times2$
The least common multiple is $5\times2\times2$, which is 20.

Teacher: If the denominator is 20, what is the new expression?

Student: $1\frac{6}{20}+2\frac{15}{20}$.

Teacher: Here, we add the integers and fractions separately.
You would have one plus two and six over twenty plus fifteen over twenty.
What do you get?

Student: Three and twenty-one over twenty.

Teacher: Twenty-one over twenty is an improper fraction. So we have to convert it to a mixed number. What is the mixed number equivalent to twenty-one over twenty?

Student: One and one over twenty is equivalent to twenty-one over twenty.

Teacher: Okay, let's find the final answer.
Three plus one, and then plus one over twenty. What do you get?

Student: Four and one over twenty.

Teacher: Correct. Therefore, the distance between these two points is four and one over twenty.

老師： 畫一條線並在上面標出兩個點。哪一個點在左邊，哪一個點在右邊？請解釋你的理由。

學生： $(-2\frac{3}{4})$ 的點在左邊。$(1\frac{3}{10})$ 的點在右邊，因為 $(1\frac{3}{10})$ 大於 $(-2\frac{3}{4})$。

老師： 因為 $(1\frac{3}{10})$ 大於 $(-2\frac{3}{4})$，所以 $(1\frac{3}{10})-(-2\frac{3}{4})$ 是兩點之間的距離。這要如何計算？

學生： 因為負負得正，所以要計算 $1\frac{3}{10}+2\frac{3}{4}$。

老師： 注意到這兩個分數有不同的分母，因此必須將這兩個分數通分。
10 和 4 的最小公倍數是多少？

學生： 因為$10=5\times2$，且 $4=2\times2$，所以最小公倍數是 $5\times2\times2$，即 20。

老師： 如果分母是 20，新的表達式要怎麼列？

學生： $1\frac{6}{20}+2\frac{15}{20}$。

老師： 這裡我們整數相加、再分數相加。你會得到 1 加 2 再加上 $\frac{6}{20}$ 加 $\frac{15}{20}$。算出來是多少呢？

學生： $3\frac{21}{20}$。

老師：$\frac{21}{20}$ 是一個假分數。因此，我們必須將它化成帶分數。$\frac{21}{20}$ 的帶分數是多少？

學生：$1\frac{1}{20}$ 等於 $\frac{21}{20}$。

老師：好的，我們來找出最後的答案。3 加 1，再加 $\frac{1}{20}$，是多少呢？

學生：$4\frac{1}{20}$。

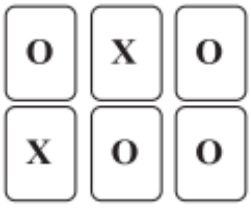
老師：沒錯，所以這兩點之間的距離是 $4\frac{1}{20}$。

例題二

說明：本題涉及到簡單的機率概念。除此之外，老師可以教導學生「一半」的概念，以及分數的約分。

（英文）Yi-Chun has 24 playing cards. Half of the playing cards are labeled with "X," and the other half are labeled with "O." The figure below shows the first six cards drawn by Yi-Chun. If she is going to draw another one out from the remaining cards and each card is equally likely to be drawn, what is the probability that the card she draws is labeled with "O?"

（中文）怡君手上有 24 張卡片，其中 12 張卡片被畫上 O 記號，另外 12 張卡片被畫上 X 記號。下圖表示怡君從手上拿出 6 張卡片放在桌面的情形，且她打算從手上剩下的卡片中抽出一張卡片。
若怡君手上剩下的每張卡片被抽出的機會相等，則她抽出 O 記號卡片的機率為何？



（104 年國中會考 12）

Teacher: According to the descriptions, how many cards are left after Yi-Chun drew the first six cards?

Student: Eighteen, because twenty-four minus six equals eighteen.

Teacher: Among the eighteen cards, how many cards are labeled with "O"? and how many are labeled with "X"? Explain your answer.

Student: Eight cards are labeled with "O" because twelve minus four equals eight.

Ten cards are labeled with “X” because twelve minus two equals ten.

Teacher: Out of the remaining 18 cards, what is the probability that the card Yi-Chun draws is labeled with “O”?

Student: Eight out of eighteen is equivalent to four-ninths.

Teacher: Great!

老師： 依據題目，怡君抽了前六張牌後，還剩下多少張牌？

學生： 18 張，因為 24 張減 6 張等於 18 張。

老師： 在這十八張牌中，有幾張標有「O」，有幾張標有「X」？請說明你的答案。

學生： 有 8 張牌被畫上「O」記號，因為十二張減去四張等於八張。
有 10 張牌標有「X」，因為十二張減去兩張等於十張。

老師： 在剩下的 18 張牌中，怡君抽到標有「O」的機率是多少？

學生： 18 張牌中有 8 張「O」，機率為 $\frac{4}{9}$。

老師： 很好！

例題三

說明：本題主要概念是分數的加法，老師也可以利用此道題目向學生介紹國際數學日。

（英文）On International Mathematics Day (Pi Day), Jane has a pizza party with her classmates in school.

(a.) Jane ate one-eighth pepperoni pizza, three-eighths cheese pizza, and one-fourth Hawaiian pizza. How much pizza did she eat in all?

(b.) Jane’s best friend, Jessica, is a vegetarian. She ate five-sixths veggie pizza. How much more pizza did Jane eat than her best friend?

（中文）在國際數學日，也就是俗稱的 π Day，珍和同學在學校舉辦了披薩派對。

(1) 珍吃了八分之一的義大利香腸披薩，八分之三的起司披薩和四分之一的夏威夷批薩。試問，珍總共吃了多少披薩？

(2) 珍最好的朋友，潔西卡是素食者。她吃了六分之五的蔬菜披薩。試問，潔西卡比珍多吃了多少披薩？

Teacher: There are three kinds of pizza eaten by Jane. We use addition most of the time when reading “in all” in the word problem.

Can you tell me how to list the expression to find the answer?

Student: One-eighth plus three-eighths and then plus one-fourth $(\frac{1}{8}+\frac{3}{8}+\frac{1}{4})$.

Teacher: One of the fractions has a denominator different from the others. So we have to convert it first. Do any of you have an idea of the conversion?

Student: Rewrite one-fourth as two-eighths.

Teacher: Correct. Now the expression becomes one-eighth plus three-eighths and then plus two-eighths. What is the sum?

Student: Six-eighths. It is equivalent to three-fourths.

Teacher: You are right. Jane ate three-fourths of the pizza.

For part b, we are going to use subtraction when reading "how much more" in the word problem. Can you tell me how to list the expression to find the answer?

Student: Five-sixths minus three-fourths.

Teacher: Since the denominators are different, let's convert the denominator to the same number. If you are unfamiliar with the least common multiple, you can multiply the first fraction by four over four and multiply the second by six over six.

It becomes twenty over twenty-four minus eighteen over twenty-four.

What is the difference in this step?

Student: Two over twenty-four. It is the same thing as one over twelve.

Teacher: Correct. Jessica ate one-twelfth more pizza than Jane did.

老師： 珍吃了三種披薩，我們在題目中讀到「總共」時，大多數情況下使用加法。你能告訴我如何列出表達式嗎？

學生： $(\frac{1}{8}+\frac{3}{8}+\frac{1}{4})$。

老師： 其中一個分數的分母與其他不同，所以我們必須先將其轉換。你們有沒有任何轉換的想法？

學生： 將$\frac{1}{4}$化成$\frac{2}{8}$。

老師： 正確。現在運算式變成了$(\frac{1}{8}+\frac{3}{8}+\frac{2}{8})$。和是多少？

學生： $\frac{6}{8}$，化簡後變成$\frac{3}{4}$。

老師： 沒錯，珍吃了$\frac{3}{4}$個披薩。

來看第(2)小題，題目中看到「多吃了多少」時，我們會使用減法。你能告訴我

如何列出表達式嗎？

學生：$(\frac{5}{6}-\frac{3}{4})$。

老師：由於分母不同，將分母轉換為相同的數字。如果不熟悉最小公倍數，可以把第一個分數乘以 $\frac{4}{4}$，第二個分數乘以 $\frac{6}{6}$，得到 $(\frac{20}{24}-\frac{18}{24})$。

兩數相減答案是多少？

學生：$\frac{2}{24}$，也就是 $\frac{1}{12}$。

老師：答對了，潔西卡比珍多吃了 $\frac{1}{12}$ 個披薩。

單元七 分數的乘法與除法
Multiplication and Division of Fractions

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■ 前言 Introduction

此單元承接分數的加法與減法，教授分數的乘法與除法。請老師提醒學生熟悉前一個單元中分數的各種英文說法，以及假分數、帶分數與小數點的轉換說法。如何將分數的除法化為分數的乘法也是教學重點之一。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
reciprocal	**倒數**	complex fraction	繁分數
undefined	**未定義的**	nonzero	非零的
ratio	比；比率	terminating decimal	有限小數
percent	百分比；百分率	repeating decimal	循環小數
scale	比率；縮尺	percent	百分之一
portion	部分	percentage	比例；百分比

percent 與 percentage 的用法

1. Percent 指「佔了多少百分比」，單字前需加數字，對應符號是 %。
   例如：You got eighty percent (80%) of the answers correct.
   你的答案中百分之八十是正確的。

2. Percentage 指「所佔的比例」，單字前可以加形容詞。
   例如：A large percentage of students participated in the competition.
   很大一部分的學生參加這場比賽。

3. What percentage of the children were absent today?
   缺席的孩童佔比是多少？
   Twenty percent of the children were absent today.
   百分之二十的孩童缺席。

4. 例題示範：Problem 8 of AMC8 in 2019
   Gilda has a bag of marbles. She gives 20% of them to her friend Pedro. Then Gilda gives 10% of what is left to another friend, Ebony. Finally, Gilda gives 25% of what is now left in the bag to her brother Jimmy. What percentage of her original bag of marbles does Gilda have left for herself?

   題目出處：
   https://artofproblemsolving.com/wiki/index.php/2019_AMC_8_Problems/Problem_8

   解答 1：$1 - 20\% = 80\%$ (Gilda has 80% left after giving out 20% to Pedro.)
   $10\% \times 80\% = 8\%$ (10% of what is left.)
   $80\% - 8\% = 72\%$ (Gilda has 72% left after giving 8% to Ebony.)
   $25\% \times 72\% = 18\%$ (25% of what is left.)
   $72\% - 18\% = 54\%$ (Gilda has 54% left after giving 18% to Jimmy.)

   解答 2：求解算式 $25\% \times 72\%$ 時，教師可以引導學生適當將25% 轉換為四分之一，相當於計算 72% 除以 4。

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Multiply the numerators and multiply the denominators.

例句(1)：To find $\frac{1}{2}\times\frac{3}{2}$ (Read as: a/one half times three halves), **multiply the numerators and multiply the denominators**. You will get $\frac{3}{4}$.

求 $\frac{1}{2}\times\frac{3}{2}$ 的值時，分子倆倆相乘與分母倆倆相乘，得到 $\frac{3}{4}$。

例句(2)：We **multiply the numerators and multiply the denominators** to find $\frac{4}{7}\times\frac{3}{4}$ (Read as: four sevenths multiplied by three quarters). You will have $\frac{12}{28}$, which is equivalent to $\frac{3}{7}$.

透過分子倆倆相乘與分母倆倆相乘，可求得 $\frac{4}{7}\times\frac{3}{4}$ 的值為 $\frac{12}{28}$。約分後得到 $\frac{3}{7}$。

❷ Write _____ (a mixed number) as an improper fraction.

例句(1)：**Write** $1\frac{1}{2}$ (read as: one and a half) **as an improper fraction**. It becomes $\frac{3}{2}$ (read as: three halves).

把 $1\frac{1}{2}$ 寫成假分數時，它會變成 $\frac{3}{2}$。

例句(2)：**Write** $2\frac{3}{4}$ and $5\frac{6}{7}$ (read as: two and three fourths and five and six sevenths) **as improper fractions.** They are $\frac{11}{4}$ (read as: eleven quarters) and $\frac{41}{7}$ (read as: forty-one sevenths) respectively.

把 $2\frac{3}{4}$ 以及 $5\frac{6}{7}$ 寫成假分數時，它們分別變成 $\frac{11}{4}$ 與 $\frac{41}{7}$。

❸ Exchange/interchange ___ (a numerator) and ___ (a denominator) to get the reciprocal of ____ (a fraction).

例句(1)：**Exchange** 2 and 3 **to get the reciprocal of** $\frac{2}{3}$ (two thirds).

把 2 和 3 交換可以得到三分之二的倒數。

例句(2)：When we **interchange** 2 and 3, we **have the reciprocal of** $\frac{2}{3}$ (two thirds).

當我們交換分子和分母的數字時，我們可以得到三分之二的倒數。

❹ The reciprocal of _____ is _____.

例句(1)：**The reciprocal of** $\frac{2}{3}$ (two thirds) **is** $\frac{3}{2}$ (three halves).

三分之二的倒數是二分之三。

例句(2)：**The reciprocal of** 2 **is** $\frac{1}{2}$ (one half).

二的倒數是二分之一。

❺ Multiply _____ by the reciprocal of _____ (, which is _____).

例句(1)：To find $\frac{2}{3} \div 2$ (two-thirds divided by two), we **multiply** $\frac{2}{3}$ (two-thirds) **by the reciprocal of** 2. Then we will get $\frac{1}{3}$ (one-third).

要求三分之二除以二的值，我們必須把三分之二乘以二的倒數。這樣我們可以得到答案為三分之一。

例句(2)：To find $\frac{2}{3} \div \frac{3}{4}$ (two-thirds divided by three-quarters), we **multiply** $\frac{2}{3}$ **by the reciprocal of** $\frac{3}{4}$ **, which is** $\frac{4}{3}$. Then we get $\frac{8}{9}$ (eight ninth) as the answer.

要求三分之二除以四分之三的值，我們必須把三分之二乘以四分之三的倒數。答案等於九分之八。

❻ How many _____ are in _____?

例句(1)：**How many** two-thirds **are in** four?

四可以被分成幾個三分之二？

例句(2)：**How many** two and one-halves **are in** three and one-quarter?

三又四分之一可以被分成幾個二又二分之一？

❼ What is _____ of _____?

例句(1)：**What is** three-quarters **of** five?
四分之三等分的五等於多少？

例句(2)：**What is** two-thirds **of** three-fifths?
三分之二等分的五分之三等於多少？

❽ _____ is the same as _____

例句(1)：Two-thirds is **the same as** two divided by three.
三分之二等同於二除以三。

例句(2)：Four over two-thirds is **the same as** four divided by two-thirds.
繁分數三分之二分之四等同於四除以三分之二。

例句(3)：Four divided by two-thirds is **the same as** four multiplied by three-halves.
四除以三分之二等同於四成以二分三。

■ 問題講解 Explanation of Problems

說明

In this section, we will cover the multiplication and division of fractions. The properties of multiplying or dividing integers will also be applied in this section. You need to remember that when a number is divided by a fraction, you can multiply the number by its reciprocal. Also remember that the order that we say a fraction in English is different from the order that we say a fraction in Chinese.

運算問題的講解

例題一

說明：此題為分數乘法的基本運算，包含分數約分。學生可以先將分數個別約分再運算，也可以先乘法運算後再約分。建議老師可以讓學生小組討論哪種方式較佳。

（英文）Evaluate $(-\frac{4}{21}) \times \frac{3}{8}$.

（中文）求 $(-\frac{4}{21}) \times \frac{3}{8}$ 的值。

Teacher: What is the sign of the product? Explain your reasoning.

Student: The answer is negative because the product of two numbers with different signs is negative.

Teacher: Where can we place the negative sign? Should it go in front of the fraction? Or is it in front of the numerator, or in front of the denominator?

Student: Either way will be fine.

Teacher: Multiply the numerators and multiply the denominators. What do you have now?

Student: $-\frac{12}{168}$。

Teacher: What is the simplest form of $-\frac{12}{168}$?

Student: Divide both the numerator and denominator by 12. The simplest form is $-\frac{1}{14}$。

Teacher: Discuss this with your classmates, and suggest a better way to find the product of the two fractions.

Student: I think we can divide out the common factor between the numerator and denominator first.

Teacher: Please demonstrate this on the board.

Student: We have $\frac{4}{8} = \frac{1}{2}$ and $\frac{3}{21} = \frac{1}{7}$.

Substituting these values, we have $-\frac{1}{2} \times \frac{1}{7} = -\frac{1}{14}$

老師： 答案是正的還是負的呢？推論看看。

學生： 答案是負的，因為兩個不同符號的數相乘會得到負數。

老師： 負號應該放在哪裡？應該放在分數的前面嗎？還是放在分子的前面或分母的前面？

學生： 都可以。

老師： 將分子相乘，將分母相乘。答案是多少？

學生： 是 $-\frac{12}{168}$。

老師： $-\frac{12}{168}$ 的最簡形式是什麼？

學生： 分子分母同除以 12，最簡分數是 $-\frac{1}{14}$。

老師： 跟同學討論一下，換個方法來求出這兩個分數的乘積。

學生： 我們可以先除掉分子和分母之間的公因數。

老師： 上來寫到黑板上。

學生： 化簡 $\frac{4}{8}$ 得 $\frac{1}{2}$，化簡 $\frac{3}{21}$ 得 $\frac{1}{7}$，代入這些值，我們就可以得到答案 $-\frac{1}{14}$。

例題二

說明：此題仍為分數乘法的基本運算，包含分數的約分。但加入了將帶分數轉換為假分數的計算。

（英文）Evaluate $2\frac{3}{4}\times 5\frac{6}{7}$.

（中文）求 $2\frac{3}{4}\times 5\frac{6}{7}$ 的值。

Teacher: Write $2\frac{3}{4}$ and $5\frac{6}{7}$ as improper fractions. What do you get?

Student: $\frac{11}{4}$ and $\frac{41}{7}$

Teacher: Multiply the numerators and multiply the denominators. What is the new numerator? What is the new denominator?

Student: The numerator is 451. The denominator is 28.

Teacher: What is the product of the two fractions?

Student: The product of the two fractions becomes $\frac{451}{28}$.

Teacher: Can we simplify this fraction? Why or why not?

Student: $\frac{451}{28}$ is in the simplest form because there is no common factor between the numerator and denominator.

老師： 將 $2\frac{3}{4}$ 和 $5\frac{6}{7}$ 寫成假分數。

學生： $\frac{11}{4}$ 和 $\frac{41}{7}$。

老師： 將分子相乘，分母相乘。新的分子是什麼？新的分母是什麼？

學生： 新的分子是 451，新的分母是 28。

老師： 相乘等於多少？

學生： 這兩個分數的乘積變成了 $\frac{451}{28}$。

老師： 我們可以簡化這個分數嗎？可以或不可以的原因是？

學生： $\frac{451}{28}$ 已經是最簡分數，因為分子和分母之間沒有公因數。

例題三

說明：此題為分數乘法與除法的混合運算，包含分數的約分。加入了將帶分數轉換為假分數的計算，也加入了「負負得正」的概念。

計算帶分數的倒數時，老師要提醒學生必須先將帶分數轉換為假分數，才可以交換分子與分母求得倒數。

在分數的除法運算部分，老師要提醒學生，除以一個分數等同於乘以它的倒數。

（英文）Evaluate $-\frac{21}{11}\times 5\frac{2}{3}\div(-1\frac{1}{3})$.

（中文）求 $-\frac{21}{11}\times 5\frac{2}{3}\div(-1\frac{1}{3})$ 的值。

Teacher: Write the mixed numbers as improper fractions. What do you get?

Student: $\frac{17}{3}$ and $-\frac{4}{3}$.

Teacher: Determine the sign of the answer.

Student: Positive.

Teacher: What is the order of operations?

Student: From left to right.

Teacher: So, we multiply $\frac{21}{11}$ and $\frac{17}{3}$ first. What is the product?

Student: The product is $\frac{119}{11}$.

Teacher: What is the reciprocal of $1\frac{1}{3}$?

Student: Because the reciprocal of $\frac{1}{3}$ is 3, I think the reciprocal of $1\frac{1}{3}$ is "one plus three", which is four.

Teacher: No, try again! Whenever you have mixed numbers, you have to rewrite it as an improper fraction and then exchange the numerator and denominator to find its reciprocal. Therefore, we multiply $\frac{119}{11}$ and $\frac{3}{4}$. What is the answer?

Student: $\frac{357}{44}$.

老師： 先將帶分數寫成假分數。

學生： $\frac{17}{3}$ 和 $-\frac{4}{3}$。

老師： 答案會是正的還是負的？

學生： 正的。

老師： 運算順序是什麼？

學生： 從左到右。

老師： 所以，我們先把 $\frac{21}{11}$ 和 $\frac{17}{3}$ 相乘，答案是多少？

學生： $\frac{119}{11}$。

老師： $1\frac{1}{3}$ 的倒數是多少？

學生： 因為 $\frac{1}{3}$ 的倒數是 3，我認為 $1\frac{1}{3}$ 的倒數是 1+3，即 4。

老師： 不對，再試一次！看到帶分數時，你必須把它化成假分數，然後分子和分母交換得到倒數。因此，$\frac{119}{11}$ 乘以 $\frac{3}{4}$，答案是多少？

學生： $\frac{357}{44}$。

∞ 應用問題 / 會考素養題 ∞

例題一

說明：此題主要為漱口水瓶蓋容量和瓶裝漱口水容量的比例換算。必要時，老師可以用圖示讓學生了解漱口水瓶蓋和瓶裝漱口水的比例關係。

（英文）The instructions on the label of a mouthwash say to use one-third of a bottle cap of mouthwash to rinse your mouth each time. Winnie bought the mouthwash and mistook one-third for a half when she read the instructions. After using it ten times, the amount of mouthwash left is three-quarters of the bottle. If Winnie starts to use the mouthwash with the correct amount according to the instructions, how many more times can she rinse before she uses up the mouthwash?

（中文）某漱口水瓶上標示正確使用方式：一次使用量為瓶蓋容量的 1/3。小瑜買了一瓶，誤將 1/3 看成 1/2，在使用 10 次後才發現錯誤，此時漱口水只剩原來的 3/4。若往後小瑜依正確方式使用完畢，則還可以用多少次？

（95 年第一次國中基測第 20 題）

Teacher: How much is used after ten oral rinses?

Student 1: $\frac{1}{2} \times 10 = 5$ caps.

Student 2: $\frac{1}{4}$ of a bottle of mouthwash.

Teacher: How many caps of mouthwash equals a bottle of mouthwash?

Student: $5 \div (1 - \frac{3}{4}) = 5 \times 4 = 20$.

Teacher: How many caps of mouthwash are left after ten oral rinses?

Student: $20 - 5 = 15$.

Teacher: How many more times can Winnie rinse before she uses up the mouthwash?

Student: $15 \div \frac{1}{3} = 15 \times 3 = 45$.

老師： 用了 10 次漱口水，是用了多少？

學生 1： $\frac{1}{2} \times 10 = 5$ 瓶蓋的量。

學生 2： $\frac{1}{4}$ 瓶漱口水。

老師： 多少個蓋子的漱口水等同於一瓶漱口水？

學生： $5 \div (1 - \frac{3}{4}) = 5 \times 4 = 20$。

老師： 漱口十次後還剩下多少漱口水？

學生： $20 - 5 = 15$。

老師： 溫妮還能再漱口幾次，才會用完漱口水？

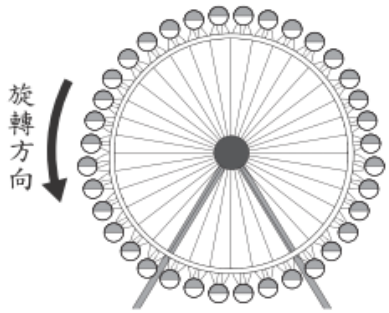
學生： $15 \div \frac{1}{3} = 15 \times 3 = 45$。

例題二

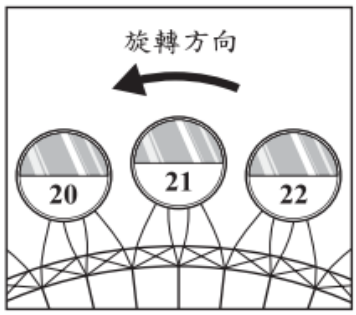
說明：本題的分數運算包含轉動**一個**車廂所需時間與轉動**數個**車廂所需時間。教學時要考慮車廂間隔數的計算方式、順時針與逆時針的判斷。必要時，可以土法煉鋼，一個車廂接一個車廂數數，先由少數車廂開始計算，再循序漸進增加車廂數的計算。

（英文）A Ferris wheel carries 36 equally-spaced cabins. The cabins are numbered from 1 to 36 clockwise. It takes 30 minutes for a complete Ferris wheel ride as it rotates counterclockwise. How many minutes does it take for the ninth cabin to be on the top after the 21st cabin passes the top of the Ferris wheel?

（中文）圖(十二) 的摩天輪上以等間隔的方式設置 36 個車廂，車廂依順時針方向分別編號為 1 號到 36 號，且摩天輪運行時以逆時針方向等速旋轉，旋轉一圈花費 30 分鐘。若圖(十三) 表示 21 號車廂運行到最高點的情形，則此時經過多少分鐘後，9 號車廂才會運行到最高點？



圖（十二）



圖（十三）

（102 年試辦會考第 6 題）

Teacher: An interval between two things is a space between them. How many intervals are there between Cabin 21 and Cabin 36 clockwise?

Student: $36 - 21 = 15$.

Teacher: How many intervals are there between Cabin 36 and Cabin 9 clockwise?

Student: $1 + (9 - 1) = 9$.

Teacher: In all, how many intervals are there between Cabin 21 and Cabin 9 clockwise?

Student: $15 + 9 = 24$.

Teacher: On the top of the Ferris wheel, how much time does it take to spin one cabin to another?

Student: $30 \div 36 = \frac{5}{6}$ minute.

Teacher: Now the 21st cabin is passing the highest point. How much time does it take for the ninth cabin to be on the top after the 21st cabin passes the top of the Ferris wheel?

Student: $\frac{5}{6} \times 24 = 20$ minutes

老師： 以順時針方向計算，第 21 號車廂和第 36 號車廂之間有多少個間隔？

學生： $36 - 21 = 15$。

老師： 以順時針方向計算，第 36 號車廂和第 9 號車廂之間有多少個間隔？

學生： $1 + (9 - 1) = 9$。

老師： 順時針數下來第 21 號車廂和第 9 號車廂之間總共有多少個間隔？

學生： $15 + 9 = 24$。

老師： 在摩天輪頂端，一個車廂轉到另一個車廂需要多長時間？

學生： $30 \div 36 = \frac{5}{6}$ 分鐘。

老師： 現在第 21 號車廂正通過最高點。21 號車廂通過摩天輪頂點後過多久，第 9 號車廂才到達頂部？

學生： $\frac{5}{6} \times 24 = 20$ 分鐘。

例題三

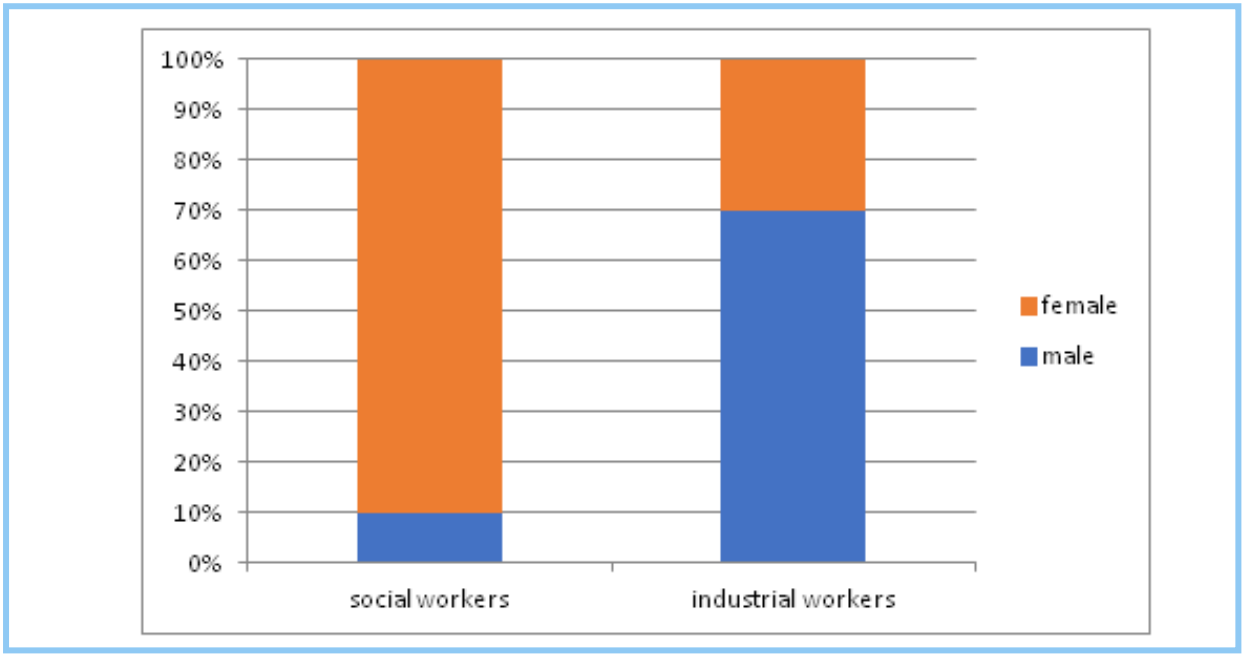
說明：本題除了分數的運算，另包含圖表的識讀。堆疊長條圖因為以比例呈現，所以兩者高度一樣，但是兩者的人數是不一樣的。學生藉由堆疊長條圖上顏色組成比例得知在各類別中男女人數比例，先求出產業外勞的人口數，再回算台灣的總人口數。老師也可以提醒學生注意英文表達三十分之一(thirtieth)和十三分之一(thirteenth)的差別。必要時，可以繪圖輔助教學。

<table>
<tr><td rowspan="2">Female
social
workers</td><td>Female industrial workers</td></tr>
<tr><td rowspan="2">Male industrial workers</td></tr>
<tr><td>Male social workers</td></tr>
</table>

（英文）Since the 1990s, most migrant workers in Taiwan have come from Indonesia, Vietnam, the Philippines, Thailand, and Cambodia. According to demographic statistics collected in September 2019, migrant workers make up one-thirtieth of the population in Taiwan. Migrant laborers in Taiwan can be categorized into two types: social workers and industrial workers. The former makes up one-third and the latter makes up two-thirds. The segmented bar chart below shows the distribution of genders in different categories of workers. For male workers, if 33.8 thousand more workers work in the industry than in social welfare, what is the population of migrant workers? What is the population of Taiwan?

（中文）自西元 1990 年起，台灣境內的外籍移工多數來自印尼、越南、菲律賓、泰國與緬甸。根據 2019 年 9 月的一項人口統計調查，外籍移工約占台灣人口的三十分之一。台灣的外籍移工主要可區分為兩類，其一為社福勞工，另一則為工業勞工。前者約佔外籍移工總人口數的三分之一，後者占三分之二。

以下的堆疊長條圖顯示各類移工的男女比例。如果男性工業勞工比男性社福勞工多 338,000 人，台灣境內外籍移工的人口數為何？台灣總人口數又為何？



Teacher: Taking all the migrant workers into account, what is the difference between the percentages of male industrial and male social workers? Is it 70% minus 10%? Explain.

Student: Male industrial workers make up 70% of all industrial workers, not 70% of all migrant workers. Similarly, male social workers make up 10% of all social workers, not 10% of all migrant workers. Therefore, we are not supposed to subtract the percentages directly.

Teacher: According to the segmented bar chart, we know that social workers make up one-third of the migrant workers, and one-tenth of all social workers are male social workers. What is the ratio of male social workers to all migrant workers?

Student: Male social workers: $\frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$.

Teacher: According to the segmented bar chart, we know that industrial workers make up two-thirds of the migrant workers and male industrial workers make up seven-tenths of social workers. What is the ratio of male industrial workers to all migrant workers?

Student: For male industrial workers it's: $\frac{2}{3} \times \frac{7}{10} = \frac{14}{30}$

Teacher: What is the difference between the percentages?

Student: $\frac{14}{30} - \frac{1}{30} = \frac{13}{30}$.

Teacher: What does this percentage represent?

Teacher: It represents the ratio of male migrant workers to all migrant workers.

Teacher: What is the population of migrant workers?

Student: $338{,}000 \div \frac{13}{30} = 338{,}000 \times \frac{30}{13} = 780{,}000.$

Teacher: What is the population of Taiwan?

Student: $780{,}000 \div \frac{1}{30} = 780{,}000 \times 30 = 23{,}400{,}000.$

老師： 就全數移工而言，男性工業勞工和男性社福勞工之間的百分比差異是多少？是70%減去10%嗎？解釋一下。

學生： 男性工業勞工佔所有工業勞工的70%，而不是所有移工的70%。同樣地，男性社福勞工佔所有社福勞工的10%，而不是所有移工的10%。因此，我們不能直接將百分比相減。

老師： 根據分段長條圖，我們知道社福勞工佔總移工的三分之一，男性社福勞工佔社福勞工總人口的十分之一。男性社福勞工佔移工總人口的多少？

學生： 男性社福勞工比例為 $\frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$。

老師： 根據分段長條圖，我們知道工業勞工佔所有移工的三分之二，男性工業勞工佔所有工業勞工的七分之一。男性工業勞工與所有移工的比例是多少？

學生： 男性工業勞工比例是 $\frac{2}{3} \times \frac{7}{10} = \frac{14}{30}$。

老師： 男性工業勞工和男性社福勞工的百分比差距是多少？

學生： $\frac{14}{30} - \frac{1}{30} = \frac{13}{30}$。

學生： 它代表男性移工在總移工人口中的比例。

老師： 移工人口有多少？

學生： $338{,}000 \div \frac{13}{30} = 338{,}000 \times \frac{30}{13} = 780{,}000$。

老師： 那麼求出台灣人口有多少？

學生： $780{,}000 \div \frac{1}{30} = 780{,}000 \times 30 = 23{,}400{,}000$人。

單元八 指數律
Properties of Exponents

國立新竹科學園區實驗高級中等學校　周慧蓮老師

■ 前言 Introduction

此單元教授數的指數運算，除了各種指數律的靈活應用，老師可以提醒學生「若有偶數個負數相乘，則其乘積為正數，若有奇數個負數相乘，則其乘積為負數」。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
exponent	**指數；冪**	**power**	**冪，次方**
base	**底數；底邊**	superscript	上標；標在某字右上角
cube	**立方；三次方**	**square**	**平方，二次冪**
cubed	**使成立方**	**squared**	**使成平方**

Order of Operations

Evaluate $10^2 \div (15 \div 3) - 4(3 - 9) + 5^3$

First	Parentheses	$10^2 \div (5) - 4(-6) + 5^3$
Second	Exponents	$100 \div (5) - 4(-6) + 125$
Third	Multiplication and Division (from left to right)	$20 + 24 + 125$
Fourth	Addition and Subtraction (from left to right)	169

Properties of Exponents

Let a and b be nonzero numbers, and let m and n be integers.

Zero Exponent	$a^0 = 1$
a to the power of zero is 1. The power 0^0 is undefined.	
Negative Exponents	$a^{-n} = \frac{1}{a^n}$
a to the power of negative n is the reciprocal of a to the power of n.	
Product of Powers	$a^m \times a^n = a^{m+n}$
Add their exponents when multiplying powers with the same base, a.	
Quotient of Powers	$a^m \div a^n = a^{m-n}$
Subtract their exponents when dividing powers with the same base, **a**.	
Power of a Power	$(a^m)^n = a^{m \times n}$
Multiply the exponents when finding the power of a power.	
Power of a Product	$(a \times b)^n = a^n \times b^n$
Find the power of each factor and then multiply when finding the power of a product.	
Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
Find the power of the numerator and the power of the denominator and then divide when finding the power of a quotient.	

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ A is **the square/cube of** B. (**The square/cube of** B is A)

例句(1)：81 is **the square of** 9.

八十一是九的平方。

例句(2)：**The cube of** 4 is 64.

四的三次方是六十四。

❷ B **squared/cubed** is A.

例句(1)：9 **squared** is 81.

九的平方是八十一。

例句(2)：4 **cubed** equals 64.

四的立方等於六十四。

❸ **Keep the base and add/subtract the exponents.**

例句(1)：To simplify $3^5 \times 3^2$, we **keep the base and add the exponents.** Then we get 3^7.

要化簡 $3^5 \times 3^2$ 時，我們保留底數並把指數相加。

例句(2)：To simplify $3^5 \div 3^2$, we **keep the base and subtract the exponents.** Then we get 3^3.

要化簡 $3^5 \div 3^2$ 時，我們保留底數並把指數相減。

❹ **Write ____ as _____.**

例句(1)：Simplifying $3^5 \times 3^2$ means **writing** the product of the two powers **as** a single power.

要化簡 $3^5 \times 3^2$ 時，把兩個次方的乘積寫成一個次方。

例句(2)：I would **write** $(2^5)^3$ **as** $2^5 \times 2^5 \times 2^5$.

我會把二的五次方的立方寫成二的五次方自乘三次。

❺ Raise *a* to the *n*th power. (Raise *a* to the power of *n*.)

例句(1)：We can **raise any number to the fourth power** by squaring it twice.

把任何一個數平方兩次就變成它的四次方。

例句(2)：**Raise 2 to the third power**. (**Raise 2 to the power of 3.**)

使二自乘三次；二的三次方。

❻ The negative sign is (not) part of the base.

例句(1)：-3^2 (read as the opposite of three squared) equals negative nine because **the negative sign is not part of the base.**

-3^2 等於負九，因為負號並不是底數的一部分。

例句(2)：$(-3)^2$(read as the square of negative three) equals nine because **the negative sign is part of the base.**

$(-3)^2$ 等於九，因為負號是底數的一部分。

❼ How many times do(es) _____ go into ____?

例句(1)：**How many times do** 2 **go into** 8?

八是二的幾倍？

例句(2)：**How many times does** one-third **go into** two?

二是三分之一的幾倍？

❽ _____ has an exponent of _____.

例句(1)：A number written without an exponent actually **has an exponent of** 1.

任何一個沒有出示指數的數字，它的指數其實是一。

例句(2)：Any nonzero number that **has an exponent of** zero is always equal to one.

任何非零的數字次方為零時，它的值永遠等於一。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

In this section, we will apply the properties of exponents to evaluate various expressions. Students have to familiarize themselves with the order of operations.

❧ 運算問題的講解 ☙

例題一

說明：此題為指數的基本運算，主要讓學生熟悉指數律的應用。

（英文）Supply the missing information. Write your answer on the space provided.

（中文）找出括弧中的解。

$9^5 \times (-3)^5 = (\quad)^5$

Teacher: In this expression, which one is the same, base or exponent?
Student: Both of the exponents are five.
Teacher: Correct. We can apply the Power of a Product Property since the exponents are the same.
Therefore, we keep the exponent and multiply the bases. What is the answer?
Student: Nine times negative three equals negative twenty-seven. The base is -27.
Teacher: Great! The answer to this expression is negative twenty-seven to the power of 5.
The space is negative twenty-seven.

老師： 這個算式中，底數或指數是相同的？
學生： 指數都是 5。
老師： 正確。由於指數相同，我們可以應用 $(a \times b)^n = a^n \times b^n$。因此，指數不變，底數相乘。答案為何？
學生： $9\times -3 = -27$。底數是-27。
老師： 很好！
這個算式的答案是$(-27)^5$，空格為-27。

例題二

說明：此題為指數的基本運算，在底數與指數均加入了負號，強化學生對於負號的認知。

（英文）Evaluate $(-2)^{-4}$.

（中文）計算 $(-2)^{-4}$。

Teacher: We can write the negative exponent as a positive exponent by rewriting the base as its reciprocal.

Therefore, the expression becomes $(-\frac{1}{2})^{4}$. Any question about this part?

Student: Double negatives make a positive.

I think we can write it as 2^4.

Teacher: The product of two negatives is positive.

In this problem, the negative sign is inside the parentheses. So it means that the negative sign is part of the base.

Here, we do not multiply -2 by -4. Therefore, it cannot be written as 2^4.

Can anyone tell me the value of $(-\frac{1}{2})^{4}$?

Student: We multiply negative one half for 4 times. It becomes one over sixteen.

Teacher: Correct.

老師：如果要把負指數變成正指數，我們可以先把底數改成它的倒數，變成$(-\frac{1}{2})^{4}$。這部分有問題嗎？

學生：負負得正。我認為題目的 $(-2)^{-4}$ 可以寫成 2^4。

老師：兩個負數的積是正數。但這題的負號在括號裡，意思負號是底數的一部分。在這裡，我們不會將 -2 跟 -4 相乘。因此不能寫成 2^4。

有人能告訴我 $(-\frac{1}{2})^{4}$ 的值嗎？

學生：把 $-\frac{1}{2}$ 乘 4 次，變成 $\frac{1}{16}$。

老師：正確。

例題三

說明：此題是比較含有乘方的分數大小，學生可以先利用乘方的奇偶來判斷值的正負。

（英文）Order $(-\frac{7}{9})^3$, $(\frac{7}{9})^3$ and $(-\frac{7}{9})^2$ from least to greatest.

（中文）將 $(-\frac{7}{9})^3$、$(\frac{7}{9})^3$、$(-\frac{7}{9})^2$ 由小到大排序。

Teacher: For convenience, let's name the cube of negative seven-ninths A, the cube of seven-ninths B, and the square of negative seven-ninths C.
Before we order these numbers, can you tell which number is negative?
Which one is positive?

Student: A is negative. B and C are positive.

Teacher: Since A is negative, it is the least number.
The square of negative seven-ninths is the same as seven-ninths squared.
How to compare the magnitude of seven-ninths cubed and seven-ninths squared?

Student: Since the base, seven-ninths, is less than one, the greater the exponent, the smaller the value. Therefore, seven-ninths cubed is smaller than seven-ninths squared.

Teacher: Correct! Let's make a conclusion.
A is smaller than B, and B is smaller than C.

老師：為了方便，我們把$(-\frac{7}{9})^3$命名為 A、$(\frac{7}{9})^3$命名為 B，$(-\frac{7}{9})^2$命名為 C。
在排列數字之前，能否分辨何者為正、何者為負？

學生：A 是負數，B 和 C 是正數。

老師：由於 A 是負數，它是最小的數字。而 $(-\frac{7}{9})^2$可以看作是 $(\frac{7}{9})^2$。
如何比較 $(\frac{7}{9})^3$ 和 $(\frac{7}{9})^2$ 的大小？

學生：由於底數 $\frac{7}{9}$ 小於 1，指數越大，數值越小。因此，$(\frac{7}{9})^3$小於$(\frac{7}{9})^2$。

老師：正確！結論：A 比 B 小，B 比 C 小。

例題四

說明：此題為分數乘除搭配指數的混合運算，教師可以教導學生先利用乘方的奇偶來判斷值的正負。面對分數的除法時，也可以改成倒數的乘法。

（英文）Evaluate $(-\frac{1}{2})^2 \div (\frac{2}{3})^3 \times (-\frac{3}{4})^5$. Leave your answer in terms of powers.

（中文）求 $(-\frac{1}{2})^2 \div (\frac{2}{3})^3 \times (-\frac{3}{4})^5$ 之解，並以指數形式表達。

Teacher: Let's figure out the sign of the values first. Which term can be converted to positive?

Student: The square of negative one-half is positive.

Teacher: Correct.

The expression becomes a half squared divided by two-thirds cubed and then multiplied by negative three-fourths to the power of five.

$(\frac{1}{2})^2 \div (\frac{2}{3})^3 \times (-\frac{3}{4})^5$.

Teacher: We do the multiplication and division from left to right.

Dividing a number by a fraction is the same as multiplying this number by the reciprocal of the fraction.

Therefore, the expression becomes a half squared multiplied by three halves cubed and then multiplied by negative three-fourths to the power of five.

$(\frac{1}{2})^2 \times (\frac{3}{2})^3 \times (-\frac{3}{4})^5$

Then, we can apply the Power of a Quotient Property.

What is the new expression?

Student: One over two squared times three cubed over two cubed, and then times negative three to the power of five over four to the power of five.

$\frac{1}{2^2} \times \frac{3^3}{2^3} \times (-\frac{3^5}{4^5})$

Student: We can write four to the fifth power as two to tenth.

$$\frac{1}{2^2} \times \frac{3^3}{2^3} \times (-\frac{3^5}{2^{10}})$$

Teacher: We then apply the product of powers. What is the simplified answer?

Student: Keep the base and add the exponents. The final answer is negative three to the power of eight over two to the power of fifteen. $-\frac{3^8}{2^{15}}$

Teacher: Correct.

老師： 首先，讓我們找出數值的符號。哪一項可以轉換為正數？

學生： $(-\frac{1}{2})^2$ 是正數。

老師： 正確。算式變為 $(\frac{1}{2})^2 \div (\frac{2}{3})^3 \times (-\frac{3}{4})^5$。

老師： 我們從左到右進行乘除。將一個數字除以一個分數等於乘以這個分數的倒數。

因此，該算式變為 $(\frac{1}{2})^2 \times (\frac{3}{2})^3 \times (-\frac{3}{4})^5$

然後，我們可以應用 $(\frac{a}{b})^n = \frac{a^n}{b^n}$。接下來怎麼列式？

學生： $\frac{1}{2^2} \times \frac{3^3}{2^3} \times (-\frac{3^5}{4^5})$。

學生： 我們可以將 4^5 寫成 2^{10}。

$\frac{1}{2^2} \times \frac{3^3}{2^3} \times (-\frac{3^5}{2^{10}})$。

老師： 然後，我們應用 $a^m \times a^n = a^{m+n}$。化簡後的答案是什麼？

學生： 保留底數並加總指數。最後答案是 $-\frac{3^8}{2^{15}}$。

老師： 沒錯！

∞ 應用問題 / 會考素養題 ∞

例題一

說明：此題主要為指數除法的應用。

（英文）How many times does 5^3 go into 5^6?
（中文）5^6是 5^3 的多少倍？　（110 年國中會考第 5 題）

Teacher: To find the multiple, we use the division. Therefore, we have to evaluate five to the power of six divided by five cubed.
Anyone knows the answer?
Student: We apply the Division of Powers Property. Keep the base and subtract the exponents.
It becomes five to the power of three. It is one hundred twenty-five.
Teacher: Correct.

老師：求倍數，我們可以使用除法。因此，我們必須計算 5^6 除以5^3。
有人知道答案嗎？
學生：我們應用 $a^m \div a^n = a^{m-n}$。保留底數並且指數相減，答案是 5^3，等於 125。
老師：答對了。

例題二

說明：本題雖然是指數的應用，也涉及科學記號的表示。老師猶有餘力時，可以向學生說明英文字對應的數字關係。One thousand equals 10^3. One million equals 10^6. One billion equals 10^9. One trillion equals 10^{12}.

（英文）On August fifteenth in the year of 2017, DaTan Power Plant had a power shortage, lacking four point three million kilowatts of electronic power. As a result, there was a power outage in many places around Taiwan. Given that one kilowatt equals one thousand watts, how many watts of electronic power equals 4.3 million kilowatts?

（中文）民國 106 年 8 月 15 日，大潭發電廠因跳電導致供電短少約 430 萬瓩，造成全臺灣多處地方停電。已知 1 瓩等於 1 千瓦，求 430 萬瓩等於多少瓦？
（108 年國中會考第 6 題）

Teacher: The mathematical expression of four point three million is 4,300,000.
(While explaining, the teacher writes "4,300,000" on the blackboard.)
To write the expression in scientific notation, move the decimal point to the left until there is only one non-zero digit to the left of the decimal point. What is the scientific notation for 4,300,000?

Student: The first non-zero digit form the left is 4, so we have to move the decimal point six places to the left. The scientific notation is four point three times ten to the power of six (4.3×10^6).

Teacher: One kilowatt equals one thousand watts. Four point three million kilowatts equals four point three million times one thousand.
We can write the expression as $4.3 \times 10^6 \times 10^3$.
What is the simplest form?

Student: Four point three times ten to the power of nine. (4.3×10^9)

Teacher: Correct. By applying the Product of Powers Property, we keep the base, 10, and add the exponents.

老師： 四百三十萬的數學表示法是 "4,300,000"。
（老師一邊講解，一邊在黑板上寫下"4,300,000"。）要將這個數寫成科學記號，我們得將小數點移到左邊，使得數字剩下一個非零整數和小數部分。

學生： 從左邊數來的第一個非零數字是 4，因此我們必須將小數點向左移動 6 位。該數的科學計數法表示為「四點三乘以十的六次方」，4.3×10^6。

老師： 一瓩等於一千瓦特。四百三十萬千瓦等於四百三十萬乘以一千。
可以寫成 $4.3 \times 10^6 \times 10^3$，化簡後是什麼？

學生： 4.3×10^9。

老師： 正確。利用指數律 $a^m \times a^n = a^{m+n}$，我們保留底數 10，指數相加。

例題三

說明：此題為指數的應用，也包含等比數列的概念。教師可以請學生拿出紙張進行紙張對折的操作，試試看最多可以將紙張對折幾次。教師亦可以適當引導學生使用電子計算機計算，進一步教育學生指數可以應用於數值較大或較小的數。

（英文）A sheet of paper is about 0.1 millimeter thick.

(a.) How thick will it be when you fold it in half once? twice? three times? (Leave your answer in terms of powers.)

(b.) If it were possible to fold the paper in half 10 times, would it be taller than you?

(c.) How many times is the thickness of a sheet of paper folded 16 times more than 6 times?

（中文）一張紙的厚度為 0.1 毫米。

(a.) 將紙張對折一次後，厚度為？對折兩次後，厚度為？對折三次後，厚度為？

(b.) 若你能夠將紙張對折 10 次，紙張的厚度會比你的身高還高嗎？

(c.) 紙張對折 16 次後的厚度，相較於紙張對折 6 次後的厚度的幾倍？

Teacher: Now, everyone has a sheet of A4-sized paper. Please fold it in half. The thickness would be 0.1×2. (read as zero point one times two.)
Any question on this part?

Student: No.

Teacher: Fold your paper again. When the paper is folded twice, any of you knows how to express the thickness in terms of the exponent of 2?

Student: Multiply the original thickness by 2. $0.1 \times 2 \times 2$, it would be 0.1×2^2. (read as zero point one times two squared.)

Teacher: In the same logic, any of you knows how to express the thickness in terms of the exponent of 2 when the paper is folded three times?

Student: It would be 0.1×2^3. (read as zero point one times two cubed.)

Teacher: For part b of this problem, if it were possible to fold the paper in half 10 times, how thick is it? Would it be greater than your height?
Explain your reasoning.

Student: The thickness is 0.1×2^{10}. Two to the tenth is 1,024 (One thousand twenty-four). It is one hundred two point four centimeters. It is shorter than my height.

Teacher: Correct!
We actually can adopt 1,000 to estimate 2^{10}. When the sheet of paper is folded ten

times, the thickness is about 100 centimeters. It ought to be less than your height.

Teacher: For part c of this problem, how to express the thickness in terms of the exponent of 2 when the paper is folded 16 times? 6 times?

Student: One is 0.1×2^{16} and the other one is 0.1×2^{6}.

Teacher: Correct.

If we want to know the number of times, it means we have to calculate the multiple. We apply the division.

What is 0.1×2^{16} divided by 0.1×2^{6} ?

Student: We can apply the Quotient of a Power Property. It becomes 2^{10}.

Teacher: Very good. Your answer is correct.

老師： 現在，每個人都有一張 A4 大小的紙。請將其對折。厚度是0.1×2。這部分有任何問題嗎？

學生： 沒有。

老師： 再次將紙對折。當紙被折兩次時，有人知道如何用 2 的指數表示厚度嗎？

學生： 把原來的厚度乘以二，$0.1 \times 2 \times 2=0.1 \times 2^{2}$。

老師： 以相同的邏輯，當紙張被折三次時，有人知道如何用 2 的指數表示厚度嗎？

學生： 0.1×2^{3}。

老師： 接著看(b)，如果能夠將紙張對折 10 次，紙張的厚度會比你的身高還高嗎？解釋你的理由。

學生： 厚度是 0.1×2^{10}。2 的十次方是 1,024。它是 102.4 公分。它比我的身高低。

老師： 正確！我們實際上可以使用 1000 來估計2^{10}。當紙張被折十次時，厚度大約為 100 公分。這應該比你的身高低。

接著第(c)小題，當紙張對折 16 次時，如何用 2 的指數表示厚度？6 次呢？

學生： 16 次是 0.1×2^{16}，6 次是 0.1×2^{6}。

老師： 正確。如果我們想知道次數，代表我們必須計算倍數，這時候就要用除法。0.1×2^{16} 除以 0.1×2^{6} 等於多少？

學生： 我們可以應用 $a^m \div a^n = a^{m-n}$，答案是 2^{10}。

老師： 非常好。你的答案是正確的。

單元九 一元一次式的運算
Operation of Linear Equations with One Variable

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■ 前言 Introduction

以英語教數學的技巧，首先建議老師務必將所有算式寫於黑板上，讓學生即使某單字或用語不熟悉的情況下，仍能掌握老師之教授內容。其次，建議老師們於本節上課前先說明分數的英語說法與中文的說法不同之處，並將數學用語之英文字彙先介紹給學生，上課解說時放慢進度，並讓學生能有時間於每個例題做完時，做 1~2 題課堂練習，以確認學生是否瞭解上課內容。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
algebraic expression	**代數式**	unknown variable	未知數
real-life problems	應用問題	**reciprocal**	**倒數**
variable	**一元**	**like terms**	**同類項**
power (first power)	**次**（一次）	parentheses	圓括號（）
term	項	square brackets	方括號 [ ]
coefficient	**係數**	commutative property	交換律
constant	常數項	associative property	結合律
symbol	符號	**distributive** property	分配律

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ An _____ has _____.

例句：**An** equilateral triangle **has** three congruent angles. An isosceles triangle has two congruent angles.

等邊三角形有三個相等的內角，而等腰三角形有兩個相等的內角。

❷ _____ represents/stands _____.

例句：A variable is a letter. It **represents/stands** a number or quantity whose value may change.

變數是一種符號，代表一個會改變的數或量。

❸ Assume_____.

例句：**Assume** that the price of one pineapple cake is x dollars.

假設一塊鳳梨酥的價錢是 x 元。

❹ How do we _____?

例句：**How do we** simplify this algebraic expression?

如何簡化此代數式？

❺ What is the product of _____ and _____?

例句：**What is the product of** (-4) **and** (-2.5)?

-4 乘以 -2.5 的積是多少？

❻ Apply _____ to _____.

例句：We will learn to **apply** the distributive property and commutative property **to** the next questions.

我們將會學到將分配律和交換律應用於接下來的題目。

❼ _____ are made up of _____.

例句：Algebraic expressions **are made up of** terms.

代數式是由項所組成。

❽ _____ be able to _____.

例句：You will **be able to** translate words into algebraic expressions.

你將有能力把文字敘述轉成代數式。

❾ _____ is equivalent to _____.

例句：Which of the following **is equivalent to** the result?

下列哪一項和答案相等？

❿ Which of the following _____ is _____?

例句：**Which of the following** statements **is** true?

下列哪一項敘述為真？

⓫ _____ remember/know/get _____?

例句：Does anyone **remember** what the distributive property is?

誰記得分配律？

■ 問題講解 Explanation of Problems

☙ 說明 ❧

Algebraic expressions are made up of terms. A term is a constant or the product of a constant and one or more variables. The constant that multiplies the variable(s) in a term is called the coefficient. The coefficient works as the number in front of the variable. For example, the coefficient of the term $3x$ is 3.

In this lesson, we will talk about the algebraic expression $ax + b$ (一元一次代數式).
In $ax + b$, a is the coefficient of x, and b is the constant. Among the algebraic expressions, $20x + 3$, $-2.5x + 100$, $\frac{1}{3}x - 10$, and $43x$ only have one variable. And the highest power of these terms is 1.

By the end of this section, you will be able to:

1. evaluate algebraic expressions;
2. identify terms, coefficients, and like terms;
3. simplify expressions by combining (putting together) like terms; and
4. translate word phrases into algebraic expressions.

代數式是由一項或多項所組成。每一項可以是一個數字（稱為常數）或數與變數的積（例如$3x$，其中 3 稱為一次項的係數）。本節將討論一元一次代數式的運算及如何將應用問題轉化為代數式。

運算問題的講解

例題一

說明：運用第一章學過的數的運算規律，例如結合律、交換律及分配律等仍可以運用於一元一次式。

（英文）Simplify the following algebraic expressions.
（中文）化簡下列各式：

(1) $(-\frac{2}{3}) \times 9x$　　(2) $35x \times (-0.4)$

Teacher: Let's do the following questions. The first example is to simplify $(-\frac{2}{3}) \times 9x$ (read as: negative two-thirds times nine x, or simplify the product of negative two-thirds and nine x.)

Here we can use the rules of operations. For example, we can use the associative property, commutative property, and distributive property. We can apply these properties to algebraic expressions.

So, for $(-\frac{2}{3}) \times 9x$, we can change it to $(-\frac{2}{3}) \times 9 \times x$. Next, get the product of $(-\frac{2}{3}) \times 9$, and then times it by x. What is the product of $(-\frac{2}{3}) \times 9$?

Student: -6.

Teacher: Great! Then we time it by x, and the result is $-6x$.

Who remembers the property of changing $(-\frac{2}{3}) \times 9x$ to $(-\frac{2}{3}) \times 9 \times x$?

Student: Commutative property.

Teacher: Good try, but that's not the answer. Check the properties that you learned in Chapter One.

Student: Associative property.

Teacher: Great! Now try another question $35x \times (-0.4)$. Please remember that you can change $x \times (-0.4)$ to $(-0.4) \times x$ with the commutative property.

(In two minutes)

Who can tell us the answer? Raise your hand.

(Pick one student)

What did you get for this question?

Student: $-14x$.

Teacher: Right on! How many of you got the same answer? Raise your hand.

(After students raise their hands.)

Good. Put your hands down. Let's move on to the next topic.

老師： 現在來解下面的題目。第一小題是化簡 $(-\frac{2}{3}) \times 9x$。可以運用規則來運算，如結合律、交換律和分配律等。這些特性可以用於下列算式。

因此，對於 $(-\frac{2}{3}) \times 9x$，我們可以將其改為 $(-\frac{2}{3}) \times 9 \times x$。接下來，取$(-\frac{2}{3})$和 9 的乘積，然後乘以$x$。$(-\frac{2}{3}) \times 9$是多少？

學生： 是 -6。

老師： 很好！然後我們乘以 x，答案是$-6x$。

有誰記得將$(-\frac{2}{3}) \times 9x$ 寫成 $(-\frac{2}{3}) \times 9 \times x$ 是什麼運算規則？

學生： 交換律。

老師： 不錯的嘗試，但是沒答對喔。再回想一下我們第一章學到的內容。

學生： 是結合律。

老師： 很棒！現在試試第二小題 $35x \times (-0.4)$。

記得，你們可以使用交換律將 $x \times (-0.4)$ 改為 $(-0.4) \times x$

（兩分鐘後）有答案的人請舉手。

（點一名學生）答案是多少呢？

學生： 是 $-14x$。

老師： 正確！有誰得到相同的答案？請舉手。

（學生舉手後）很好，手放下，我們接下來看下一部份。

例題二

說明：老師接下來指導學生分配律和交換律在一元一次式的運算。我們在前面章節學過「除以一個數」其運算結果等於「乘以這個數的倒數」。接下來的步驟和前一題類似，運用分配律和結合律，讓學生用兩分鐘的時間求出這一題的答案。

（英文）Simplify the following expressions:
（中文）化簡下列各式

(1) $-5(2x-8)$　　(2) $(-4x+2)\div(-\frac{2}{5})$

Teacher: We will learn to apply the distributive property and commutative property to the next questions. The first one is "negative five times (two x minus eight)".
To remove the parentheses/round brackets, you need to use the distributive property. So, distribute or break down "negative five" in both "two x and negative eight". Then you'll get "negative five times two x" and "negative five times negative eight". So, what's the answer?

Student: Negative ten x plus forty.

Teacher: Great! The next question is $(-4x+2)\div(-\frac{2}{5})$. (Read as: Negative four x plus two, then divided by negative two-fifths.)
"Divided by a number" is the same as "multiplied by the reciprocal of that number."
What is the reciprocal of negative two-fifths?

Student: Negative five over two.

Teacher: Good. So, the next step is like what we saw in the previous question.
$(-4x+2)\times(-\frac{5}{2})$ (Read as: Negative four x plus two, and then multiplied by/times negative five over two.)
Then use the distributive property and commutative property. Now, please finish the rest. I will check if you get it right in two minutes.

老師：我們將學到如何應用分配律和交換律來解決下一題。
第一小題是$-5(2x-8)$。為了消除括號，你需要使用分配律。所以，把-5乘進去括號中的$2x$和-8。然後，你會得到$(-5)\times 2x$和$(-5)\times(-8)$。那麼答案是什麼呢？

學生：$-10x+40$。

老師： 不錯喔！第二小題是$(-4x+2)\div(-\frac{2}{5})$。記得「除以一數」相當於「乘以該數的倒數」。那$(-\frac{2}{5})$的倒數是多少？

學生： 是 $-\frac{5}{2}$。

老師： 很好。接下來就如同前一個問題的步驟。使用分配律和交換律來計算$(-4x+2)\times(-\frac{5}{2})$。現在，請完成剩下的部分。兩分鐘後我來看你們的答案對不對。

∞ 應用問題 / 會考素養題 ∞

例題一

（英文）The price of a visit to a dentist is $200. If the dentist fills any cavities, one has to pay an additional charge of $50 per cavity. If the dentist finds n cavities, what's the cost (, including the price of the visit)? Tommy has 5 cavities to be filled. Find the total cost for him.

（中文）牙醫診所每次掛號費為 200 元，補蛀牙每顆部分負擔為 50 元。請問牙醫補 *n* 顆牙齒的費用（含掛號費）為何？如果湯米有 5 顆蛀牙需要補，請問共需付多少錢？

Teacher: In this problem, you need to spend 200 dollars on a visit to the dentist. So, you still have to pay 200 dollars even if the dentist doesn't find any cavities. If there is one cavity, the extra charge is 50 dollars. If there are two cavities, the extra charge is 50 times two (50×2) dollars. If there are n cavities, then what is the extra charge?

Student: $50\times n$.

Teacher: Great! Now, how much is the total cost if it includes the fee of the visit?

Student: $200+50n$.

Teacher: Correct! Now Tommy has 5 cavities. That means *n* equals five. So, you need to put 5 where *n* is. Find the cost.

Student: 450 dollars.

Teacher: Great! Let's see the next question.

老師：如題所示，牙醫診所每次掛號費為 200 元，代表就算你沒有蛀牙，也需要付 200 元。如果有一顆蛀牙，需要再付 50 元補蛀牙；如果有兩顆蛀牙，額外費用是 100（50×2）元。如果有 n 顆蛀牙，那麼額外費用是多少呢？

學生：$50\times n$。

老師：很好！現在，如果包括掛號費，總費用是多少呢？

學生：$200+50n$。

老師：正確！現在湯米有 5 顆蛀牙需要補，也就是 n 等於 5，所以要將 n 代換成 5。算算看是多少。

學生：450 元。

老師：很棒！現在我們來看下一個問題。

例題二

說明：老師用 $2x-3$ 和 $3x+1$ 取代式中的 A 和 B，示範完如何計算 $3A+B$，讓學生練習。並簡化 $A-3B$。接下來，運用第一章簡化分數學習的通分，將 $\frac{3A+B}{2}-\frac{A-3B}{3}$ 通分後引導學生自行練習如何簡化一元一次式。

（英文）If A and B stand for the polynomials $2x-3$ and$3x+1$, then which of the following is the same as the result of $\frac{3A+B}{2}-\frac{A-3B}{3}$?

（中文）若 A、B 分別表示多項式 $2x-3$ 與 $3x+1$，則 $\frac{3A+B}{2}-\frac{A-3B}{3}$ 與下列哪一個式子相同？

(A) $\frac{13x-12}{6}$　(B) $\frac{41x-12}{6}$　(C) $\frac{13x-36}{6}$　(D) $\frac{41x-36}{6}$

（102 年試辦會考 6）

Teacher: First, let's use $2x-3$ and $3x+1$to replace A and B. Then, $3A+B$ will be $3(2x-3)$ plus $(3x+1)$.

By using the distributive property, we get $(6x-9)+(3x+1)$. Next, we add the like terms. The algebraic expression is equal to $9x-8$. In the same way, you'll find "$A-3B$" is equal to $(2x-3)-3(3x+1)$. Could you simplify $(2x-3)-3(3x+1)$?

Student: Yes, the result is $-7x-6$.

Teacher: Correct. Now, we can see that $\frac{3A+B}{2}-\frac{A-3B}{3}$ (Read as: Three A plus B over two

minus A minus 3B over three) is equal to $\frac{9x-8}{2} - \frac{-7x-6}{3}$ (Read as: nine x minus eight over two, then subtracted by negative seven x minus six over three).

Now we just simplify the two fractions by changing them to the same denominator. What is the least common multiple (LCM) of 2 and 3?

Student: 6.

Teacher: Very good. So, the fraction $\frac{9x-8}{2}$ is equal to $\frac{3(9x-8)}{6}$ and $\frac{-7x-6}{3}$ is equal to $\frac{2(-7x-6)}{6}$.

Now, please simplify $\frac{3(9x-8)}{6} - \frac{2(-7x-6)}{6}$ by yourselves.

You have three minutes. (Wait three minutes…) Time's up.

(Pick one student.) What did you get?

Student: (B) $\frac{41x-12}{6}$

Teacher: That's right! Raise your hand if you got the same answer. Well done!

老師： 首先，我們用 $2x - 3$ 和 $3x + 1$ 來取代 A 和 B。

那麼，$3A + B$ 就會是 $3(2x - 3) + (3x + 1)$。

利用分配律，我們可以得到 $(6x - 9) + (3x + 1)$。接下來，我們把同類項合併得到 $9x - 8$。同樣地，$A - 3B$ 等於 $(2x - 3) - 3(3x + 1)$。

你們會化簡 $(2x - 3) - 3(3x + 1)$ 嗎？

學生： 會，答案是 $-7x - 6$。

老師： 正確。現在，我們看到 $\frac{3A+B}{2} - \frac{A-3B}{3}$ 等於 $\frac{9x-8}{2} - \frac{-7x-6}{3}$。

現在我們只需要將這兩個分數化簡成相同的分母。那 2 和 3 的最小公倍數是什麼？

學生： 6。

老師： 非常好。所以，$\frac{9x-8}{2}$ 擴分變成 $\frac{3(9x-8)}{6}$，而 $\frac{-7x-6}{3}$ 等於 $\frac{2(-7x-6)}{6}$。

現在，請自行簡化 $\frac{3(9x-8)}{6} - \frac{2(-7x-6)}{6}$。給你們三分鐘。

（等待三分鐘...）時間到。

（選一個學生）答案是多少呢？

學生： (B) $\frac{41x-12}{6}$

老師： 答對了！答案算出來一樣的同學請舉手。很棒！

例題三

說明：老師可以利用此例來說明如何將應用問題的文字敘述轉成代數式，並運用一元一次式的運算法則。在這題應用問題中，一塊鳳梨酥的價錢為 x 元，問學生一小盒（6 個）包含盒子的價錢是多少元來引導學生思考。再提問一大盒（12 個）含盒子需多少錢？Tony 買 3 小盒及 3 大盒一共需多少錢？2 小盒及 4 大盒一共需多少錢？

（英文）Tony goes to Sunny Bakery to buy six boxes of pineapple cakes for his foreign friends. The price of each pineapple cake is NT$ x. There are two different sizes of boxes: a small box can fit six pineapple cakes inside and a large box can carry 12 pineapple cakes. If Sunny Bakery charges NT$5 for a small box and NT$8 for a large box, how much will Tony need to pay for 3 small boxes and 3 large boxes of pineapple cakes? What is the total price of 2 small boxes and 4 large boxes of pineapple cakes?

（中文）Tony 要去晴光烘焙坊買 6 盒鳳梨酥送給外國友人，一塊鳳梨酥售價 x 元（台幣）。禮盒有兩種款式：小盒每盒含 6 塊鳳梨酥，大盒每盒含 12 塊鳳梨酥。因為成本考量，包裝盒小盒需加收 5 元而大盒則為 8 元。如果 Tony 決定買 3 盒小盒鳳梨酥及 3 盒大盒鳳梨酥，共需付多少錢？如果買 2 盒小盒鳳梨酥及 4 盒大盒鳳梨酥，共需付多少錢？

Teacher: In this word problem, if each pineapple cake costs NT$ x, then what is the price of six pineapple cakes? Think about NT$5 for each small box. Then what's the total price for a small box of pineapple cakes?

Student: $6x + 5$.

Teacher: Great! In the same way, what's the cost for a large box of 12 pineapple cakes?

Student: $12x + 8$.

Teacher: Correct! Now consider how to translate the algebraic expression for the total cost if Tony wants to buy three small boxes and three large boxes of pineapple cakes.

Student: $3(6x + 5) + 3(12x + 8)$.

Teacher: If Tony changes his mind and buys 2 small boxes and 4 large boxes, then what's the total cost?

Student: $2(6x + 5) + 4(12x + 8)$.

Teacher: Now, please simplify the expressions. Remember to remove the parentheses and put together the like terms.

(In a few minutes...)

Teacher: What's your answer?

Yes, you are right. The answer is $60x + 42$.

The numbers of your homework questions are listed on the blackboard. Remember the homework is due this Friday. Class dismissed.

老師： 如題所示，每塊鳳梨酥價格是 x 元，那麼六塊鳳梨酥的價格是多少？加上每個小盒子的價格是 5 元。一盒小盒的鳳梨酥的價格是多少？

學生： $6x + 5$。

老師： 很好！同樣地，一盒有 12 個鳳梨酥的大盒鳳梨酥費用是多少？

學生： $12x + 8$。

老師： 正確！如果 Tony 想要買 3 盒小盒鳳梨酥及 3 盒大盒鳳梨酥，共需付多少錢？

學生： $3(6x + 5) + 3(12x + 8)$。

老師： 如果 Tony 改變主意，買 2 盒小盒鳳梨酥及 4 盒大盒鳳梨酥，共需付多少錢？

學生： $2(6x + 5) + 4(12x + 8)$。

老師： 現在，請化簡。記得去括號且合併同類項。

（幾分鐘後）答案是多少？

沒錯，答案是$60x + 42$。

你們的作業題號寫在黑板上了，請記得要在這個星期五前完成。下課。

單元十 一元一次方程式的列式與求解
Attributing and Solving a Linear Equation

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■ 前言 Introduction

此單元為一元一次方程式列式、檢驗某數是否為方程式的解、與利用等量公理求解。教師在教授一元一次方程式的列式，將文字敘述改為數學式的教學時，須留意英文文法的部分，尤其是以英語表達誰比誰大、又大多少的句型。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
solution	解	**unknown variable**	**未知數**
equation	方程式	**equivalent equation**	**等價方程式**
expression	表示式	linear equation in one variable	一元一次方程式
like terms	**同類項**	right-hand side of the equation	右式
check	檢驗	left-hand side of the equation	左式
in balance	等量關係	inverse operations	反運算
balance scale	天平		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ Write an equation.

例句：**Write** and solve **an equation**.

列出方程式並求解。

❷ Solve the equation _____.

例句：Please **solve the equation** for *x*.

請求出方程式中未知數 *x* 的解。

❸ Check the solution.

例句：**Check** if the given number is a solution of the equation.

檢驗該解是否為方程式的解。

❹ _____ is the solution to _____ (an equation).

例句：3 **is the solution to** $2x-6=0$.

3 是方程式 $2x-6=0$ 的解。

❺ A is ____ (a number) less than B.

例句(1)：A **is** 3 **less than** B.

A 是比 B 小 3 的數。

例句(2)：10 **less than** *d* equals two times *d*.

$d-10=2d$。

❻ A divided by B

例句：20 **divided by** 4 equals 5.

20 除以 4 等於 5。

❼ A is twice as high/large/big/great as B.

例句：8 **is twice as large as** 4.

8 是 4 的兩倍。

❽ Add ____ (a number) on each side

例句：**Add** 5 **on each side**.

兩邊同加 5。

❾ Subtract ____ (a number) from each side.

例句：**Subtract** 5 **from each side**.

兩邊同減 5。

❿ Multiply (Divide) each side by ____ (a number).

例句：**Multiply (Divide) each side by** 10, and then simplify.

兩邊同乘 10（兩邊同除以 10）後化簡。

■ 問題講解 Explanation of Problems

∞ 說明 ∞

In this section, we will learn how to list an equation with one unknown variable. We will use inverse operations to solve the equation. “Listing an equation and looking for its solutions” is essential in algebra. This is very useful in real life.

There are lots of word problems in the textbook. Before solving them, you have to deal with the first problem: How do you translate these English words into math language? How do you write the math equation and solve it with math operations?

You always assume an unknown variable, such as x, to stand for the missing value in the questions. Then use math operations to find out the value for x. The equation with the equal sign is like a balance scale. Every action you take on one side should be repeated on the other side to

keep the balance. The ultimate goal is to isolate the unknown variable. Let "x" only exist on one side, with numbers on the other side. Then you will have the value for x.

This is just a brief introduction to this section. Later we will go over more examples.

∞ 運算問題的講解 ∞

例題一

說明：老師提醒學生，看到文字敘述中的某些關鍵字，應使用適當的運算。尤其是減法時，還要特別留意區分被減數與減數。
老師解釋「2 倍的 x 等於 5 倍的 x 少 10」的列式。

（英文）Convert the following statements into equations.
（中文）將下列文字敘述列出一元一次方程式。

中文敘述	Statements	Equations (Answers)
比 x 多 3 的數是 5	3 added to a number is equal to 5.	$3 + x = 5$
比 x 少 3 的數是 5	3 subtracted from a number is equal to 5	$x - 3 = 5$
比 x 的 3 倍少 2 的數是 5	3 times a number decreased by 2 is 5.	$3x - 2 = 5$
2 倍的 x 等於 5 倍的 x 少 10	Twice the number is 10 less than 5 times the number.	$2x = 5x - 10$

Teacher: Let's convert these English statements into math equations. This translation helps us communicate in math language.
Here are some tips for you.
When you see "is", "is equal to" or "equals", that means an equal sign "=."
When you see "be added to" or "be increased by", that means addition.
When you see "be subtracted from" or "be decreased by", that means subtraction.
Please be careful with subtraction. You have to identify the minuend (被減數) and the subtrahend (減數). The minuend is the number in front of the minus sign, and the subtrahend is the one behind. The order is important.

Teacher: Take "Twice the number is 10 less than 5 times the number" as an example.

You can use x to stand for the unknown number. "Twice the number" means "two times the number", so you should write $2x$. Do you see the word "is"? This means an equal sign.

"10 less than 5 times the number means 10 less than $5x$". Is 10 the minuend or subtrahend? Should you write $10 - 5x$ or $5x - 10$?

Student: $5x - 10$

Teacher: Correct. The next step is to put everything together: $2x$ equals $5x - 10$. That is the answer!

老師：讓我們將這些英文敘述轉換成數學方程式。這樣的翻譯有助於我們用數學語言進行溝通。老師給你們一些提示。

看到 "is"、"is equal to" 或是 "equals"，代表等於「=」；

看到 "be added to" 或是 "be increased by"，代表加法；

看到 "be subtracted from" 或 "be decreased by"，代表減法。請注意減法，你必須判斷哪個是被減數，哪個是減數。被減數是減號前的數字，而減數則是減號後面的數字，順序很重要喔。

老師：以 "Twice the number is 10 less than 5 times the number" 為例。

你可以使用 x 代表未知數。Twice the number 的意思是「兩倍的 x」，因此你應該寫 $2x$，然後你看到 "is" 了嗎？"is"代表等於。

"10 less than 5 times the number" 的意思是「比 $5x$ 少 10」。問大家一下，這邊的 10 是被減數還是減數？要寫成 $10 - 5x$ 還是 $5x - 10$？

學生：$5x - 10$。

老師：正確。接著全部合在一起：$2x = 5x - 10$，就是答案！

例題二

說明：把 4 帶入方程式，檢驗 4 是否為方程式的解。

（英文）Is 4 the solution to $x + 4 = 4(x + 2)$?

（中文）4 是否為方程式 $x + 4 = 4(x + 2)$ 的解？

Teacher: 1. To check if 4 is the solution to this equation, you can substitute x for 4 and check the value for both sides.

2. The expression on the left-hand side is $x + 4$.

4 plus 4 is 8. The expression on the right-hand side is 4 times (x plus 2). 4 times 6 equals 24. 8 is not equal to 24.

Therefore, 4 is not the solution to the equation.

老師： 將 4 代入方程式，檢查兩邊的值是否相等，來確認 4 是否為此方程式的解。等號左邊為 $x+4$，4 加上 4 等於 8；等號右邊為 $4\times(x+2)$，4 乘以 6 等於 24。由於 8 不等於 24，所以 4 不是此方程式的解。

例題三

（英文）Is 3 the solution to $3x-4=\frac{1}{6}x+\frac{9}{2}$ (Read as: $3x$ minus 4 equals one sixth x plus 9 halves)? Explain your reason.

（中文）3 是否為方程式 $3x-4=\frac{1}{6}x+\frac{9}{2}$ 的解？解釋之。

Teacher: Now, let's try another exercise. Is 3 the solution to $3x-4=\frac{1}{6}x+\frac{9}{2}$?

Please explain your reason. I will give you 2 minutes to complete this question.

Student: Ok.

(The teacher circulates in the classroom and observes students' work for 2 minutes)

Teacher: Time's up. Who would like to share their answer and explain their reason?

Student: Me. 3 is the solution. When I plug in 3, I have the same answer on both sides of this equation.

Teacher: Thanks for sharing. You are correct. 3 is the solution. Let me briefly go through this question again. You can substitute x for 3. Check the answers of both sides.

3 times 3 minus 4 equals 5. You will get 5 on the left-hand side. One sixth times 3, then plus 9 halves equals 5. 5 is equal to 5. So 3 is the solution to this equation.

Well done! Everyone got the correct answer.

老師： 現在，讓我們試試這一題。3 是否為方程式 $3x-4=\frac{1}{6}x+\frac{9}{2}$ 的解？請解釋。我會給你 2 分鐘時間完成這個問題。

學生： 好的。

（老師下去巡同學做題目）

老師： 時間到。有人要分享答案順便解釋嗎？

學生： 我。答案是 3。當我代入 3 時，等式的兩側答案相同。

老師： 謝謝分享，沒錯，這題的解是 3。讓我簡要地再講一遍這個問題。

你可以代入 3 來替換 x，確認兩側的答案。3 乘以 3 減 4 等於 5，左側得到 5；

$\frac{1}{6}$ 乘以 3，然後加上 $\frac{9}{2}$ 等於 5。5 等於 5。因此，3 是這個方程式的解。

做得好！大家都答對了。

例題四

說明：教師教等量公理求方程式的解。

（英文）Solve $6x - 8 = 10$.

（中文）解方程式 $6x - 8 = 10$。

Teacher: You can use transformation to rewrite the original equation, and isolate the variable on one side of the equation. Then you will get the answer.

You can think of an equation as a balance scale. "$6x - 8$" and "10" are on the two sides of the balance scale and they are in balance. Any transformation you make must keep the equation in balance. That is, when you do something on the left, you should do the same thing on the right. After every transformation, the equation is equivalent to the original one. We call it "equivalent equations."

Ok, your goal is to isolate the "x" on one side of the equation.

So, your first step is to add "8" on both sides and undo the subtraction "−8." Subtraction and addition are inverse operations of each other. They can cancel each other out, and the value will become "0."

$6x - 8 + 8 = 10 + 8$

You will get $6x = 18$, because -8 and $+8$ equals 0.

When dividing each side by 6, you will get $\frac{6x}{6} = \frac{18}{6}$. (Read as: $6x$ over 6 equals 18 over 6.)

Simplify it, and you get $x = 3$.

Use 3 for each x to see whether both sides have the same value.

老師： 你可以把方程式轉換一下，並將未知數 x 放到同一側，再得出答案。

你可以將方程式看成天秤，$6x-8$ 和 10 在天秤的兩側且保持平衡。在進行轉換時，必須保持方程式的平衡。也就是說，當你在左側進行某些操作時，也必須在右側進行相同的操作，每次轉換後，方程式都跟原始方程式等價，我們稱之為「等價方程式」。

好的，你的目標是將 x 放到方程式的同一側。因此，你的第一步是在兩側加上 8，將 -8 抵銷。減法和加法是互相相反的操作，可以相互抵消，數值會變成 0。$6x-8+8=10+8$ 你會得到 $6x=18$，因為 -8 和 $+8$ 相加等於 0。

接下來，當你將每一側除以 6 時，你會得到 $\frac{6x}{6}=\frac{18}{6}$ 。化簡後，你會得到 $x=3$。

最後再把 3 帶回去 x，檢查等號兩側是否相同。

∞ 應用問題 / 會考素養題 ∞

例題一

說明：老師解釋收入、支出、收支平衡的基本概念，引導學生找出解題的重要資訊。學生設未知數，教師引導學生解方程式。

（英文）John and his friends are baking brownies for their club fundraising. It costs NT$28 to make each brownie, and a one-time cost of NT$250 for the equipment rental fees. John plans to sell each brownie for NT$35. How many brownies does John need to sell to break even?

（中文）John 和他的朋友為了替社團募款，打算烤布朗尼蛋糕來賣。每個布朗尼蛋糕的成本是 28 元，器材租借費為 250 元，John 打算每個布朗尼蛋糕出價 35 元。請問他要賣多少個才能收支平衡？

Teacher: To “break even” means the costs and revenue are equal. That is, you don’t lose or make money. The money that you spend on, such as brownie powder, sugar, and equipment rental fee are the costs. The equipment rental fee is a one-off cost (一次性費用). So, what is the total cost?

Student: Can I assume that John needs to sell x brownies?

Teacher: Yes.

Student: The total cost should be $28x+250$.

Teacher: Correct. The revenue is the total value of sold brownies. Does anyone know the total revenue?

Student: $35x$?

Teacher: Correct. Because the cost and the revenue should be equal, the equation will be $28x + 250 = 35x$. What's the next step for solving the equation?

Student: Isolate the x. So $250 = 7x$, and x equals 35.7.

Teacher: Can he sell 35.7 brownies?

Student: No, he can't!

Teacher: The number of brownies needs to be an integer, so you need to round up. John needs to sell 36 brownies to break even.

老師：「收支平衡」指的是成本和收益相等，也就是說不賺不虧。支出包括蛋糕粉、糖和器材租借費等費用。器材租借費採單次繳納。那麼，總費用是多少呢？

學生：我可以假設 John 需要賣出 x 個布朗尼嗎？

老師：可以。

學生：那麼總成本應該會是 $28x + 250$。

老師：對。收益是售出布朗尼的總價值。有人知道總收益嗎？

學生：是 $35x$ 嗎？

老師：正確。因為成本和收益應該相等，所以方程式為 $28x + 250 = 35x$。下一步該怎麼做來解決這個方程式呢？

學生：將 x 都移到等號同一邊，得到 $250 = 7x$，x 等於 35.7。

老師：他可以賣 35.7 個布朗尼嗎？

學生：不行！

老師：布朗尼的數量需要是整數，因此需要四捨五入。答案是 John 需要賣 36 個布朗尼才能收支平衡。

例題二

說明：老師教學生設未知數，寫出方程式，利用等量公里解題，用 google map 查台北動物園到深坑老街的距離，判斷是否能負擔計程車的費用。不過本題題幹是不考慮計程車延滯計費，但實際情形是必須考慮延滯計費的。

（英文）Here are the taxi rates in Taipei City and New Taipei City

	Normal tariff
Base fee/ initial rate	NT$70 for the first 1250 meters
Distance rate	**Then** NT$5 per 200 meters

The waiting time is not included.

(1) Please calculate how far you can go by taxi if you have NT$300?

(2) You are planning a day trip in Taipei City. After visiting Taipei Zoo, you would like to go to Shenkeng Old Street for some stinky tofu. Can you afford to take a taxi from Taipei Zoo to Shenkeng Old Street if you only have NT$300?

（中文）雙北地區計程車費率如下表

	日間收費方式
起跳金額與里程	前 1250 公尺為 70 元
續跳金額與里程	接續每 200 公尺為 5 元

不考慮延滯計費。

(1) 在你有 300 元的情形下，請問搭計程車可以到達多遠的距離？

(2) 你正在規劃一個台北小旅行，從台北動物園離開後想前往深坑老街吃臭豆腐。如果你只有 300 元的話，可以搭計程車去深坑嗎？

Teacher: (1) Assume the total distance is x meters. The taxi charges NT$70 for the first 1250 meters and NT$5 per 200 meters for the following distance. You would like to know how many “200 meters” there are, so $(x-1250)$ divided by 200.

Because you need to add the initial rate, the total taxi rate will be

$\frac{x-1250}{200} \times 5 + 70$. (read as: x minus 1250, divided by 200, multiplied by 5, plus 70)

You can write the equation $\frac{x-1250}{200} \times 5 + 70 = 300$ representing the total money.

Now you are going to find the solution. Remember to use inverse operations, which help you to isolate the variable, x.

Here are the steps to solve the equation:

$\frac{x-1250}{200} \times 5 + 70 - 70 = 300 - 70$ .....**Subtract 70 from each side.**

$\frac{x-1250}{200} \times 5 = 230$.....**Simplify.**

$\frac{x-1250}{200} \times 5 \div 5 = 230 \div 5$ .....**Divide each side by 5.**

$\frac{x-1250}{200} = 46$ .....**Simplify.**

$\frac{x-1250}{200} \times 200 = 46 \times 200$ .....**Multiply each side by 200.**

$x - 1250 = 9200$.....**Simplify.**

$x - 1250 + 1250 = 9200 + 1250$ .....**Add 1250 to each side.**

$x = 10450$.....**Simplify.**

After these steps, your unknown variable x is isolated. By taxi, you can go 10,450 meters, or 10.45 km.

Teacher: (2) You have to know the distance from Taipei Zoo to Shenkeng Old Street, so first check the distance on Google Maps. The map suggests three different ways, and the longest one is about 5 km.

In the first question, you found that the taxi can go 10.45 km for NT$300. Since 10.45 km is greater than 5 km, you should be able to take the taxi to Shenkeng Old Street!

(However, in reality, you would still need to pay for the waiting time, and you would need some money left over for stinky tofu.)

老師： (1) 設總路程為「x」公尺。計程車收費為前 1250 公尺為 70 元，接續每 200 公尺為 5 元。你想知道有幾個「200 公尺」，所以是 $\frac{x-1250}{200}$。因為需要加上起始費

率，所以計程車費用總共是 $\frac{x-1250}{200}\times 5+70$。目前你有 300 元，因此可以列出方程式 $\frac{x-1250}{200}\times 5+70=300$。

現在你要找出解法。記得使用反運算，有助於你把未知數 x 單獨列出。以下是解方程式的步驟：

$\frac{x-1250}{200}\times 5+70-70=300-70$ …… **兩邊減去 70**

$\frac{x-1250}{200}\times 5=230$ …… **簡化**

$\frac{x-1250}{200}\times 5\div 5=230\div 5$ …… **兩邊除以 5**

$\frac{x-1250}{200}=46$ …… **簡化**

$\frac{x-1250}{200}\times 200=46\times 200$ …… **兩邊乘以 200**

$x-1250=9200$ …… **簡化**

$x-1250+1250=9200+1250$ …… **兩邊加上 1250**

$x=10450$ …… **簡化**

完成這些步驟後，你的未知變數 x 已經單獨在一邊。搭計程車可以走 10450 公尺，或 10.45 公里。

老師： (2) 你必須知道從台北市立動物園到深坑老街的距離，因此先在 Google 地圖上查距離。地圖提供三種不同的路線，最長的約為 5 公里。

第一小題，我們已經算出來花 300 元，計程車可以行駛 10.45 公里。由於 10.45 公里大於 5 公里，所以可以搭計程車到深坑老街！

（不過實際上，你會有「時間成本」，而且還需要留下一些錢買臭豆腐。）

單元十一 線對稱圖形
Shapes with Lines of Symmetry

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■ 前言 Introduction

本單元有關日常生活中常見的許多線對稱圖形。老師介紹本單元時，可以在介紹線對稱的定義、線對稱及對稱軸等英文用語及英文常用句型後，讓學生在課本或講義的範例中找出線對稱圖形的對稱軸，並以英語敘述該圖形是否有對稱軸。接著讓學生就生活經驗提出具線對稱的圖形，來加深學生對線對稱圖形的概念。然後老師讓學生分組討論，並畫出一些較複雜圖形的對稱軸。最後由老師引導學生複習一些常見圖形的對稱軸，以加深學生對線對稱圖形的了解。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
symmetry	**對稱**	shape	圖形
line symmetry	**線對稱**	dashed line	虛線
axis of symmetry	**對稱軸**	isosceles trapezoid	等腰梯形
vertical	垂直的	rectangle	長方形
horizontal	水平的	equilateral triangle	正三角形
oblique	斜的	square	正方形
infinite	無限的	scalene triangle	不等邊三角形
symmetric	**對稱的**	pentagon	五邊形

bisect	平分	hexagon	六邊形
perpendicular	垂直的	regular polygon	正多邊形

※備註：對稱軸(an axis of symmetry)又稱為對稱線(a line of symmetry)。美國中小學課程內容也常用「對稱線(a line of symmetry)」此稱呼。

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ divides _____ into _____.

例句：An axis of symmetry is a line that **divides** a figure **into** two reflected parts.
對稱軸就是將圖形分成完全對稱兩半的一條直線。

❷ Similarly, _____.

例句：**Similarly,** the isosceles trapezoid also has an axis of symmetry.
同樣地，等腰梯形也有一條對稱軸。

❸ _____ is equal to _____.

例句：The number of lines of symmetry in a regular polygon **is equal to** the number of sides.
正多邊形的對稱線數等於邊數。

❹ infinitely many

例句：A circle has **infinitely many** axes of symmetry
任一圓形有無限多條對稱軸。

❺ _____ is symmetric with _____.

例句：Which of the following point **is symmetric with** point *B*?
下列哪一點和 *B* 點對稱？

■ 問題講解 Explanation of Problems

☙ 說明 ❧

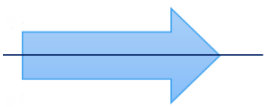
Let's talk about the contents of this section: Shapes with **Lines of Symmetry**.
What is **a line of symmetry**? A line of symmetry is a line that cuts a shape exactly in half. If you fold the shape along a line of symmetry, the two halves would match exactly. This type of symmetry is called "**line symmetry**", and this kind of line is called "a line of symmetry" or "an **axis of symmetry**." In this section, we will discuss the shapes with lines of symmetry.

本章節討論線對稱圖形。所謂線對稱圖形就是該圖形可繪出至少一條直線將其分成完全對稱的兩半，或者將圖形沿著某一條直線對摺，可使直線兩側圖形完全重疊，這樣的圖形稱為「線對稱圖形」，而該直線稱為「對稱軸」。

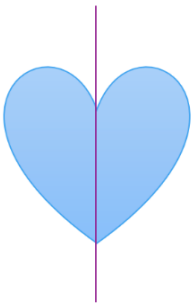
Let's see the following three figures:
In part (a), the horizontal line is a line of symmetry of the figure because you can find that it divides the figure into two **symmetric** parts.
In part (b), the line of symmetry is vertical.
In part (c), there are five lines of symmetry in a regular pentagon. Also, any line of symmetry goes through one vertex and perpendicularly bisects the opposite side.

在下列範例(a)箭號圖形的對稱軸為一條水平直線；範例(b)心形圖形的對稱軸為一條鉛直線；範例(c)該正五邊形圖形的對稱軸共有五條。所有通過頂點且垂直平分對邊的直線均為該圖形的對稱軸。

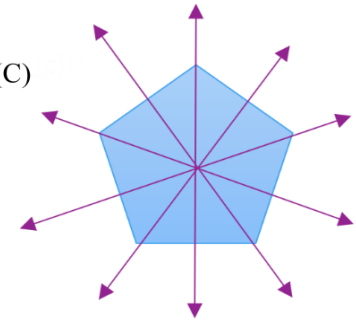
(A)

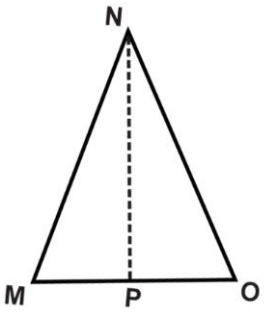


(B)



(C)





Refer to the isosceles triangle $\triangle NMO$ (read as triangle *NMO*) at the right. If you fold $\triangle NMO$ and make the sides $\overline{NM}$ (read as segment *NM*) and $\overline{NO}$ overlapped, you will get the folding line $\overleftrightarrow{NP}$. You can also find that $\overline{MP}$ and $\overline{OP}$ are overlapped as well.

We can identify that $\overleftrightarrow{NP}$ (read as line NP) is an axis of symmetry of $\triangle NMO$. In this figure, Point *M* and Point *O* are called "symmetric points." The two pairs of overlapping sides $\overline{NM}$ & $\overline{NO}$ ($\overline{NM} = \overline{NO}$) (read as segment *NM* is equal to segment *NO*) or $\overline{MP}$ & $\overline{OP}$ ($\overline{MP} = \overline{OP}$) are called "symmetric segments." The three pairs of overlapping angles $\angle M$ & $\angle O$($\angle M = \angle O$), $\angle MNP$ (read as angle MNP) & $\angle ONP$ ($\angle MNP = \angle ONP$) (read as angle *MNP* is equal to angle *ONP*), and $\angle MPN$ & $\angle OPN$ ($\angle MPN = \angle OPN$) are called "symmetric angles."

將等腰 $\triangle NMO$ 的兩腰 $\overline{NM}$ 和 $\overline{NO}$ 對摺重疊時會形成摺線 $\overleftrightarrow{NP}$，同時兩線段 $\overline{MP}$ 和 $\overline{OP}$ 也會重疊，故等腰三角形是線對稱圖形，其中 $\overleftrightarrow{NP}$ 稱為對稱軸，重疊的 *M* 點和 *O* 點稱為對稱點，重疊的邊 $\overline{NM} = \overline{NO}$ 或 $\overline{MP} = \overline{OP}$ 稱為對稱線段，重疊的角 $\angle M = \angle O$ 或 $\angle MNP = \angle ONP$ 稱為對稱角。

You can find that an isosceles triangle has a line of symmetry. And you can conclude the following properties of an isosceles triangle:

(1) The base angles are congruent. (Because $\angle M = \angle O$)

(2) The axis of symmetry $\overleftrightarrow{NP}$ bisects the vertex angle $\angle N$. (Because $\angle MNP = \angle ONP$)

(3) The axis of symmetry $\overleftrightarrow{NP}$ is the perpendicular bisector of the base $\overline{MO}$.
(Because $\angle MPN = \angle OPN = 90°$ and $\overline{MP} = \overline{OP}$)

在上述的線對稱圖形中，可以得到等腰三角形的性質如下：

(1)兩底角相等 （因為 $\angle M = \angle O$）

(2)對稱軸 $\overleftrightarrow{NP}$ 是頂角 $\angle N$ 的角平分線（因為 $\angle MNP = \angle ONP$）

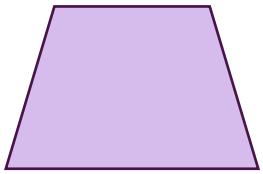
(3)對稱軸 $\overleftrightarrow{NP}$ 是底邊 $\overline{MO}$ 的垂直平分線（因為 $\angle MPN = \angle OPN = 90°$ 且 $\overline{MP} = \overline{OP}$）

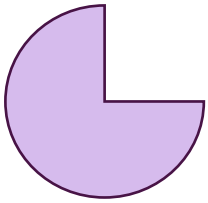
∞ 運算問題的講解 ∞

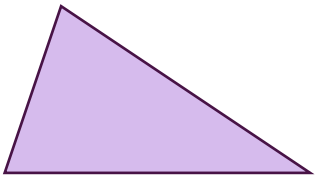
例題一

說明：讓學生學習畫出對稱軸以瞭解線對稱圖形。
老師引導學生找出各圖形的對稱軸，第(3)小題的圖形為不等邊三角形，該圖無法畫出任何對稱軸，所以不等邊三角形不是線對稱圖形。第(1)小題和第(2)小題則都可以分別找出一條對稱軸。

（英文）Draw the lines of symmetry of these figures.
（中文）畫出下列圖形的對稱軸。

(1) 

(2) 

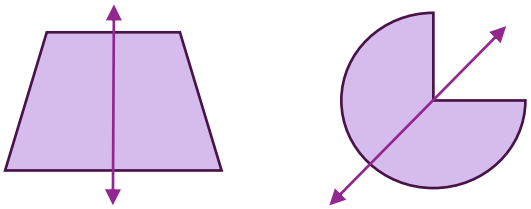
(3) 

Teacher: As we just mentioned, a shape has an axis of symmetry (line of symmetry) when the two sides must be the same if we fold the shape along the line. In part (1), is there any line of symmetry in the isosceles trapezoid?

Student: Yes, there is one line of symmetry.

Teacher: Yes, you are right. You can draw a vertical line that passes through the midpoints of the two bases of the trapezoid. Great!
Let's see part (2). Can you find any axis of symmetry for this shape?

Student: Yes, I can find one axis of symmetry.



Teacher: Correct. If you draw an oblique line through the center, you'll find that the two halves are symmetric about the line. Let's see part (3). Is there any line of symmetry in the scalene triangle?

Student: No, there is no axis of symmetry at all.

Teacher: Yes, you are right. A scalene triangle doesn't have any axis of symmetry.

老師： 如我們剛剛提到的，當我們沿著一條線折疊圖形時，如果兩側圖形都相同，該線就是對稱軸。第(1)小題中，該等腰梯形中有任何對稱軸嗎？

學生： 是，有一條對稱軸。

老師： 答對了。你可以畫一條垂直線，通過等腰梯形的兩個底的中點。不錯喔！接著看第(2)小題，你能找到這個圖形的任何對稱軸嗎？

學生： 是的，我能找到一條對稱軸。

老師： 正確。如果你畫一條斜線穿過中心，你會發現沿著這條線的兩邊圖形是對稱的。在第(3)小題，這個不等邊三角形有任何對稱軸嗎？

學生： 沒有，它沒有對稱軸。

老師： 是的，沒錯。不等邊三角形沒有任何對稱軸。

例題二

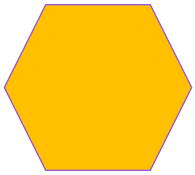
說明：本題是讓學生瞭解一些線對稱圖形具有不只一條對稱軸，正 n 邊形具有 n 條對稱軸，圓形則有無限多條對稱軸。

上述例題一的三個圖形中，有些圖形沒有線對稱或僅有一條對稱軸。例題二將討論具有不只一條對稱軸的圖形，例題二第(1)小題的圖形有水平及垂直的兩條對稱軸，第(2)小題的正六邊形則具有 6 條對稱軸。此時，老師可以強調**任一正 *n* 邊形具有 *n* 條對稱軸**。在第(3)小題，因為任一圓的直徑都可以將該圓形分成對稱的兩個半圓，故任一圓形均有無限多條對稱軸。

（英文）Determine the number of lines of symmetry for each figure.

（中文）找出下列圖形分別有多少條對稱軸？

(1) 

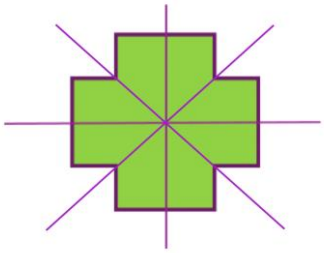
(2) 

(3) 

Teacher: In the first example, we know that some of the graphs do not have any line of symmetry, and some have exactly one line of symmetry. In part (1), we can find that there is more than one line of symmetry. How many lines of symmetry can you find in all?

Student: Two lines of symmetry.

Teacher: Good try. A lot of you might think there are only two lines of symmetry. One is vertical, and the other is horizontal. But there should be four lines of symmetry in the figure of part (1). Please see the correct figure on the board.



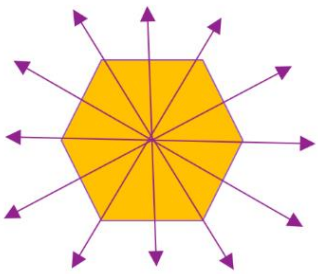
Now, please use three minutes to figure out the number of lines of symmetry and draw them out if possible for the rest parts. You can work and discuss with your partner.

(After three minutes……)

Teacher: Ok, time's up. Please tell me how many lines of symmetry there are in a regular hexagon.

Student: Three.

Teacher: Really? If you draw three diagonals that go through any vertex and its opposite vertex, you can get three lines of symmetry. But if you draw the perpendicular bisector of any side, it also cuts the regular hexagon into symmetric halves. So actually, there are six different lines of symmetry in a regular hexagon.



If you draw more regular polygons, you can find that a regular polygon with n sides has exactly n lines of symmetry. For example, a square has four axes of symmetry. A regular pentagon has five lines of symmetry. **The number of lines of symmetry in a regular polygon is equal to the number of sides.**

Teacher: Finally, how many lines of symmetry are there in a circle?

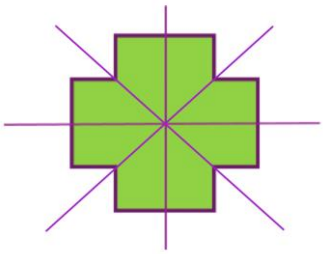
Student: A lot.

Teacher: Yes, a circle has a lot of lines of symmetry. If you draw any diameter of the circle, it divides the circle into two symmetric halves. So, a circle has infinitely many axes of symmetry.

老師： 在例題一中，我們知道有些圖形沒有對稱軸，而有些圖形只有一條對稱軸。在這題的第(1)小題，我們可以發現該圖形有不止一條的對稱軸。你能找到幾條對稱軸呢？

學生： 共有兩條對稱軸。

老師： 不錯的嘗試喔。很多人可能會認為只有兩條對稱軸，一條是垂直的，另一條是水平的。但事實上，在第一部分的圖形中應該有四條對稱軸。請看白板上的正確圖形。

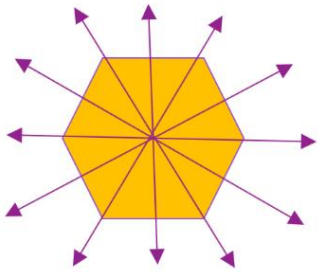


現在，利用三分鐘找出第(2)、(3)小題圖形的所有對稱軸，也可以試著把它們畫出來或跟旁邊的同學討論。

老師： （三分鐘後）好的，時間到。請告訴我一個正六邊形有幾條對稱軸。

學生： 3 條。

老師： 真的嗎？如果你畫三條從任何一個頂點到其對面頂點的對角線，你就可以得到 3 條對稱軸。但如果你畫出任何一邊的垂直平分線，它也會將正六邊形分成對稱的兩半。因此，一個正六邊形中有 6 條不同的對稱軸。



如果你畫出其他的正多邊形，你會發現一個正 n 邊形恰好有 n 條對稱軸。例如，正方形有 4 條對稱軸、正五邊形有 5 條對稱軸。一個正多邊形的對稱軸的數量剛好等於邊的數量。

老師： 最後，一個圓有多少條對稱軸？

學生： 很多條！

老師： 是的，一個圓有很多條對稱軸。如果你畫出圓的任何直徑，它就會將圓分成兩個對稱的半圓。因此，一個圓有無限多條對稱軸。

例題三

說明：本題是讓學生分組討論英文字母的線對稱性質，另外動動腦想想有什麼單字也能找出對稱軸，增加課程的趣味性及互助合作的態度。

讓學生分組討論英文 26 個大寫字母是否具有線對稱性質，並找出其中具有兩條對稱軸的 4 個字母。在學生瞭解 26 個大寫字母哪些不具線對稱性質，哪些有一條對稱軸，哪些有兩條對稱軸後，試著找出具有線對稱的單字 WOW and MOM。

（英文）Consider the line of symmetry of the English alphabet: A, B, C, D, E, F, etc. Try to find:

(1) Four letters which have two lines of symmetry.

(2) One word that has a line of symmetry.

（中文）想一想英文大寫字母中，哪些字母有線對稱性質？並試著找出

(1) 4 個具有兩條對稱軸的字母。

(2) 1 個具有線對稱性質的字彙。

Teacher: Let's check the line of symmetry of the English alphabet. First, look at the letter A. It's not hard for you to find that the letter A has a vertical axis of symmetry. Similarly, the letter B has a horizontal axis of symmetry if you think the top and the bottom are about the same.

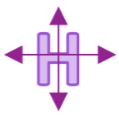
Now, please discuss the 26 capital letters with your partner and find out the four letters which have two axes of symmetry.

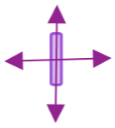
You have three minutes for this question.

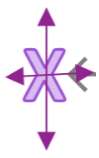
(After three minutes……)

Did you find the answer? Please tell me the four letters in order.

Student: H, I, O, and X







Teacher: Wow, you are so excellent! Now, please think about part (2). Can you find any word that also contains a line of symmetry?

Student: WOW!

Teacher: Very good. Any other word?

Student: MOM.

Teacher: Yes, you are right. The words WOW and MOM both have a vertical axis of symmetry.

老師： 讓我們來算英文字母的對稱軸有幾條。首先，看看字母 A。很容易發現字母 A 有一條垂直對稱軸。同樣地，如果你覺得字母 B 的上下部分相似，則字母 B 有一條水平對稱軸。

現在，請和你的搭檔一起討論 26 個大寫字母，找出有兩條對稱軸的四個字母。給你們三分鐘。

（三分鐘後）找到了嗎？請按順序告訴我這四個字母。

學生： H、I、O 和 X。

老師： 哇，你們很棒！現在，請想想第(2)小題。你能找到任何也包含對稱軸的英文詞彙嗎？

學生： WOW！

老師： 很好。還有其他的嗎？

學生： MOM。

老師： 很好，沒錯。WOW 和 MOM 這兩個單字都有一條垂直對稱軸。

例題四

說明：本題是跟學生討論線對稱圖形－箏形的對稱軸、對稱線段、對稱角，及對稱點的連線段會被對稱軸垂直平分。

因箏形的對稱軸為 $\overline{EG}$ 為 $\angle EFH$ 和 $\angle FGH$ 的角平分線，同時也是對稱點 F 點和 H 點連接線段 $\overline{FH}$ 的垂直平分線。

（英文）Let's fold a piece of paper and cut a triangle from it. Then you can get a kite when you unfold the triangle. Refer to the kite *EFGH* below. Find the answers to the following questions.

(1) Which of the following is the axis of symmetry of kite *EFGH*?

(A) $\overline{EG}$ (B) $\overline{FH}$ C) $\overline{FG}$ (D) $\overline{EH}$

(2) Which of the following points is symmetric with point *F*?

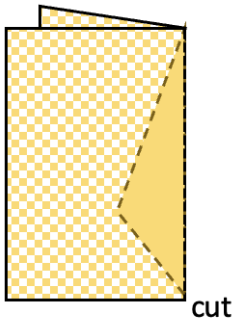
(A) point *E* (B) point *G* (C) point *H* (D) point *O*

(3) Which of the following segments is symmetric with $\overline{FG}$?

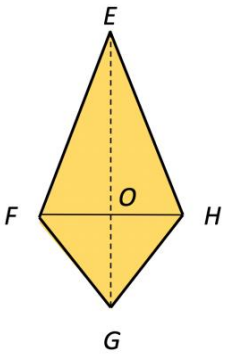
(A) $\overline{EF}$ (B) $\overline{EH}$ (C) $\overline{FH}$ (D) $\overline{HG}$

(4) Which of the following angles is symmetric with $\angle EFG$?

(A) $\angle EFH$ (B) $\angle EHG$ (C) $\angle FEH$ (D) $\angle FGH$



(Before the paper is cut)



(After the paper is cut)

（中文）如果將一張紙對摺後剪出一個三角形（如圖），將其展開後為一個箏形。試回答下列問題：

(1) 該箏形的對稱軸為：(A) $\overline{EG}$ (B) $\overline{FH}$ (C) $\overline{FG}$ (D) $\overline{EH}$

(2) *F* 點的對稱點為：(A) *E* 點 (B) *G* 點 (C) *H* 點 (D) *O* 點

(3) 與 $\overline{FG}$ 對稱的線段為：(A) $\overline{EF}$ (B) $\overline{EH}$ (C) $\overline{FH}$ (D) $\overline{HG}$

(4) 角*EFG* ($\angle EFG$)的對稱角為：

(A) $\angle EFH$ (B) $\angle EHG$ (C) $\angle FEH$ (D) $\angle FGH$

Teacher: What is the answer for part (1)?

Student: (A) segment EG

Teacher: Good. The kite has one axis of symmetry, which is the segment EG (denoted $\overline{EG}$). Then, which point is symmetric with point F?

Student: Point H.

Teacher: Yes, you are right. The answer is (C) Point H. And the axis of symmetry $\overline{EG}$ is the perpendicular bisector of $\overline{FH}$. For part (3), what is the answer?

Student: It's (D) $\overline{HG}$.

Teacher: Excellent! Let's see the last part. Find the answer for part (4).

Student: The answer is (B) $\angle EHG$

Teacher: Wow, you did a great job.

老師： 第(1)小題的答案是什麼？

學生： (A) $\overline{EG}$。

老師： 不錯，菱形有一條對稱軸線，就是線段 EG ($\overline{EG}$)。
那麼，哪一個點跟 F 點對稱？

學生： H 點。

老師： 對，答對了，答案是 (C) H 點。而對稱軸線 $\overline{EG}$ 是 $\overline{HG}$ 的垂直平分線。
那第(3)小題的答案是什麼？

學生： (D) $\overline{HG}$。

老師： 很好喔！最後一小題，知道答案嗎？

學生： 答案是 (B) $\angle EHG$。

老師： 哇，你們很棒！

例題五

說明：求正多邊形的對稱軸。要學生判別比利時、美國、日本及烏克蘭國旗圖形的對稱軸數，並找出相同的對稱軸數，故答案為(A) 與 (D)。

（英文）Which of the following national flags have the same number of lines of symmetry?

（中文）下列哪二個國家的國旗圖形有相同的對稱軸數？

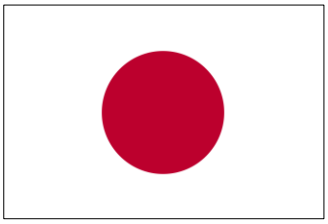
(A)

The national flag of Belgium

(B)

The national flag of USA

(C)



The national flag of Japan

(D)

The national flag of Ukraine

Teacher: Try to figure out the lines of symmetry for each national flag. Which ones of the following national flags have the same number of lines of symmetry?

Student: (A) and (D).

Teacher: Yes, you are right. Both of the national flag figures of Belgium and Ukraine have exactly one line of symmetry.

老師： 好的，現在讓我們來看看其他國家的國旗吧。請找出每一個國旗的對稱軸。哪些國旗具有相同數量的對稱軸呢？

學生： (A)和(D)。

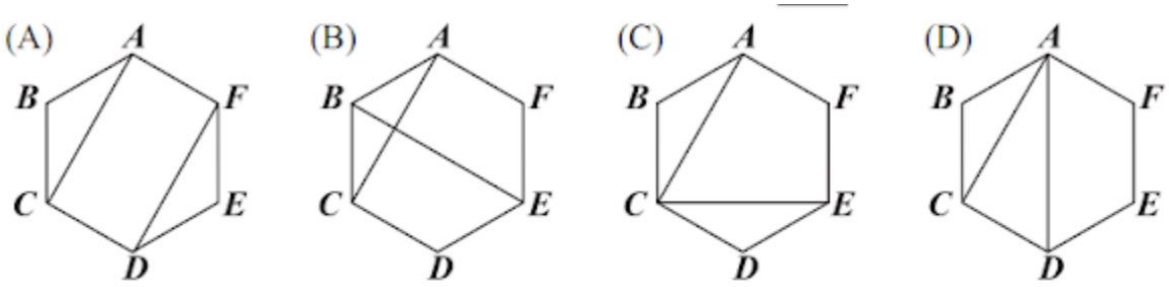
老師： 沒錯，答對了。比利時和烏克蘭的國旗圖案都只有一條對稱軸。

∞ 應用問題 / 會考素養題 ∞

例題一

說明：帶學生瞭解線對稱圖形的變化。

（英文）Kuan used four different ways to draw two diagonals of a regular hexagon. Refer to the diagram below and find which of the following figures is not a graph of line symmetry.



（中文）若阿光以四種不同方式連接正六邊形 *ABCDEF* 的兩條對角線，連接後的情形如下列選項中的圖形所示，則下列哪一個圖形不是線對稱圖形？

（106 年國中會考 4）

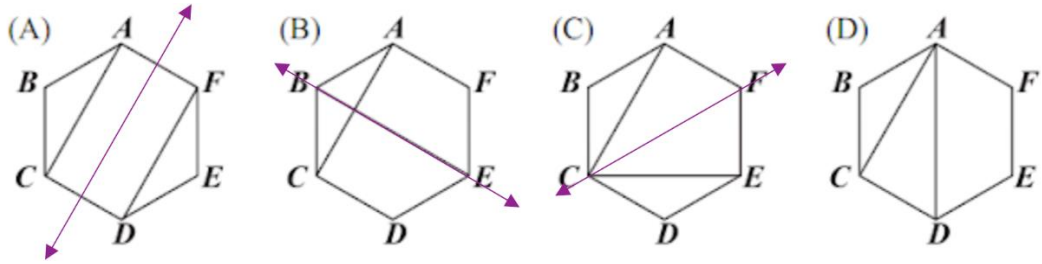
Teacher: Look at the four answer choices and draw the axis of symmetry. You have two minutes to find out the answer.

(After two minutes)

Who can solve this question? (Teacher can choose a student to show the work on the blackboard.)

Student: I can do it.

Teacher: Very good. Please come here and draw the axes of symmetry of the figures on the board.



(After the student demonstrates his/her work on the board.)

Excellent. You can see that only the figure in choice (D) has no axis of symmetry.

That's all for this section.

老師： 請看下列四個選項，然後畫出對稱軸。給大家兩分鐘的時間。

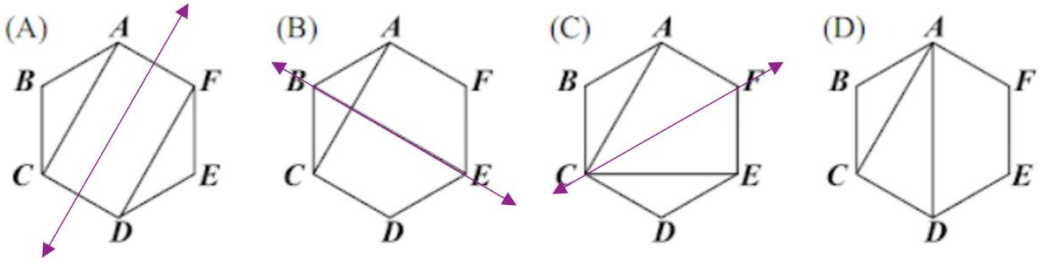
（兩分鐘後）這題誰能回答？

（老師可以選擇一名學生上台寫解題過程。）

學生： 我可以。

老師： 很好，請在黑板上畫出這些圖形的對稱軸。

（學生在黑板上寫解題過程）



太棒了。你可以看到只有選項(D)的圖形沒有對稱軸。

下課囉。

單元十二 三視圖
Three-View Drawing

國立新竹科學園區實驗高級中等學校　印娟娟老師

■ 前言 Introduction

本章節三視圖的重點是透過立體圖形的觀察，來體認視圖的需求。同時理解不同位置的視圖的繪製與呈現。老師在介紹本單元時可以在介紹三視圖、前視圖、上視圖及右視圖等英文用語及英文常用句型後，就可以實際例題讓學生判別或畫出各立體圖形的前視圖、上視圖及右視圖。最後老師可以較複雜的立體圖讓學生以分組活動畫出各視圖以加深學生們對三視圖的瞭解。

■ 詞彙 Vocabulary

※粗黑體標示為此單元重點詞彙

單字	中文	單字	中文
three-view drawing	**三視圖**	**front view**	前視圖
top view	上視圖	**right view**	右視圖
left view	左視圖	**back view/rear view**	後視圖
bottom view	下視圖/底視圖	three-dimensional	立體的
two-dimensional	平面的/二維的	object	物體
column	行	net	網/展示圖
triangular	三角的	prism	角柱（體）
cube	立方體	expand	展開

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ _____ be represented on _____ .

例句：There are a number of ways in which 3D objects can **be represented on** paper.
有多種方式可以將 3D 立體圖形呈現在紙上。

❷ refer to _____.

例句：She spoke for an hour without once **referring to** her notes.
她講了一個小時都完全沒看講稿。

❸ _____ be formed of _____.

例句：The table **was formed of** two large slabs of stone.
這桌子由兩塊石板組成。

❹ according to _____

例句：Students are all put in different groups **according to** their abilities.
所有學生都依據能力進行分組。

❺ from left to right

例句：Please write down the notes **from left to right**.
請將筆記從左到右地寫下來。

❻ demonstrate _____.

例句：Students **demonstrated** the three-view drawing of this question on the blackboard.
學生們在黑板上演示了該題的三視圖。

❼ In order to _____, _____.

例句：In order to pass the exam, I suggest that you spend more time studying.
為了通過考試，我建議你多花一些時間唸書。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

There are a number of ways in which 3D objects can be represented on paper. Usually, there are six different views: Front view, back view, top view, bottom view, right view, and left view. Although six different views can be drawn, three of these views usually give enough information for a three-dimensional object. In this section, we will discuss three-view drawings. What is a three-view drawing? As a rule, they show an object from three different angles of views. (Usually, we use the front, top, and right views.) Each view is drawn in a 2-D (two-dimensional) drawing. Different people can see an object from different angles. It is important to know where the front side is when you draw the three-view drawing.

一般立體圖形通常可以從六個不同方向（前、後、左、右、上、下）的視圖來完全表達一個立體圖形。但許多時候僅從前視圖與後視圖、左視圖與右視圖、上視圖與下視圖中分別各挑一個即可情楚表達一個立體圖形。本章節將討論如何畫出「三視圖」，一般多用前視圖、上視圖及右視圖。

∞ 運算問題的講解 ∞

例題一

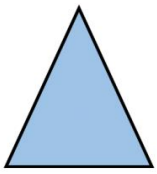
說明：根據三視圖來判別立體圖形。

依據三視圖，因前視圖為一個三角形，所以(A)正方體不符合；另外，右視圖則為一個長方形，所以(C)三角錐不符合；檢查(B)三角柱的上視圖，自上方看該三角柱，會看到左邊和右邊兩個長方形相交的直線，所以答案為 (B)三角柱。

（英文）Which of the following objects has the front, top, and right views as shown below?

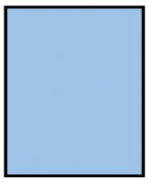
（中文）哪一個立體圖形的三視圖符合下列所示？

Front View

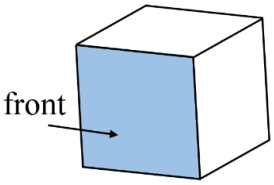


Top View

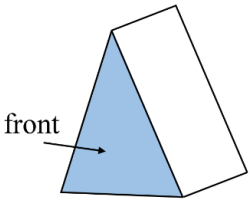
Right View



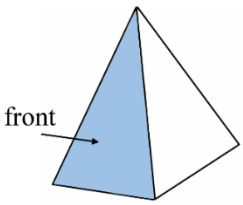
(A) A cube



(B) A triangular prism



(C) A triangular pyramid



(D) none of these

Teacher: As we just mentioned, the three-view drawing includes the front, top, and side views. First, what do you see from the front side?

Student: A triangle.

Teacher: Correct. So, we can delete (A) because the front side of a cube is a square. How about the right side?

Student: A rectangle.

Teacher: Very good. So, you can delete (C) because its right side is also a triangle. Now, try to figure out the top view of (B) a triangular prism. If you look from the top, it will look like the bottom view, which is a square. But please notice that you will see a line that shows the intersection of the left and right sides in the middle from the top. Then, we can find the answer is (B).

Teacher: Let's see the next example.

老師： 正如我們剛才提到的，三視圖包括前視圖、上視圖和側視圖。首先，你看到立體圖形的前視圖是什麼？

學生： 是一個三角形。

老師： 正確。所以我們可以排除選項 (A)，因為立方體的前視圖面是一個正方形。那題目中該立體圖形的右視圖呢？

學生： 是一個矩形。

老師： 很好。所以選項(C)可以排除，因為它的右視圖也是一個三角形。現在，試著找出三角柱體 (B) 的俯視圖。

如果你從上面看，它會看起來會像下視圖一樣，是一個正方形。但是，左右兩平面會在中央交會成一條線。所以，我們可以得出答案是 (B)。

老師： 接著看下一題。

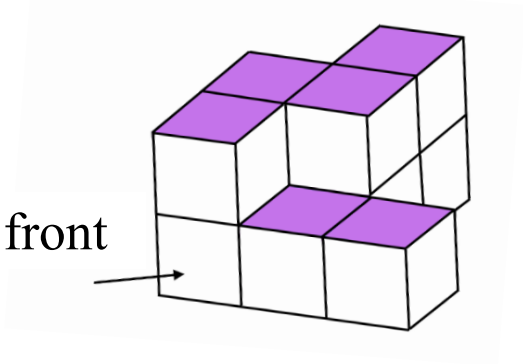
例題二

說明：立體圖形的觀察，畫出三視圖。

當我們畫三視圖時，需要考慮到是否有隱藏(沒看到)的方塊。已知該立體是由 11 個方塊組成，但自前方看過去僅能看到 10 個方塊，所以必定有一個方塊在看不到的對角。現在讓我們一起畫出上視圖。

（英文）Draw the top view of the object which is formed of 11 same cubes.

（中文）下列圖形是由 11 個相同大小的立方體組合而成的，請畫出該物體的上視圖。



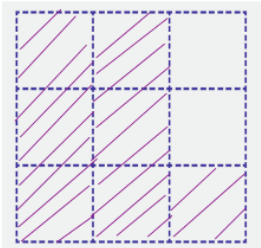
Teacher: According to the given information, this object is formed by 11 same cubes. Only ten cubes can be seen from the front view. So, there must be one cube which is in the left back side. Now, let's draw the top view from left to right. In the first column, all the three squares should be shaded since they are occupied by cubes. Then, how

to draw the second column?

Student: It is the same as the left column.

Teacher: Yes, you are right.

Let's do the last column. It is obvious that only the one close to the front side with a cube. So, here's the drawing of top view:

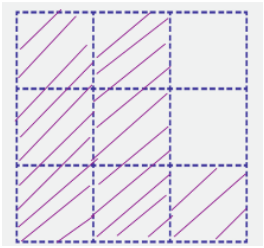


老師：已知這個圖形是由 11 個相同的立方體組成，但我們從正面只看到十個立方體，也就是說，在左後方必定有一個立方體。現在，讓我們從左到右繪製上視圖。在第一行中，所有三個正方形都應該被填滿，因為由立方體占據。那麼，第二行呢？

學生：它與左邊那行相同。

老師：是的，沒錯。讓我們畫最後一行。

明顯地，只有一個立方體在最前面，所以上視圖應該這樣畫：

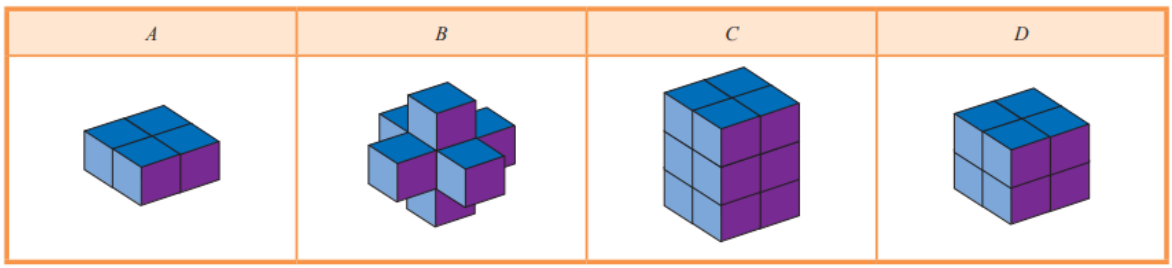


例題三

說明：探討數個立體圖形不同位置的視圖。

（英文）Which of the following objects has a different top view from the others.

（中文）下面哪一個立體圖形的上視圖與其他立體圖形不同？



（圖形取自國家教育研究院所出版之「三視圖—從哪裡看」教師手冊第 16 頁）

Teacher: Now, please work with your partner to figure out the top view of each graph.

(After three minutes)

Did you figure out the top views of the four objects? Which one is different from

the others?

Student: Yes. The top view of (B) is different from the others.

Teacher: Good job. I found that most of you got the correct answers. The top views of the objects of A, C, and D are the same square. So, only (B) is different from the others.

老師： 現在，請跟旁邊的同學一起找出每個立體圖形的上視圖。

（三分鐘後）你們找出這四個立體圖形的上視圖了嗎？哪一個跟其他的不同？

學生： 找到了。B 的上視圖與其他的不同。

老師： 很好。我發現你們大多都找對了。A、C 和 D 的物體的俯視圖都是相同的正方形。因此，只有 B 與其他的不同。

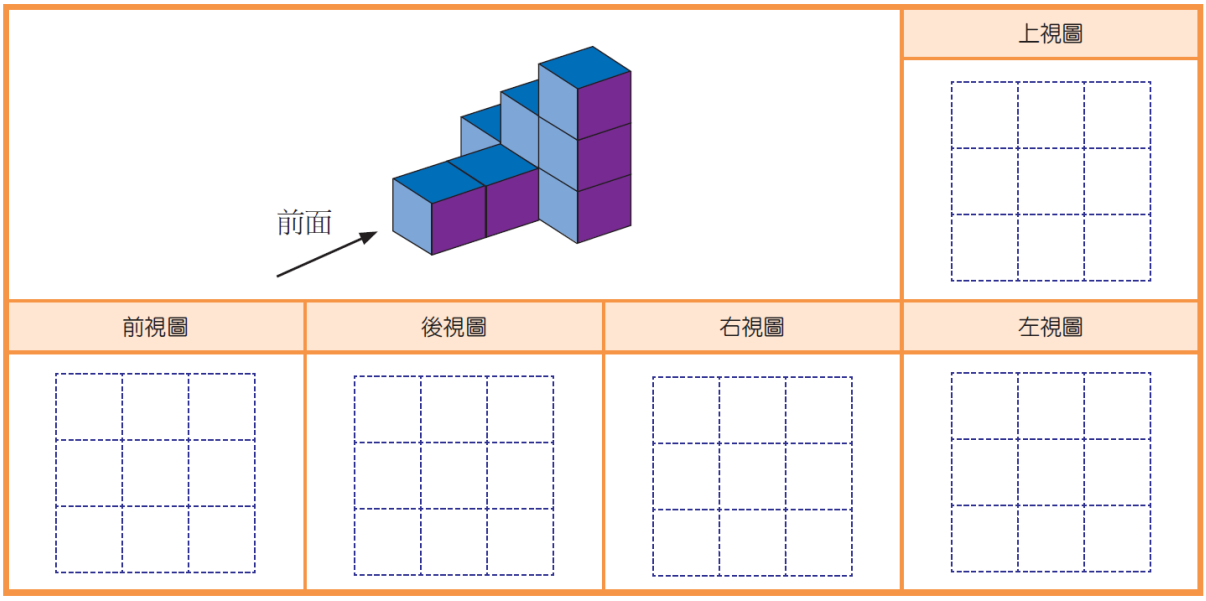
例題四

說明：探討立體圖形的觀察位置與視圖的關係。

當我們畫上視圖時，需要考慮到自上方看下來的圖形，老師示範完上視圖後。接下來請學生分組畫出前視圖、後視圖、右視圖及左視圖，並請一組學生上台示範。老師可以在示範結束後提醒學生前視圖及後視圖、右視圖及左視圖等圖形的對稱性質。同時引導學生思考下視圖是否會有類似性質，可發現上視圖及下視圖兩圖是完全相同的。

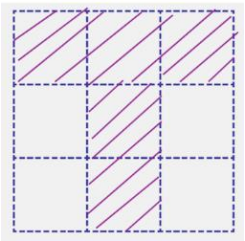
（英文）Draw the top, front, back, right, and left views of this object.

（中文）畫出下面立體圖形的上視圖、前視圖、後視圖、右視圖及左視圖。



（取自國家教育研究院所出版之「三視圖—從哪裡看」教師手冊第 18、19 頁）

Teacher: In order to draw the different views of this object, you need to look from the correct sides. If we look at this object from the top, the graph should be like this:

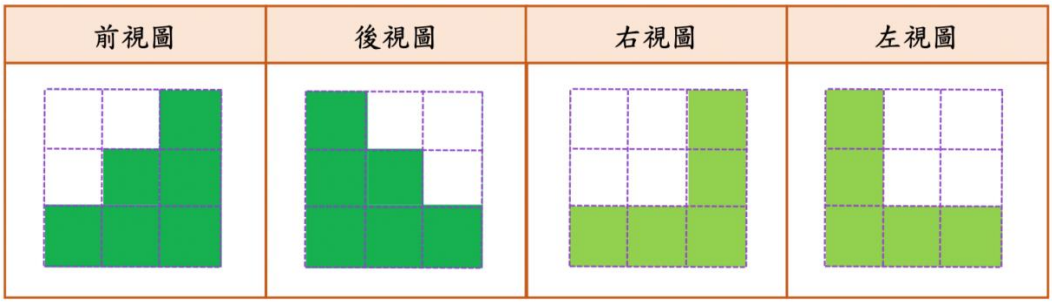


Now, please work with your partner to do the rest parts. Six minutes.

(After six minutes)

Group 3, please come to draw your graphs on the blackboard, and the other people please check your graphs with the drawings that group 3 have done on the board.

(Students demonstrated their work on the blackboard.)

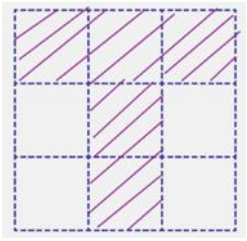


Teacher: Wow, group three did a great job. Now, please notice that the front view and the back view are different here but symmetric with each other. Likewise, the right view and the left view have a similar situation. Now, please tell me the answer. Will the top view and the bottom view be different and symmetric as well?

Student: No, the top view and the bottom view are the same and symmetric.

Teacher: Yes, you are right.

老師： 為了繪製不同視圖，需要從正確的方向觀察。如果我們從頂部觀察這個物體，圖形應該是這樣的：



老師： 現在，請跟旁邊的同學一起在六分鐘內完成其他的部分。

（六分鐘後）第 3 組，請來到黑板前把你們的答案畫出來，其他人請比較一下你們跟第 3 組畫的視圖。

（學生在黑板上畫上答案。）

老師： 哇！第 3 組做得很好。現在，請注意，這裡的前視圖和後視圖是不同的，但是左右對稱的。同樣，右視圖和左側視圖也有類似的情況。現在，請告訴我，上視圖和下視圖也會不同且對稱嗎？

學生： 不是，上視圖和下視圖是相同且對稱的。

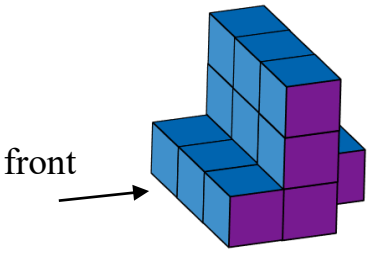
老師： 答對了。

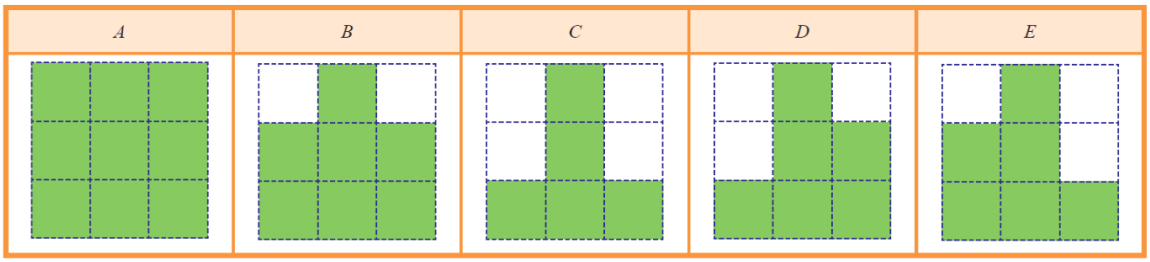
例題五

說明：判別各立體圖形的視圖。已知該立體是由 14 個方塊組成，但自前方看過去能看到 12 個方塊，所以必定有 2 個方塊在最後一排。從該立體圖形隱約可看到第 13 個方塊，所以根據圖形第 14 個方塊應該在後方的最下層，所以該題的右視圖為 C。而該立體的左視圖亦為 C，因為左視圖和右視圖為對稱圖形，圖 C 的反面和正面是重合的。

（英文）Which of the following is the right view of the object which is formed of 14 same cubes?

（中文）如圖，由 14 個相同的小立方體組成的立體圖形，則下列哪一個選項是他的右視圖？





（取自國家教育研究院所出版之「三視圖—從哪裡看」教師手冊第 27 頁）

Teacher: Look at the figure of the object which is formed of 14 same cubes. Can you tell which one is the right view of it?

Student: C

Teacher: Very good.

Student: Why the answer is not D?

Teacher: Good question. According to the given, there are 14 cubes to form this object. Then, you can count that there should be two cubes in the back row and both on the bottom. Therefore, D is not the answer. Did you get it?

Student: Yes, I got it.

Teacher: Excellent. Now, can you find the left view among these choices?

Student: The answer is still C.

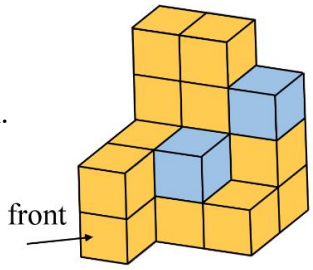
Teacher: Yes, you are definitely right. The symmetric view of C (Left View) is the same as C itself (Right View).

老師： 看這個由 14 個相同的立方體組成的物體，你能說出哪一個是它的右視圖嗎？
學生： 是 C。
老師： 非常好。
學生： 為什麼答案不是 D 呢？
老師： 好問題。根據所給的條件，這個物體由 14 個立方體組成。然後，你可以數一數，後排應該只有兩個立方體，都在底部。因此，D 不是正確答案。這樣懂了嗎？
學生： 是的，我懂了。
老師： 太好了。現在，看看左視圖是哪一個呢？
學生： 一樣是 C。
老師： 是的，沒錯。左視圖是右視圖 C 的對稱視圖，它的圖形還是 C。

例題六

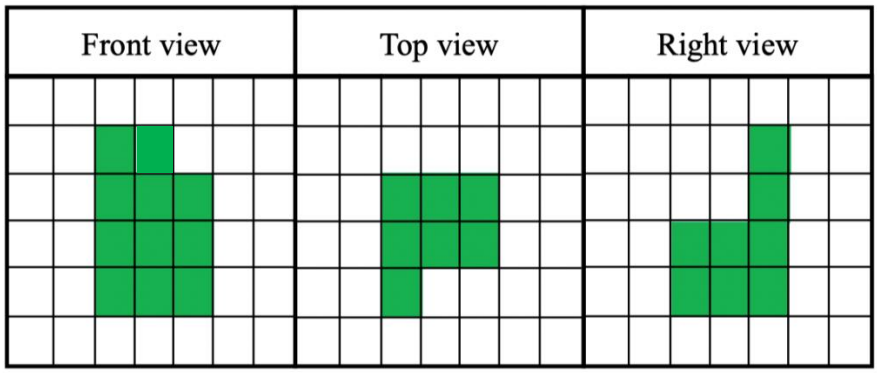
說明：當立體方塊的組合變動時，觀察並判別三視圖的變化。
老師先示範畫出第(1)小題立體圖形的前視圖、上視圖及右視圖。然後讓學生依照(1)的方式做第(2)小題：拿掉兩個藍色小方塊的三視圖。讓學生比較看看其中哪幾個圖形會變得不同。

（英文）(1) Draw the front, top, and right views of the object shown below.
(2) Draw the new three views if the two blue cubes are removed.
Are the three views the same as the original? Explain.

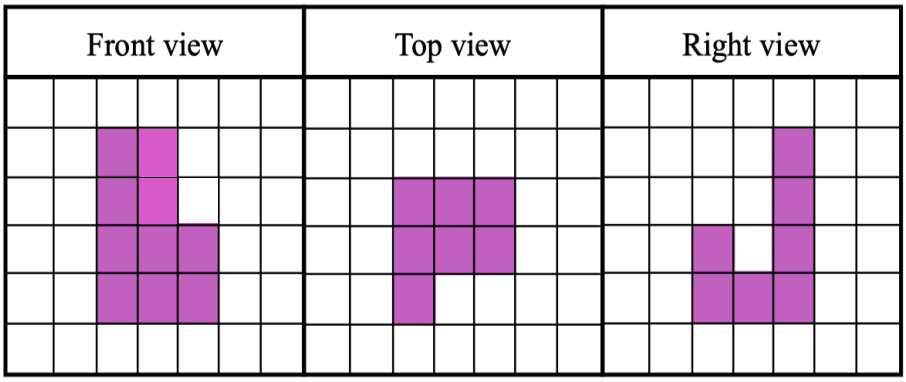


（中文）(1)畫出右方立體圖形的前視圖、上視圖及右視圖。
(2)若拿掉兩個藍色小方塊，
則三視圖中哪幾個圖形會變得不同？

Teacher: Let's see the question. When we draw the front view, the height for each column is 4, 4, and 3 cubes from left to right. Then we look from the top and the right view to finish the three-view drawings.



Teacher: Now, it's your term to do the three-view drawings after taking off the blue cubes. Please compare the new three views with the original graphs
You have five minutes to do this part.
(After five minutes)
Ok. Time's up. Please show me your work. (Teacher goes around and checks students' work.)

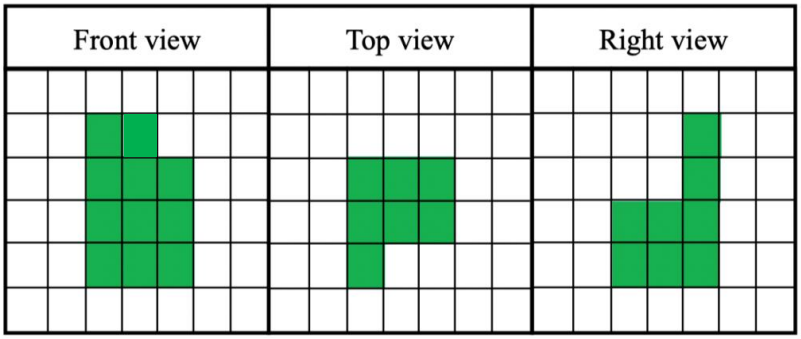


Teacher: Very good. A lot of you did a great job. Now, compare the three views in both parts. Is there any view is different from the original?

Student: Yes, the front view.

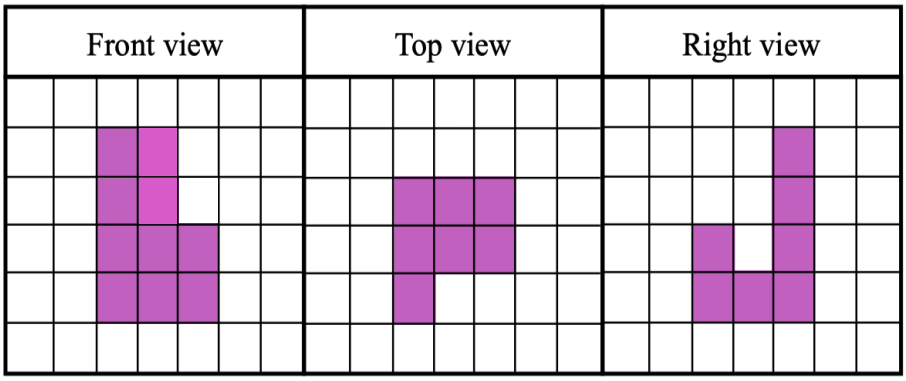
Teacher: Excellent. Let's see the last example.

老師： 接下來看例題六。當我們畫前視圖時，從左到右每行的高度分別為 4、4 和 3 個立方體。然後，我們從上視圖和右視圖繼續完成三個視圖的繪製。



老師： 現在換成去掉藍色立方體後繪製三個視圖，並將新的三個視圖與原本的圖形進行比較。有五分鐘的時間完成。

（五分鐘後）好的，時間到了。讓老師看看你們畫的三視圖。（老師走過去看學生畫的圖。）



老師： 非常好。大家都做得很好。現在比較兩批三個視圖，有任何視圖與原始圖形不同？

學生： 有，前視圖不同。

老師： 很好。現在來看最後一題。

例題七

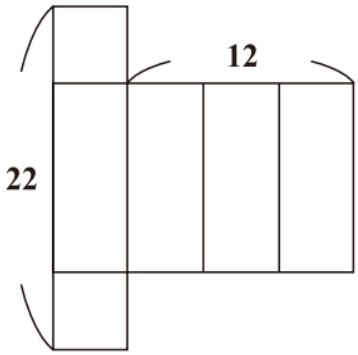
說明：根據長方立體的展示圖，還原長方體圖形。

（英文）The graph shown below is a rectangular prism which is cut along a line and expanded. Find the volume of the prism if the base is a square.

（中文）

圖（二）為一個長方體的展開圖，且長方體的底面為正方形。根據圖中標示的長度，求此長方體的體積為何？

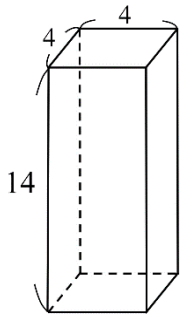
(A) **144**
(B) **224**
(C) **264**
(D) **300**



圖（二）

（111 年國中會考 4）

Teacher: Let's figure out the figure of the rectangular prism by folding it back.
The graph should like this:
By the given information, the base is a square. Can you tell the side length of the square?



Student: The side length is 4.

Teacher: Good. You can find the length 12 is exactly the length of three side lengths. So, each side is 4. Then, what is the height of the object?

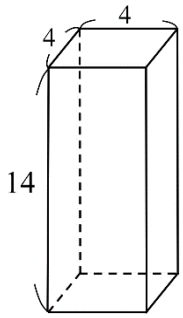
Student: The height is 14.

Teacher: You are right. 22 is the sum of the height and two side lengths of the square base. So, the height is 14. Base on what we have found and figure out the volume now.

Student: The volume is $4 \times 4 \times 14$ which is 224.

Teacher: Excellent job. That's the end of the class. Class is dismissed.

老師： 我們把展開圖折回來，找出長方體的圖形。圖形應該是這樣的：根據所給條件，底面是一個正方形。你能告訴我正方形的邊長嗎？



學生： 邊長是 4。

老師： 很好。你可以發現 12 正好是三個邊長的長度。因此，每邊長是 4。那麼，這個物體的高是多少？

學生： 高是 14。

老師： 答對了。22 是正方形底面的兩邊長和高的總和。因此，高是 14。現在根據我們找到的來算出體積。

學生： 體積是 $4 \times 4 \times 14$，即 224。

老師： 很好。課程結束，下課。

國內外參考資源 More to Explore

國家教育研究院樂詞網 查詢學科詞彙 https://terms.naer.edu.tw/search/	
教育雲：教育媒體影音 為教育部委辦計畫雙語教學影片 https://video.cloud.edu.tw/video/co_search.php?s=%E9%9B%99%E8%AA%9E	
Oak Teacher Hub 國外教學及影音資源，除了數學領域還有其他科目 https://teachers.thenational.academy/	
CK-12 國外教學及影音資源，除了數學領域還有自然領域 https://www.ck12.org/student/	
Twinkl 國外教學及影音資源，除了數學領域還有其他科目，多為小學及學齡前內容 https://www.twinkl.com.tw/	

Khan Academy 可汗學院，有分年級數學教學影片及問題的討論 https://www.khanacademy.org/	
Open Textbook (Math) 國外數學開放式教學資源 http://content.nroc.org/DevelopmentalMath.HTML5/Common/toc/toc_en.html	
MATH is FUN 國外教學資源，還有數學相關的小遊戲 https://www.mathsisfun.com/index.htm	
PhET: Interactive Simulations 國外教學資源，互動式電腦模擬。除了數學領域，還有自然科 https://phet.colorado.edu/	
Eddie Woo YouTube Channel 國外數學教學影音 https://www.youtube.com/c/misterwootube	

國立臺灣師範大學數學系陳界山教授網站	
國高中數學雙語教學相關教材 https://math.ntnu.edu.tw/~jschen/index.php?menu=Teaching_Worksheets	
2023 年第四屆科學與科普專業英文(ESP)能力大賽	
科學專業英文相關教材，除了數學領域，還有其他領域 https://sites.google.com/view/ntseccompetition/%E5%B0%88%E6%A5%AD%E8%8B%B1%E6%96%87%E5%AD%B8%E7%BF%92%E8%B3%87%E6%BA%90/%E7%9B%B8%E9%97%9C%E6%95%99%E6%9D%90?authuser=0	

國中數學領域雙語教學資源手冊：英語授課用語

[七年級上學期]

A Reference Handbook for Junior High School Bilingual Teachers in the Domain of Mathematics: Instructional Language in English

[7th grade 1st semester]

- 研編單位：國立臺灣師範大學雙語教學研究中心
- 指導單位：教育部師資培育及藝術教育司
- 撰稿：印娟娟、吳珮蓁、陳立業、周慧蓮
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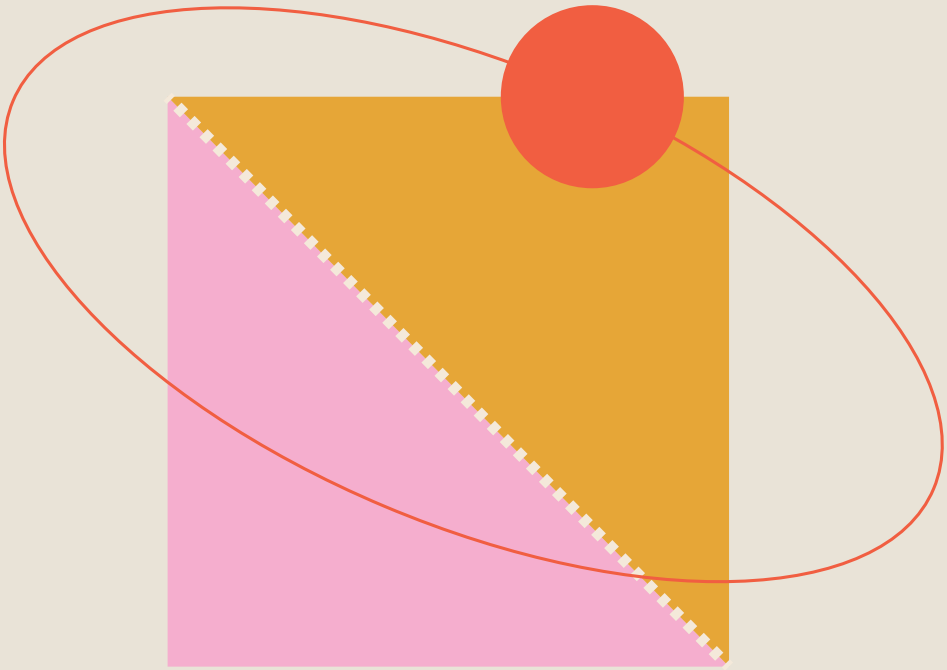
- 語言諮詢：李壹明
- 綜合規劃：王宏均
- 編輯排版：吳依靜
- 封面封底：JUPE Design



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