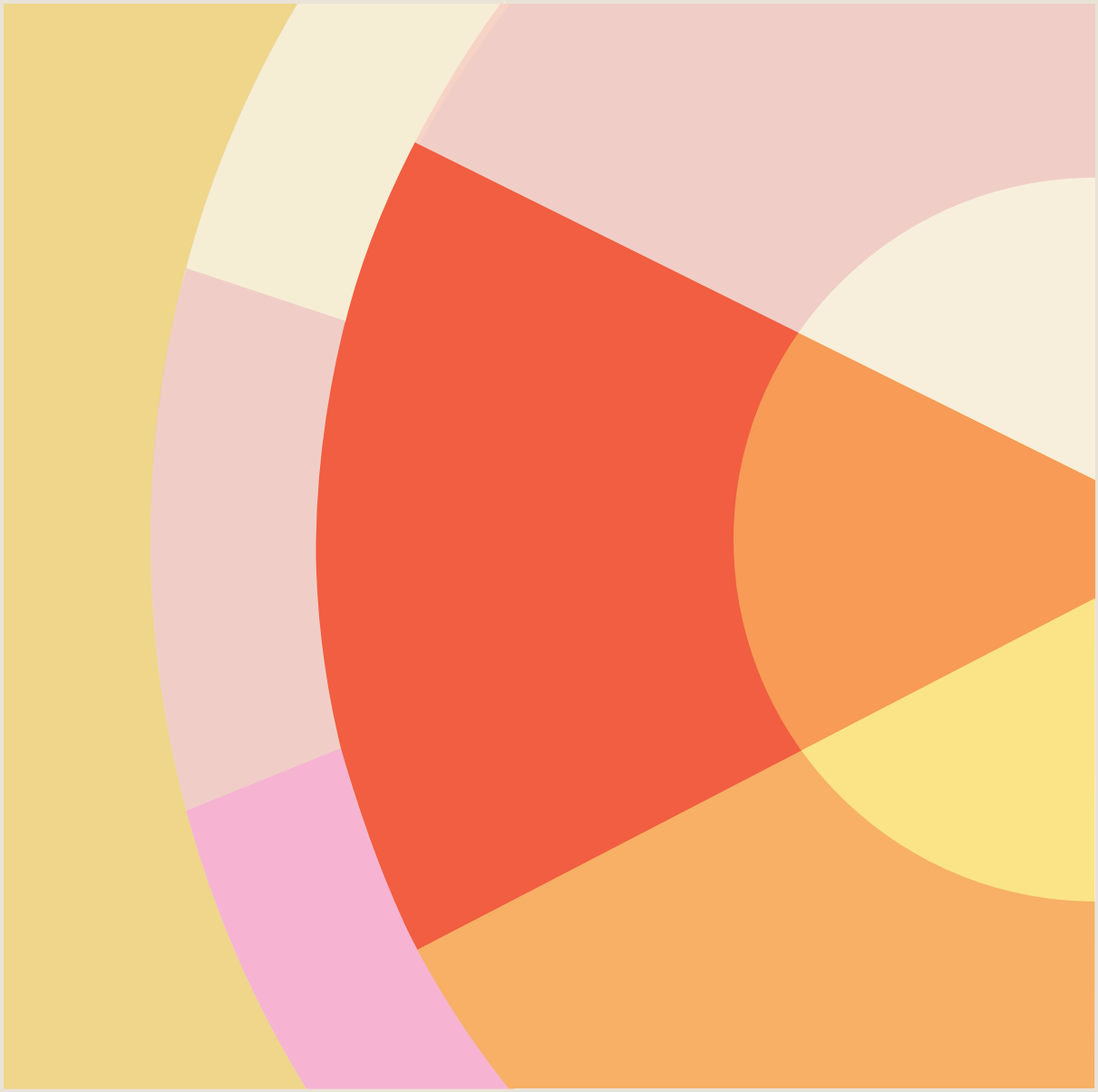
國中數學領域

雙語教學資源手冊 英語授課用語

A Reference Handbook for Junior High School Bilingual Teachers in the Domain of Mathematics: Instructional Language in English

〔九年級下學期〕



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目次 Table of Contents

單元一　簡易二次函數圖形
Simple Quadratic Function Graph

國立彰化師範大學數學系　盧昶元、邱奕銘

■ 前言 Introduction

本章的學習目標是讓學生能由生活中觀察二次函數現象（規律）及由具體情境認識二次函數，並在二次函數圖形繪製時，著重描點的過程，且適時運用透明片或資訊融入等教學活動，順利習得二次函數都會形成平滑曲線的概念，進而讓學生透過描繪二次函數的圖形，理解二次函數的標準式，熟知開口方向、大小、頂點、對稱軸與最大值、最小值等問題。

■ 詞彙 Vocabulary

單字	中文	單字	中文
function	函數	plane coordinates	平面坐標
linear function	線型函數	function value	函數值
quadratic function	二次函數	axis of symmetry	對稱軸
vertex	頂點	perpendicular line	垂直線
absolute value	絕對值	parabola	拋物線

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ **Put $x =$ _____ into function $y = ax^2 + b + c$, the function value become _____.**

例句(1)：**Put $x = 0$ into function $y = x^2 + 3x + 2$, the function value become 2.**

將 $x = 0$ 帶入方程式 $y = x^2 + 3x + 2$，其函數值為 2。

例句(2)：**Put $x = 2$ into function $y = x^2 + 3x + 2$, the function value become** 12.

將 $x = 2$ 帶入方程式 $y = x^2 + 3x + 2$，其函數值為 12。

❶ **As the absolute value of coefficients of the x^2 is _____, the opening of parabola is _____.**

例句：**As the absolute value of coefficients of the x^2 is** greater/smaller, **the opening of parabola is** greater/smaller.

當 x^2 的係數的絕對值愈大/小，它的拋物線開口就會愈小／大。

■ 問題講解 Explanation of Problems

➿ 運算問題的講解 ➿

例題一

說明：本題是為了讓學生熟悉使用二次函數的兩對稱點性質和線段方程式關係。

Students can get familiar with the relationship between the axis of symmetry and the quadratic formula from the worked example.

（英文）In the diagram, point A and point B are on the graph of $y = x^2$ and $\overline{AB} \perp y$-axis. If $\overline{AB} = 6$, what is the equation of $\overline{AB}$?

(A) $y = 3$

(B) $y = 6$

(C) $y = 9$

(D) $y = 36$

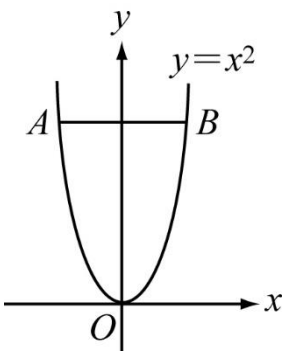
（中文）如圖，A、B 分別為 $y=x^2$ 上兩點，且 $\overline{AB}\perp y$ 軸。若 $\overline{AB}=6$，則直線 $\overline{AB}$ 的方程式為何？

(A) $y=3$

(B) $y=6$

(C) $y=9$

(D) $y=36$



（91 年第二次國中基測 17）

Teacher: Here, we are given three clues

Point A and point B are on the graph of $y=x^2$

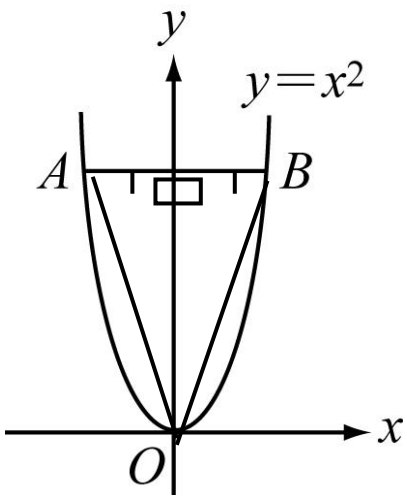
$\overline{AB}\perp y$-axis

$\overline{AB}=6$

Teacher: Now let's go to the first clue about this parabola equation $y=x^2$. Where are the vertex and the axis of symmetry of this parabola?

Student: The vertex is $(0,0)$ and the axis of symmetry sits right on the y-axis.

Teacher: That is correct. Because the axis of symmetry of this parabola sits right on the y-axis and $\overline{AB}\perp y$-axis, point A and point B are equidistant from the y-axis. It tells us that the coordinates for point A and point B are $(-3, y_1)$ and $(3, y_2)$ respectively.



Teacher: Now, we substitute them back into the original equation $y=x^2$, and the values of both y_1 and y_2 are 9.

That also tells us that $\overline{AB}$ is a horizontal line.

So, $y=9$ on the line over $\overline{AB}$.

老師：來~先看到這題他給了我們三個線索：

A、B 分別為 $y=x^2$ 上兩點；$\overline{AB}\perp y$ 軸；$\overline{AB}=6$

老師：首先看到第一個線索 $y=x^2$，那麼 $y=x^2$ 圖形的頂點、對稱軸分別在哪裡？

學生：頂點在(0 , 0)；對稱軸在 y 軸。

老師：沒錯～因為 $y=x^2$ 圖形對稱在 y 軸，$\overline{AB}\perp y$ 軸，所以 A、B 兩點與 y 軸的距離相等，所以我們可以知道 A、B 兩點座標為$(-3 , y_1)$、$(3 , y_2)$。

老師：將$(-3 , y_1)$、$(3 , y_2)$代回 $y=x^2$，得到 y_1、y_2 皆為 9
因此我們可以發現 $\overline{AB}$ 是一條水平線
所以 $y=9$ 在 $\overline{AB}$ 這條線上。

例題二

說明：本題是讓學生熟悉使用二次函數的代換關係，並可依據題目線索自設一次函數。
From the worked example, students can get familiar with how to solve quadratic systems by substitution and by defining a linear function with the given clues on their own.

（英文）On the coordinate plane, a parabola given by $y=x^2$, point A and point B are on the parabola. The x values of the two points are 2 and 4 respectively. Two lines are drawn, one through point A, and parallel to the y-axis and the other line through point B and parallel to the x-axis. The two lines are met at one point, marked C. Find the coordinates of C.

(A)（2 , 16）
(B)（2 , 2）
(C)（4 , 2）
(D)（4 , 2）

（中文）坐標平面上，二次函數 $y=x^2$ 的圖形通過 A、B 兩點，其中 A、B 兩點的 x 坐標分別為 2、4。若自 A 作 y 軸的平行線，自 B 作 x 軸的平行線，且兩線交於 C 點，則 C 點坐標為何？

(A)（2 , 16）
(B)（2 , 2）
(C)（4 , 2）
(D)（4 , 2）

（99 年第二次國中基測 26）

Teacher: Firstly, we substitute the x values of point A and point B back into the given function $y=x^2$. That gives us the coordinates for point A and point B, which are

(2 , 4) and (4 , 16) respectively.

Teacher: Now, we draw the two lines based on the instruction. One passes through point *A*, parallel to the *y*-axis and that gives us the value of the *x*-intercept, which is 2. The other line passes through point *B*, parallel to the *x*-axis, and we can come to the value of the *y*-intercept. That is 16. Now, we know that the coordinates of *C* are (2 , 16).

老師： 首先將 A、B 兩點帶回 $y=x^2$，得到 A、B 兩點座標分別為(2 , 4)、 (4 , 16)。

老師： 再依據題目所示，自 A 作 y 軸的平行線，得到方程式 $x=2$，自 B 作 x 軸的平行線，得到方程式 $y=16$，因此直線的交點為 (2 , 16)。

單元二　二次函數圖形的最大值與最小值

Maximum and Minimum Values of Quadratic Function Graph

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■ 前言 Introduction

本節先由 $y = ax^2$ 的圖形與 $y = ax^2 + k$ 的圖形平移關係，理解 $y = ax^2 + k$ 的圖形開口方向、大小、頂點（最高點和最低點）、對稱軸。

然後再進一步理解 $y = ax^2$ 圖形與形如 $y = a(x - h)^2$、$y = a(x - h)^2 + k$ 這種已配方的二次函數圖形間的平移關係，以及這些二次函數的圖形特性。

■ 詞彙 Vocabulary

單字	中文	單字	中文
coordinate	坐標	apex	最高點
vertex	頂點	nadir	最低點
coincidence	疊合	axis of symmetry	對稱軸
translation	平移	intersection point	交點
x-axis	x 軸	y-axis	y 軸

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ In the function _____, the *y* value has the maximum/minimum ___ as the *x* value becomes _____.

例句(1)：**In the function** $y = -5(x+3)^2 + 1$, **the *y* value has the maximum** 1 **as the *x* value becomes** -3.

在方程式 $y = -5(x+3)^2 + 1$ 中，當 x 值為 -3 時，y 有最大值 1。

例句(2)：**In the function** $y = -3(x-1)^2 + 2$, **the *y* value has the maximum** 2 **as the *x* value becomes** 1.

在方程式中 $y = -3(x-1)^2 + 2$ 中，當 x 值為 1 時，y 有最大值 2。

❷ The number of the intersections between the quadratic function _____ and *x*-axis is _____.

例句(1)：**The number of the intersections between the quadratic function** $y = 2(x-5)^2 - 4$ **and *x*-axis is** two.

二次函數 $y = 2(x-5)^2 - 4$ 與 x 軸的交點個數會有 2 個。

例句(2)：**The number of the intersections between the quadratic function** $y = -(x+3)^2$ **and *x*-axis is** one.

二次函數 $y = -(x+3)^2$ 與 x 軸的交點個數會有 1 個。

■ 問題講解 Explanation of Problems

∞ 運算問題的講解 ∞

例題一

說明：本題是讓學生熟悉使用二次函數公式，並瞭解如何找出對稱圖形。

Students can understand the quadratic equations and know how to find the symmetric figure.

（英文）Mei placed a transparent sheet with a parabola onto the coordinate plane. Point (2 , 3) is on the vertex of the parabola. When the parabola opens upwards, the graph represents the quadratic function $y = 2(x-2)^2 + 3$ as shown in picture 1. If she turns the sheet upside-down to make the parabola open downward and keeps the vertex remaining at the same place (as shown in picture 2), what quadratic function would the graph in picture 2 be?

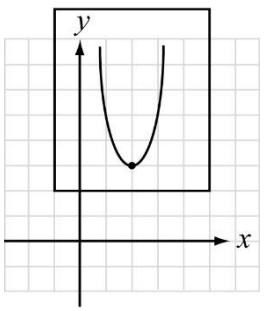
（中文）小梅將一張畫有拋物線的透明片擺到坐標平面，將拋物線頂點與點(2 , 3)重合，開口向上時，此拋物線為二次函數 $y = 2(x-2)^2 + 3$ 的圖形，如圖一。若她將透明片反轉，使得開口向下且頂點的位置不變，如圖二，則圖二的拋物線為下列哪一個二次函數的圖形？

(A) $y = -2(x-2)^2 + 3$

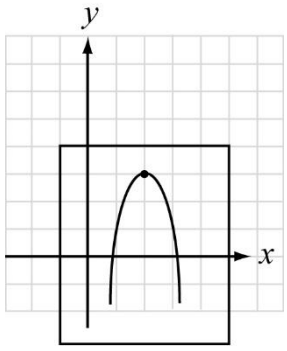
(B) $y = -2(x-2)^2 - 3$

(C) $y = -2(x+2)^2 + 3$

(D) $y = -2(x+2)^2 - 3$



圖一



圖二

（97 年第二次國中基測 14）

Teacher: Now we see that the function of the parabola is $y = 2(x-2)^2 + 3$. Can you tell me what function the graph is?

Student: It is a quadratic function graph.

Teacher: Yes, what form can a graph of a quadratic function be?

Student: $y = a(x-h)^2 + k$, and (h, k) is the vertex of the graph.

Teacher: Great. Now compared to $y = 2(x-2)^2 + 3$, what are a, h, and k?

Student: $a = 2$, $h = 2$, and $k = 3$.

Teacher: Okay. Now reversing the transparent sheet means that the graph is made symmetric to the graph of $y = 2(x-2)^2 + 3$ and its axis of symmetry is $y = 3$.

Teacher: What did we learn from the book to do when the graph is upside-down from the original quadratic function?

Student: Make a negative and remain h and k still.

Teacher: Yes. So, 2 becomes -2 and others remain still. Therefore, the function would be $y = -2(x-2)^2 + 3$.

老師： 老我們可以看到拋物線的函數為 $y = 2(x-2)^2 + 3$ ，那同學們可以告訴我這是個幾次函數的圖形嗎？

學生： 二次函數圖形。

老師： 沒錯，那我們知道二次函數圖形可以寫成甚麼形式？

學生： $y = a(x-h)^2 + k$，(h, k)為圖形頂點。

老師： 很好喔～那我們對照題目 $y = 2(x-2)^2 + 3$ 的 a、h、k 值分別為多少？

學生： $a = 2$、$h = 2$、$k = 3$。

老師： OK，那現在透明片反轉，也就是做一個與 $y = 2(x-2)^2 + 3$ 對稱於 $y = 3$的圖形。

老師： 課本有說我們要做一個與原二次函數做一個上下顛倒的圖形時要怎麼做嗎？

學生： 將 a 值加上負號；h、k 其值不變。

老師： 沒錯，所以 2加上負號，得到 -2，其他數字不變；所以得到變換後的函數為 $y = -2(x-2)^2 + 3$。

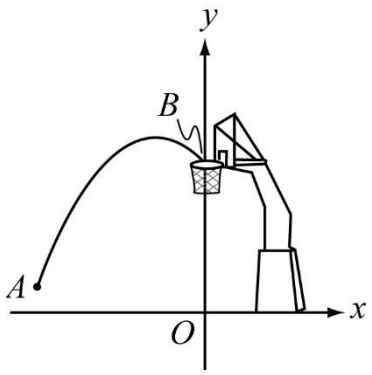
例題二

說明：本題是讓學生熟悉使用二次函數公式，並了解二次函數參數的幾何意義。
Students can get familiar with the quadratic equation and understand the geometric meaning of quadratic equation.

（英文）It is a coordinate plane on the right. Point B represents a basket and is on the axis y. A player throws a ball into the basket through point B from point A and makes a parabola. If the parabola is a graph of a function, what function is it?

（中文）圖是一坐標平面。已知籃框位置 B 點在 y 軸上，今有一選手將球從 A 點的位置投出，球經過的路徑是拋物線，由 B 點空心進籃。若此拋物線是下列某一函數的圖形，則此函數為何？

(A) $y = 6 - (x + 2)^2$
(B) $y = 6 - (x - 2)^2$
(C) $y = 6 + (x - 2)^2$
(D) $y = 6 + (x + 2)^2$



（92 年第二次國中基測 28）

Teacher: First, it says the path of the projected ball is a parabola, what function is it?
Student: Quadratic function.
Teacher: Yes. What is the equation of quadratic function?
Student: $y = a(x - h)^2 + k$, and (h , k) is the vertex of the graph.
Teacher: Great. We know that a is positive or negative affects the direction the parabola opens. When it opens upwards, $a > 0$; when it opens downwards, $a < 0$. Since the path of the ball is a parabola that opens downwards, $a < 0$.
Teacher: Next, we can see that the vertex of the graph is located on Quadrant II, so $h < 0$, and $k > 0$.
Teacher: So the answer is (A).

老師： 首先題目說球經過的路徑是拋物線，所以是個什麼函數？
學生： 二次函數！

老師： 沒錯，那二次函數都可以寫成什麼形式？

學生： $y = a(x-h)^2 + k$，(h, k)為圖形頂點。

老師： 很好，我們知道 a 的正負值和開口方向有關係，當開口向上時，$a > 0$；開口向下時，$a < 0$，所以我們觀察球經過路徑，是個開口向下的拋物線圖形，所以 $a < 0$。

老師： 接著觀察圖形頂點為於第二象限，所以得到 $h < 0$，$k > 0$。

老師： 因此只有(A)符合我們要求，所以答案選(A)。

單元三　四分位數與盒狀圖
Quartiles and Boxplots

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■ 前言 Introduction

此單元利用平均數以外的其他數值讓學生了解數據的分布。請老師適當協助學生複習平均數、中位數與長條圖的概念與詞彙。其中，四分位數與中位數的關係也應加以說明。

■ 詞彙 Vocabulary

單字	中文	單字	中文
quartile	四分位數	median	中位數
boxplot	盒狀圖	variation	變化的程度
outlier	離群值	range	範圍
minimum	最小值	mean	平均值
quarter	四分之一	first	第一
Box-and-whisker plot	盒鬚圖	second	第二
whisker	鬚	third	第三
maximum	最大值	half	一半
interquartile range	四分位距	odd	奇數
distribution	分布	even	偶數

繪製盒狀圖
1. Draw a number line using the minimum and maximum values as the endpoints. 2. Draw a box using the first quartile and third quartile as the boundaries. 3. Draw a line through the median inside the box. 4. Draw whiskers from the box to the minimum and maximum values.

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ _____ range of a dataset is _____.

例句(1)：The range **of a data set is** the difference between the maximum value and the minimum value.

數據裡的最大值與最小值的差是該組數據的範圍。

例句(2)：The interquartile **range** (IQR) **of a dataset is** the difference between the third quartile and the first quartile.

數據裡的第三四分位數與第一四分位數的差是該組數據的四分位距。

❷ _____ divide the dataset into _____.

例句(1)：The median (second quartile, Q2) **divides the data set into** two halves.

中位數將一組數據分成兩半。

例句(2)：Quartiles **divide the dataset into** four equal parts.

四分位數將一組數據分成四等份。

❸ What percentage of the data is represented by _____?

例句(1)：**What percentage of the data is represented by** the upper / lower whisker in the boxplot?

盒狀圖的長鬚代表整體資料中的多少比例？

例句(2)：**What percentage of the data is represented by** the box in the boxplot?

盒狀圖的盒子代表整體資料中的多少比例？

❹ Because there are ___ in the dataset, _____.

例句(1)：**Because there are** 24 numbers **in the dataset**, each group should have 6 numbers.
因為資料集裡有 24 個數，每一等份會有 6 個數。

例句(2)：**Because there is** an odd number of values **in the dataset,** the lower half should have $\frac{n-1}{2}$ number of values.
因為資料集裡有奇數個數值，數據較小的那一半應該有 $\frac{n-1}{2}$ 個數值。

❺ Make a boxplot that represent _____.

例句(1)：**Make a boxplot that represents** the heights of the ninth-graders.
製作一個代表九年級身高的盒狀圖。

例句(2)：We use the five-number summary to **make a boxplot that represents** the lengths of the songs.
我們用五數概括法來製作歌曲長度的盒狀圖。

❻ Order _____ from least to greatest (from greatest to least).

例句(1)：The first step of making a boxplot is to **order** the data values **from least to greatest.**
繪製盒狀圖的第一步是將資料集裡的數值從小排到大。

例句(2)：The minimum, first quartile, median, third quartile, and maximum are **ordered from least to greatest.**
最小值、第一四分位數、中位數、第三四分位數和最大值是由小至大排列。

❼ …data value(s) from the set.

例句(1)：The first quartile and the third quartile are not necessarily the **data values from the set.**
第一和第三四分位數的數值不一定是資料集裡的數值。

例句(2)：The outlier is an extreme **data value from the set.**
離群值是資料集裡的極端值。

❽ The lengths of the whiskers are_____.

例句(1)：Even though the same number of data values (25% of the data values) is represented by each whisker, **the lengths of the whiskers are** very different.
即便盒狀圖的左右兩條長鬚都代表四分之一等份資料集的數值，鬚的長度卻是相當不同。

例句(2)：If the distribution of the data values is symmetric, **the lengths of the whiskers are** about the same.
如果資料是對稱分布，盒鬚圖左右兩條鬚的長度會幾乎一樣長。

❾ … takes up at least _____ of the data values.

例句(1)：The number of data values less than or equal to the first quartile **takes up at least one-quarter of the data values.**
小於或等於第一四分位數(Q1)的資料筆數至少占全部資料筆數的四分之一。

例句(2)：The number of data values greater than or equal to the first quartile **takes up at least three-quarters of the data values.**
大於或等於第一四分位數(Q1)的資料筆數至少占全部資料筆數的四分之三。

❿ How many _____ are in _____?

例句(1)：**How many** data values **are in** the dataset?
資料集裡共有幾筆資料？

例句(2)：**How many** data values **are in** the upper half of the dataset?
資料集裡的前半部共有幾筆資料？

■ 問題講解 Explanation of Problems

☙ 說明 ❧

We learned the mean and median before. However, these two values are not enough to demonstrate the distribution of the dataset. In this section, we will learn the concept of quartiles. We then adopt the five-number summary to make a boxplot representing the dataset. When the boxplot is constructed, we interpret the data.

❧運算問題的講解 ☙

例題一

說明：此題為奇數個數值的四分位數求法。

（英文）The following data values represent the prices (in New Taiwan Dollars) of drinks in the school store.
40, 25, 22, 28, 30, 35, 26, 29, 20, 25, 40
Find the three quartiles.

（中文）以下數據代表某校內商店的飲料價格（以新台幣計），試找出第一、第二和第三四分位數。
40, 25, 22, 28, 30, 35, 26, 29, 20, 25, 40

Teacher: The first step in finding quartiles is to order the data values from least to greatest. What are these ordered numbers?

Student: They are 20, 22, 25, 25, 26, 28, 29, 30, 35, 40, 40.

Teacher: Correct.
There are eleven values here. Eleven is an odd number.
Because there is an odd number of values in the data set, both the lower and upper halves have five numbers. Six is half of the quantity eleven plus one. So, the sixth number—twenty-eight—represents the median of the dataset.
Try to use similar logic to find the first quartile.

Student: There are five values in the lower half. Half of the quantity five plus one is three. The median of the lower half is the third number. It is twenty-five.

Teacher: Good job! Anyone knows the third quartile?

Student: The first quartile is the third number. The third quartile is the third number counting from the end. It is thirty-five. Thirty-five is the third quartile.

Teacher: Very good.

老師： 求四分位數的第一步是將數值由小到大排序。請問排列後的數列？

學生： 20、22、25、25、26、28、29、30、35、40、40。

老師： 正確。

這裡有 11 個數字。11 是奇數。

因為數值的數量是奇數個，所以前半部分和後半部分各有五個數字。

6 是 11＋1 的一半。因此第六個數字 28，28 代表這些數值的中位數。

我們要試著用類似的方法求出第一四分位數。

學生： 前半部分有五個值。5+1的一半是 3。前半部分的中位數是指第三個數字，也就是 25。

老師： 做得好！那有人知道第三四分位數嗎？

學生： 第一四分位數是第三個數字。第三四分位數是從結尾開始倒數的第三個數字。它是 35。35 是第三四分位數。

老師： 很好！

例題二

說明：此題為偶數個數值的四分位數求法。

（英文）The following data values represent the ages of the members of a teachers' sports club.

24, 32, 29, 23, 25, 22, 38, 24, 28,　30, 45, 27

Find the three quartiles.

（中文）以下數據代表某校參加運動社團的教師年齡，試找出第一、第二和第三四分位數。

24, 32, 29, 23, 25, 22, 38, 24, 28, 30, 45, 27

Teacher: The median is the second quartile. It divides the dataset into halves. How many numbers will be in the upper half if there are 12 numbers in the data set? How many are in the lower half?

Student: Twelve divided by two is six. So, there are six numbers in the upper half. There are

six numbers in the lower half.

Teacher: Because twelve divided by two is six, the median would be the average of the sixth and seventh numbers. What is it?

Student: Twenty-two plus thirty-eight is sixty. Sixty divided by two is thirty. Therefore, the median is thirty.

Teacher: Does anyone disagree with this answer? Explain your reasoning.

Student: I think thirty is not the median because the data values are not ordered from least to greatest.

Teacher: Great!
Now, let's order these data values first. What are they?

Student: They would be 22, 23, 24, 24, 25, 27, 28, 29, 30, 32, 38, 45.
The median is twenty-seven point five.

Teacher: The first quartile is actually the median of the lower half. What is it?

Student: There are six numbers in the lower half. When we divide six by two, we get three. Therefore, the first quartile is the average of the third and fourth numbers. It is twenty-four.

Teacher: The third quartile is actually the median of the upper half. What is it?

Student: There are six numbers in the upper half. Therefore, the third quartile is the average of the third and fourth numbers counting from the end. It is thirty-one.

Teacher: Thanks for sharing. Very good.

老師： 中位數是第二四分位數，會把資料分成兩半。如果數據資料總共有 12 個數字，上半部會有多少個數字？下半部有多少個數字？

學生： $\frac{12}{2} = 6$。因此上半部有六個數字，下半部也有六個數字。

老師： 因為$\frac{12}{2} = 6$，中位數會是第六個和第七個數值的平均值，是多少呢？

學生： $22 + 38 = 60$，$\frac{60}{2} = 30$。因此，中位數是 30。

老師： 有人有不同意見嗎？說看看。

學生： 我認為 30 不是中位數，因為數據沒有由小排到大。

老師： 很好！現在，將數值重新排列。

學生： 得出 22、23、24、24、25、27、28、29、30、32、38、45。中位數是 27.5。

老師： 第一四分位數是前半部的中位數。這是多少？

學生：前半部有六個數字，$\frac{6}{2}=3$。因此，第一四分位數等於第三個和第四個數的平均，為 24。

老師：那麼，第三四分位數是後半部的中位數，是多少呢？

學生：後半部一樣有六個數字，$\frac{6}{2}=3$。因此，第三四分位數等於倒數第三個和第四個數字的平均，為 31。

老師：謝謝同學的回答，很棒喔。

例題三

說明：此題為已分組資料的四分位數計算。

（英文）The frequency table below summarizes the daily internet usage (hours) for Class 902 on the weekend. Find the interval where the quartiles are.

（中文）以下的次數分配表摘要了九年二班學生於週末的網路使用時數。請找出第一、第二與第三四分位數坐落的區間。

daily internet usage (hr)	0 ~ 1	1 ~2	2 ~ 3	3 ~ 4	4 ~ 5	5 ~ 6	6 ~ 7
frequency	2	10	5	6	3	3	1

Teacher: How many students are in Class 902 in total?

Student: The sum of these seven cells is thirty. There are 30 students in Class 902.

Teacher: If we order the numbers from least to greatest, which one is the median?

Student: Thirty divided by two is fifteen. The median is the average of the fifteenth and sixteenth numbers.

Teacher: Try to find the cumulative sum of the cells. In which interval do the fifteenth and sixteenth number locate?

Student: The median locates in the interval, two to three, because the cumulative sum of the first three cells is seventeen. It includes the fifteenth and sixteenth numbers.

Teacher: There are fifteen numbers in the first half. Where is the median of the first half located?

Student: The eighth number is the median of the first half because half of the quantity fifteen plus one is eight.

Teacher: Check the cumulative sum. Where is the median of the first half located?

Student: It is in the interval, one to two.

Teacher: Great!

It means that the first quartile is in the interval of one to two.

Where is the median of the second half located?

Student: Three-quarters of the dataset is twenty-two point five ($\frac{3}{4} \times 30 = 22.5$).

Twenty-three is the smallest integer that is greater than twenty-two point five. So, the twenty-third number is the median of the second half.

Teacher: Can any of you tell me the value of the third quartile based on the cumulative sum?

Student: It is right in the interval of three to four.

Teacher: Good job! If you count backward, this interval is the interval where the eighth number is located. So, it is in the interval of three to four.

老師： 九年二班總共有多少學生？

學生： 這七格的和是 30。九年二班有 30 個學生。

老師： 如果我們將數字從小到大排序，哪一個是中位數？

學生： $\frac{30}{2} = 15$，中位數是第 15 和第 16 個值的平均值。

老師： 加總格子內的數值。第 15 和第 16 個值在哪個區間裡？

學生： 前三個格子累計是 17，包含第 15 和第 16 個值，因此中位數在 2-3 小時的區間內。

老師： 前半部有 15 個數字，前半部中位數在哪裡？

學生： 前半部的中位數是第 8 個，因為 $\frac{15+1}{2} = 8$。

老師： 檢查累計的和。前半部的中位數在哪裡？

學生： 它在 1 到 2 的區間內。

老師： 好！代表第一四分位數在 1~2 小時的區間內。

後半部的中位數在哪裡呢？

學生： 全部資料數的四分之三是 22.5。23 是大於 22.5 的最小整數。所以後半部的中位數為第 23 個數字。

老師： 有人可以告訴我根據累積和計算的第三四分位數的值嗎？

學生： 落在 3~4 小時的區間。

老師： 好！你也可以從後面數回來，倒數第 8 個值所在的區間也會位於 3~4 小時的區間內。

ଌ應用問題 / 會考素養題ଓ

例題一

說明：本題涉及盒狀圖的簡易判讀。針對學習落後的同學，教師可以從盒狀圖的端點、盒子邊界的統計意義切入。

（英文）There are the same number of students in Jia and Yi classes. Below are the boxplots of their math scores. Let a and b be the medians of Jia's and Yi's math scores.

Let c and d be the number of students with math scores above 80. Which of the following is the correct order of a and b? Which of the following is the correct order of c and d?

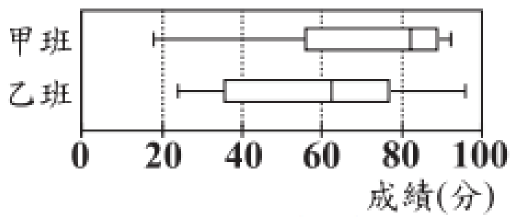
（中文）已知甲、乙兩班的學生人數相同，圖（九）為兩班某次數學小考成績的盒狀圖。若甲班、乙班學生小考成績的中位數分別為 a、b；甲班、乙班中小考成績超過 80 分的學生人數分別為 c、d，則下列 a、b、c、d 的大小關係，何者正確？

(A) $a > b$，$c > d$

(B) $a > b$，$c < d$

(C) $a < b$，$c > d$

(D) $a < b$，$c < d$



圖（九）

（107 年國中會考 19）

Teacher: The median is the bar inside the box. Check the boxplots: which bar has the greater value?

Student: As we can see from the boxplots, Jia's bar is around 80 and Yi's bar is around 60. Jia's bar is greater than Yi's bar. So, a is greater than b.

Teacher: Great.

Next, we would like to check the percentage of students with math scores above 80. In Jia's class, more than 50% of students have scores above 80 because the median is above 80 How about Yi's class?

Student: In Yi's class, I think less than 25% of students have scores above 80 because it only has a short whisker.

Teacher: You are correct. So, from c and d, which is greater?

Student: C is greater.

Teacher: Based on what we said above, the correct choice is (A).

老師： 中位數是盒狀圖內的直線。請觀察哪個盒狀圖內線段的值較大？

學生： 從盒狀圖我們可以看出，甲班的線段約位在 80 左右，而乙班的約在 60 左右。甲班的中位數大於乙班中位數，所以 $a > b$。

老師： 太好了！接下來，我們要求小考成績超過 80 分的學生人數百分比。甲班有超過 50%的學生得分高於 80 分，因為中位數在 80 以上。那乙班呢？

學生： 我認為乙班只有不到 25%的學生得分超過 80 分，因為盒部右側橫線非常短。

老師： 你對。那麼 c 和 d 哪個比較大呢？

學生： C 比較大。

老師： 很好，正確答案是(A)。

例題二

說明：本題涉及已分組成績的判讀。針對學習落後的同學，教師可以先複習平均數、四分位數的意義。

（英文）The table shows the statistics of 55 students' math scores in Class Ji. Comparing males' with females' math scores, which of the following is correct?

(A)The interquartile range is greater for males than females.

(B)The interquartile range is smaller for males than females.

(C)The mean math score is greater for males than females.

(D)The mean math score is smaller for males than females.

（中文）表（一）為甲班 55 人某次數學小考成績的統計結果，關於甲班男、女生此次小考成績的統計量，下列敘述何者正確？

(A)男生成績的四分位距大於女生成績的四分位距

(B)男生成績的四分位距小於女生成績的四分位距

(C)男生成績的平均數大於女生成績的平均數

(D)男生成績的平均數小於女生成績的平均數

表（一）

成績(分)	50	70	90
男生(人)	10	10	10
女生(人)	5	15	5
合計(人)	15	25	15

（105 年國中會考補考 16）

Teacher: The mean is the total scores divided by the number of students. Please calculate the total scores for male and female students.

Student: The sum total of scores for all male students is: $50 \times 10 + 70 \times 10 + 90 \times 10$ which equals 2100.

The sum total of scores for all female students is: $50 \times 5 + 70 \times 15 + 90 \times 5$ which equals 1750.

Teacher: Great. We then divide the total scores by the number of students.

What is the mean score of males' and females' math scores?

Student: For males, the mean score is the quotient of 2100 divided by 30. It is 70.

For females, the mean score is the quotient of 1750 divided by 25. So, it is 70 as well.

Teacher: In this case, the males' mean score is the same as the females' mean score.

Let's compare the interquartile range. We have to find the first and third quartiles each for the males' and for the females' math scores.

Who wants to give it a try?

Student: Me. I would like to start with males' math scores. There are 30 male students in all. There are 15 students in the lower half. The first quartile would be the 8^{th} score.

It is 50 because 10 male students have 50 as their math scores. If we count the number of students with the greatest score, the third quartile is 90 because 10 male students have 90 as their math scores. Therefore, the interquartile range is 40.

Teacher: Very good. Who wants to share your calculations for the interquartile range of females' scores?

Student: Let me try.

There are 25 female students. It means there are 12 students in the lower half. The first quartile would be the average of the 6^{th} and 7^{th} scores. It is 70 because there are only5 students with50 as their math score.

Similarly, if we count it backwards, the third quartile would be 70 as well because there are only5 students with 90 as their math score. So, in this case, the interquartile range is zero.

Teacher: Based on what student X (student's name) and student Y (student's name) said, the correct choice is A.

老師： 平均值是總分數除以學生人數。請求出男、女生成績的總分。

學生： 男生成績總分是：$50 \times 10 + 70 \times 10 + 90 \times 10=2100$；

女生成績總分是：$50 \times 5 + 70 \times 15 + 90 \times 5=1750$。

老師： 很棒，接下來將總分數除以學生人數。

學生： 男生成績平均分數是 2100 除以 30 等於 70 分；
女生成績平均分數是 1750 除以 25 等於 70 分。

老師： 在這種情況下，男生成績的平均數與女生成績的平均數相同。
現在讓我們來比較四分位距。我們要找到男生和女生數學成績的第一四分位數和第三四分位數。誰要試一下？

學生： 我。想從男生成績開始。男生總共有 30 人，其中有 15 人的成績在中位數以下。第一四分位數會是第 8 個數字，也就是 50 分，因為男生有 10 位的成績為 50 分。如果我們計算最高分的學生數量，第三四分位數是 90 分，因為男生有 10 位的成績為 90 分。因此，四分位距為 40。

老師： 非常好。誰想要試著找出女生成績的四分位距？

學生： 我試試看。女生總共有 25 人，因此前半部有 12 個學生。第一個四分位數將是第 6 和第 7 個分數的平均值，也就是 70 分，因為只有 5 位學生成績是 50 分。同樣地，如果我們從後往前數，第三四分位數也是 70 分，因為只有 5 位學生成績是 90 分。因此，在這種情況下，四分位距為 0。

老師： 根據剛才兩位同學的說法，正確答案是(A)。

例題三

說明：此題主要為利用盒狀圖判斷學生在數學成績的長期表現。教師可以讓學生透過解讀圖表來求取對應數值。

（英文）Below are Min's and May's math test scores and the related boxplots.

(a.) Find the ranges of Min's and May's math test scores.

(b.) Find the medians of Min's and May's math test scores.

(c.) Use the medians, ranges, and boxplots to decide who had better performance overall.

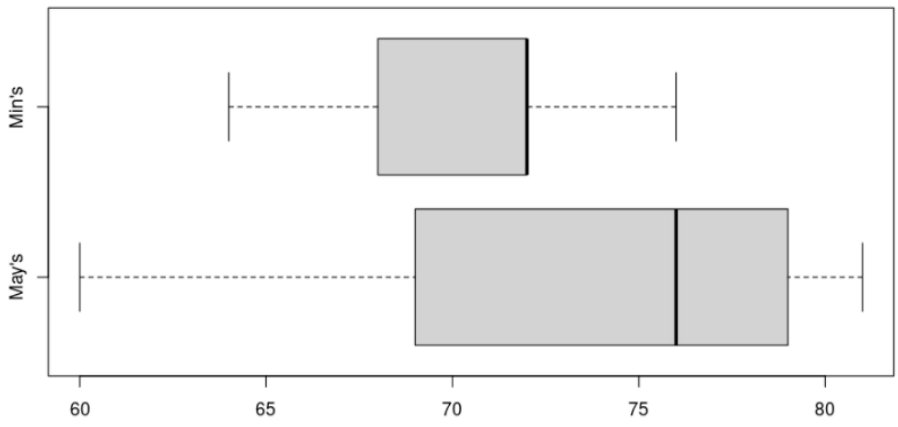
（中文）小明和小美兩人 10 次數學小考分數如下，

(a.) 小明和小美這 10 次數學小考分數的全距各為多少分？

(b.) 小明和小美這 10 次數學小考分數的中位數各為多少分？

(c.) 利用中位數、全距和盒狀圖判斷，誰的數學考試表現比較好？

	Min	May
1	73	60
2	68	79
3	72	80
4	76	81
5	71	69
6	72	78
7	64	69
8	72	74
9	72	74
10	65	78



Teacher: The range is the difference between the maximum value and the minimum value. They are the endpoints of the boxplot. For Min's boxplot, the minimum is below 65, and the maximum is above 75. Please look for the exact values of minimum and maximum.

Student: The maximum is 76, and the minimum is 64. So, the range is the difference between 76 and 64, which is 12.

Teacher: We can use similar logic to find the range of May's math score. Can anyone tell me your answers?

Student: The maximum of May's score is greater than 80, and the minimum is 60. Looking at the values in the table, the maximum of May's score is 81, and the minimum is

60. So, the range is 21.

Teacher: We can get the values of the median by looking at the boxplots. Min's median is above 70, and May's is above 75. If we order their scores from lowest to highest, we have to find the average between the 5th and 6th values. Who wants to try?

Student: The lower half of Min's score is 64,65,68, 71, and 72. Then the upper half is 72, 72,72,73, and 76. So Min's median is 72.

Teacher: Great. Can any of you tell us the exact value of May's median?

Student: If we order May's score from lowest to highest, the first six numbers are 60, 69,69,74,74, and78. May's median is the average between 74 and 78. It is 76, and it is definitely above 75.

Teacher: Great job! Who do you think has a better performance, May or Min, in the math test?

Student: May's median is greater than Min's. So, I think May had the better performance.

Teacher: Based on the ranges, who had the more consistent performance?

Student: The range of May's score is greater than Min's. It means that May has more variable test performances than Min.

Teacher: The last step is to check the boxplots. In general, the math scores are much higher for May than Min. May exceeds Min's maximum math score in over 50% of tests. So, we can see that May has better performance generally.

老師： 最大值和最小值的差我們稱為全距（range），是盒狀圖的兩個端點。小明的盒狀圖，最小值在 65 以下，最大值在 75 以上。請找出最小值和最大值的確切數值。

學生： 最大值是 76，最小值是 64。所以全距是 76 和 64 的差，為 12。

老師： 我們可以用類似的邏輯找到小美的數學分數全距。誰會？

學生： 小美的最大值大於 80，最小值為 60。從表格中看，小美的最大值為 81，最小值為 60。所以全距為 21。

老師： 我們可以觀察盒狀圖來找出中位數。小明的中位數在 70 以上，小美的中位數在 75 以上。如果我們將他們的分數從低到高排序，我們需要找到第 5 和第 6 個值之間的平均值。誰想試試？

學生： 小明的分數的前半部分是 64、65、68、71 和 72。後半部分是 72、72、72、73 和 76。所以小明的中位數為 72。

老師： 太棒了。那有人能告訴我小美的中位數數值嗎？

學生： 如果我們將小美的分數從低到高排序，前六個數字是 60、69、69、74、74 和 78。

小美的中位數是 74 和 78 的平均值。它是 76，高於 75。

老師： 做得好！根據中位數，誰表現比較好？

學生： 小美的中位數比小明的高，所以小美表現比較好。

老師： 根據全距，誰的成績更穩定？

學生： 小美的全距比小明大，代表小美的考試成績比較不穩定。

老師： 最後是檢查盒狀圖。整體來說，小美的數學分數遠高於小明。小美有超過 50%的考試分數高於小明的最高分數。因此，我們可以看出，小美的整體表現比較好。

單元四　機率與樹狀圖
Probability and Tree Diagrams

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■ 前言 Introduction

此單元教授機率的定義與計算方式，涉及分數的各種英文說法。教師可以視情況複習分數的英文說法。常見的機率題型，包含丟擲硬幣、骰子與抽取撲克牌。建議教師與學生熟悉其相關用詞，以利課堂講解。

■ 詞彙 Vocabulary

單字	中文	單字	中文
trial	試驗	probability	機率
event	事件	experiment	試驗
outcome	結果	sample space	樣本空間
tree diagram	樹狀圖	likely	很可能的
occur	發生	unlikely	不可能的
fair	公正的	coin	硬幣
toss	丟擲	head	（硬幣的）正面
dice	骰子	tail	（硬幣的）反面
draw	抽、取出	roll	滾動

playing card(s)	紙牌	suit	（紙牌的）花色
spade	（紙牌的）黑桃	diamond	（紙牌的）紅磚
club	（紙牌的）黑梅花	heart	（紙牌的）紅心
face card	人頭牌	joker	鬼牌

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶The _____ is a measure of_____.

例句(1)：**The** probability of an event **is a measure of** how likely it is an event will occur.
一個事件的發生機率是指該事件會發生的可能性。

例句(2)：**The** probability of precipitation (rain) **is a measure of** the confidence that precipitation will occur in a given area.
降雨率是測量特定區域會下雨的信心。

❷ Toss/Flip/Roll/Draw a _____.

例句(1)：**Toss/ Flip a** fair coin.
丟擲一枚公正的硬幣。

例句(2)：**Roll a** six-sided dice.
滾動一個六面的骰子。

例句(3)：**Draw a** card from a standard deck of playing cards.
從標準的撲克牌組裡抽取一張牌。

❸ How many possible outcomes are in the sample space _____?

例句(1)：**How many possible outcomes are in the sample space** when we roll a six-sided dice?
丟擲一顆公正的骰子時，樣本空間裡共有幾種可能的結果？

例句(2)：**How many possible outcomes are in the sample space** when we toss three coins?
丟擲三個硬幣時，樣本空間裡共有幾種可能的結果？

❹ What is the total number of possible outcomes _____?

例句(1)：**What is the total number of possible outcomes** when we roll a six-sided dice?
丟擲一顆公正的骰子時，共有幾種可能的結果？

例句(2)：**What is the total number of possible outcomes** when we flip three coins?
丟擲三枚公正的硬幣時，共有幾種可能的結果？

❺ The probability of _____.

例句(1)：**The probability of** a student guessing exactly one answer correctly is one-fourth.
學生猜對答案的機率是四分之一。

例句(2)：Find **the probability of** each event.
找出每一個事件的機率。

❻ What is the probability that _____?

例句(1)：**What is the probability that** you roll an even number?
你獲得偶數字的機率為何？

例句(2)：**What is the probability that** none of the cards you drew is black?
你沒抽到黑色卡的機率為何？

❼ the sum of the _____.

例句(1)：**The sum of the** probabilities of all possible outcomes is 1.
所有可能結果的機率總和為一。

例句(2)：Find the probability that **the sum of the** two numbers rolled is 7.
找出數字和為七的機率。

❽ _____ branch of the tree diagram _____.

例句(1)：The first **branch of the tree diagram** is a list of all possible outcomes for the first event.
樹狀圖的第一層分支列示事件的所有可能結果。

例句(2)：Each **branch of the tree diagram** represents a possible outcome of the experiment.
樹狀圖的每一個分支代表實驗的可能結果。

■ 問題講解 Explanation of Problems

☙ 說明 ❧

In this section, we will cover the basic calculation of probabilities. Students learn to list the possible outcomes to help them determine the size of the sample space. A tree diagram serves as another aid for students to find the probability.

❧ 運算問題的講解 ☙

例題一

說明：此題為機率的基本題型，包含樣本空間的計算。

（英文）Roll a six-sided dice.

(a) List the possible outcomes in the sample space.
(b) Find the probability of rolling a 3.
(c) Find the probability of rolling an odd number.
(d) Find the probability of rolling a 10.

（中文）滾動一顆六面的骰子。

(1) 列出樣本空間裡的可能結果。
(2) 找出結果為“3”的機率。
(3) 找出結果為奇數點的機率。
(4) 找出結果為“10”的機率。

Teacher: Here I have a six-sided dice. How many possible outcomes are there if I roll this dice?
Student: Six outcomes. They are 1, 2, 3, 4, 5, and 6.
Teacher: Among these possible outcomes, how many times does “3” occur?
Student: Once. Three occurs once.

Teacher: The probability of an event is the quotient of the number of favorable outcomes divided by the number of total outcomes. What is the probability of rolling a 3?

Student: The probability is one over six.

Teacher: Very good.
As for part (c), which of the possible outcomes are the odd numbers?

Student: An odd number is not divisible by 2. So, 1, 3, and 5 are odd numbers.

Teacher: Using the definition of probability, what is the probability of rolling an odd number?

Student: The probability is 3 over 6. It can be simplified as one-half.

Teacher: Great! Similarly, the probability of rolling an even number is one-half.
The last question is to find the probability of rolling a 10.
Is 10 an element in the sample space?

Student: No.

Teacher: The probability of rolling a 10 is zero.
This means it will not occur.

老師： 我有個六面骰子。如果投擲這個骰子，有多少種可能的結果？

學生： 有六種可能的結果，它們分別是 1、2、3、4、5 和 6。

老師： 在這些可能的結果中，數字「3」有幾次？

學生： 出現一次。

老師： 事件的機率是符合條件的結果數除以總結果數。
擲出「3」的機率是多少？

學生： 機率是 $\frac{1}{6}$。

老師： 好。針對第三小題，在這些可能的結果中，哪些是奇數？

學生： 奇數不可被 2 整除，所以 1、3、5 是奇數。

老師： 利用機率的定義，擲出奇數的機率是多少？

學生： 機率是 $\frac{3}{6}$，可以簡化為 $\frac{1}{2}$。

老師： 很好！同理，擲出偶數的機率也是 $\frac{1}{2}$。
最後一個問題是找出擲出 10 的機率。10 是樣本空間裡的可能結果嗎？

學生： 不是。

老師： 因此，擲出 10 的機率是零。表示不可能發生。

例題二

說明：此題仍為機率的基本題型，教師可以利用實體撲克牌加以解說。

（英文）There are fifty-two cards in a standard deck of playing cards. The four suits are spades, clubs, hearts, and diamonds. Each suit has 13 cards. They are Ace, 2, 3, 4, 5, 6, 7, 8, 8, 10, Jack, Queen, and King. If you draw a card randomly from the playing cards,

(a) What is the probability of getting a black card?

(b) What is the probability of getting a face card?

（中文）一副標準的撲克牌有 52 張卡。四種花色分別為黑桃、梅花、紅心和方塊。每種花色各有 13 張，分別是 A、2、3、4、5、6、7、8、9、10、J、Q、K，從撲克牌中任取 1 張

(1) 抽到黑卡的機率為何？

(2) 抽到人頭卡的機率為何？

Teacher: Half of the playing cards are black, and the other half are red. Each card is equally likely to be drawn. What is the probability of drawing a black card?

Student: One half.

Teacher: Well done! Of course, we can consider it as twenty-six divided by fifty-two. The answer is the same as one half. Are there any questions?

Student: No.

Teacher: The face cards are Jack, Queen, and King. There are four suits. How many face cards in in total?

Student: Three times four is twelve. So, in total, we will have 12 face cards.

Teacher: What is the probability of having a face card when you draw a card from the deck?

Student: Twelve divided by fifty-two, which is three over thirteen.

老師：一副撲克牌有一半花色是黑的，另一半是紅的。假設每張牌被抽中的機會相等。抽到黑卡的機率是多少？

學生：$\frac{1}{2}$。

老師：做得好！當然，我們可以將它視為 $\frac{26}{52}$，跟 $\frac{1}{2}$ 一樣。到這邊有任何問題嗎？

學生：沒有。

老師：接著，人頭卡是 J、Q、K，並且有四種花色。總共有多少張人頭卡？

學生：$3 \times 4 = 12$。所以人頭卡總共有 12 張。

老師：從牌堆中抽一張牌時，出現人頭卡的機率是多少？

學生：$\frac{12}{52}$，也就是 $\frac{1}{13}$。

例題三

說明：此題為應用樹狀圖計算機率的基本題。

（英文）Draw a tree diagram to find

(a) the probability of getting heads when you flip the coin.

(b) the probability of getting heads at the second flip when you flip a coin twice.

（中文）利用樹狀圖找到

(1) 丟擲一枚銅板時結果正面的機率。

(2) 丟擲一枚銅板兩次時，第二次結果為正面的機率。

Teacher: The possible outcomes of flipping a coin are heads or tails. They are equally likely. What is the probability of getting heads?

Student: It is one over two, which is a half.

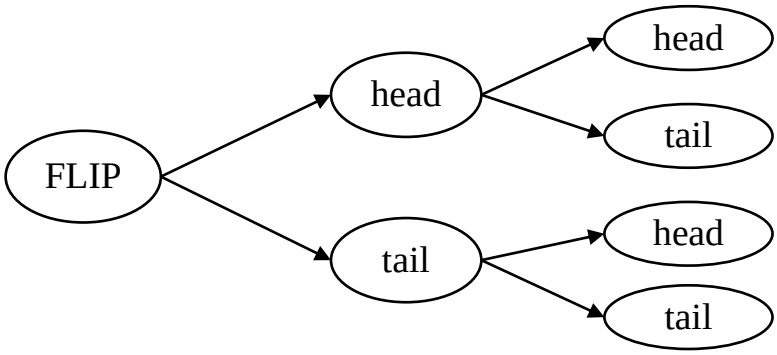
Teacher: Very good. Do you think the answer to the second question will be different from a half?

Student: Of course. If you get heads on the first flip, there is less chance of getting heads again.

Teacher: There is a problem with your reasoning because you might also get tails on the first flip.

Student: How do we list the possible outcomes?

Teacher: We can use a tree diagram to list the possible outcomes.
There are four possible outcomes, including (H, H), (H, T), (T, H) and (T, T).
(H, H) represents getting heads on the first flip, and heads on the second flip.
Among these outcomes, how many outcomes get heads on the second flip?



Student: There are two outcomes that get heads on the second flip.

Teacher: What is the probability of getting heads on the second flip?

Student: Two over four, which is a half.

Teacher: Very good!

老師： 丟一枚硬幣的可能結果有正面和反面，兩者的出現的機率相等。
那麼結果是正面的機率是多少呢？

學生： $\frac{1}{2}$，也就是一半

老師： 非常好。你們認為第(2)小題的答案會是一半嗎？

學生： 不會，如果第一次丟到正面，第二次再出現同為正面的機率會降低。

老師： 你的解釋不太正確，因為你在第一次也可能得到反面呀。

學生： 我們要怎樣列出可能的結果？

老師： 我們可以使用樹狀圖。
有（正，正）、（正，反）、（反，正）和（反，反）四個可能的結果。（正，正）代表在第一次擲幣結果為正面，且第二次擲幣結果也是正面。
在這些結果中，有多少個結果是第二次擲幣擲到正面？

學生： 有兩個結果是第二次擲幣擲到正面。

老師： 那這樣機率是多少？

學生： $\frac{2}{4}$，也就是一半。

老師： 非常好！

ꕥ應用問題 / 會考素養題ꕥ

例題一

說明：此題為計數原理應用於日常生活的實例。

（英文）You have 4 different colors of wrapping paper and 3 shapes of boxes. The colors are red, yellow, baby blue, and light green. The boxes come in rectangular, square and, circular shapes.

(a) How many different ways are there to use one shape of a box and one color of wrapping paper to wrap the package?

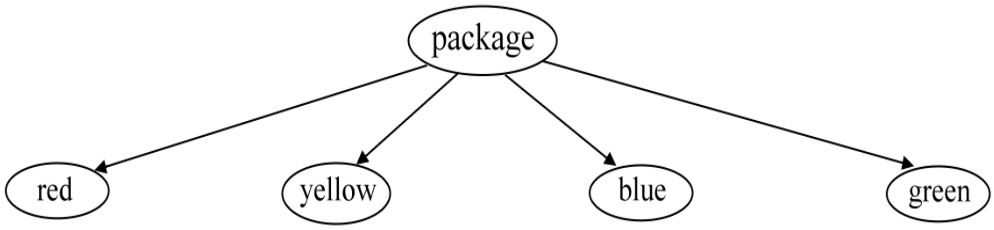
(b) What is the probability of wrapping the rectangular box with yellow paper?

（中文）你有四種顏色的包裝紙和三種形狀的盒子。包裝紙的顏色包括紅色、黃色、淡藍色和淡綠色。盒子有長方形、正方形和圓形三種。

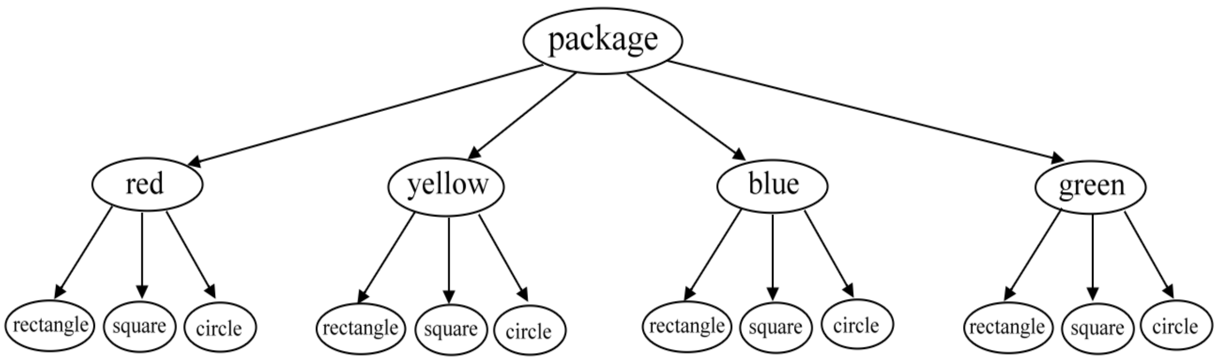
(1) 如果想選擇一個顏色的包裝紙和一個形狀的盒子，總共有幾種選擇方式？

(2) 採用長方形盒子和黃色包裝紙進行包裝的機率為何？

Teacher: There are 4 colors of wrapping paper and 3 shapes of boxes.
Here we can see the first branch of the tree diagram. Who can draw the next branch?



Student: Let me try. There are the three kinds of boxes following each color.



Teacher: Looking at the tree diagram – how many different combinations are there in total?
Student: There are 12 combinations.

Teacher: Very good! Twelve is the size of the sample space.
What is the probability of using a rectangular box with yellow wrapping paper in this situation?
Student: From the tree diagram, there is one situation where we wrap the rectangular box with yellow wrapping paper. The probability should be one over twelve.
Teacher: Great!

老師： 這裡的包裝紙有四種顏色，盒子有三種形狀。現在看到樹狀圖的第一個分支。誰可以畫下一個分支呢？
學生： 讓我試試看。每個顏色都有三種盒子形狀。
老師： 看一下畫出來的樹狀圖，總共有多少種不同的組合呢？
學生： 總共有 12 種。
老師： 很好！12 就是樣本空間的大小。這種情況下，用長方形盒子和黃色包裝紙進行包裝的機率是多少？
學生： 從樹狀圖中可以看出，只有一種情況是長方形盒子和黃色包裝紙進行包裝。機率為 $\frac{1}{12}$。
老師： 太棒了！

例題二

說明：本題涉及到簡單的機率概念。除此之外，老師可以教導學生分數的約分。

（英文）The zoo provides 100 scratch-off lottery tickets for the first 100 customers who enter the zoo. Among the tickets, there are 32 tickets with prizes. The table below shows the types and amounts of each prize. If Bob is the first visitor to enter the zoo and each ticket is equally likely to be selected, what is the probability that Bob receives the doll?

（中文）動物園準備了 100 張刮刮樂，打算送給開幕當日的前 100 位遊客每人一張，其中可刮中獎品的刮刮樂共有 32 張，表（一）為獎品的種類及數量。若小柏為開幕當日的第一位遊客，且每張刮刮樂被小柏拿到的機會相等，則小柏刮中玩偶的機率為何？

(A) $\frac{1}{2}$　(B) $\frac{1}{16}$

(C) $\frac{8}{25}$　(D) $\frac{1}{50}$

表（一）

獎品	數量
北極熊玩偶一個	1
獅子玩偶一個	1
造型馬克杯一個	10
紀念鑰匙圈一個	20

（110 年國中會考 11）

Teacher: What is the size of the sample space?

Student: There are 100 tickets. So, the size of the sample space is 100.

Teacher: Great! How many dolls are in the prize pool?

Student: There is one polar-bear doll and one lion doll. So, there are two dolls in all.

Teacher: The probability is defined as the quotient from the number of favorable outcomes divided by the size of the sample space. What is the probability?

Student: Two out of one hundred, which is one over fifty.

Teacher: Great! The correct answer is D.

老師： 這題的樣本數是多少？

學生： 有 100 張刮刮樂，所以樣本數為 100。

老師： 太好了！獎品中有多少玩偶？

學生： 有一個北極熊玩偶和獅子玩偶，所以總共有兩個玩偶。

老師： 機率的定義是符合條件的結果的數量除以總結果數，那麼機率是多少？

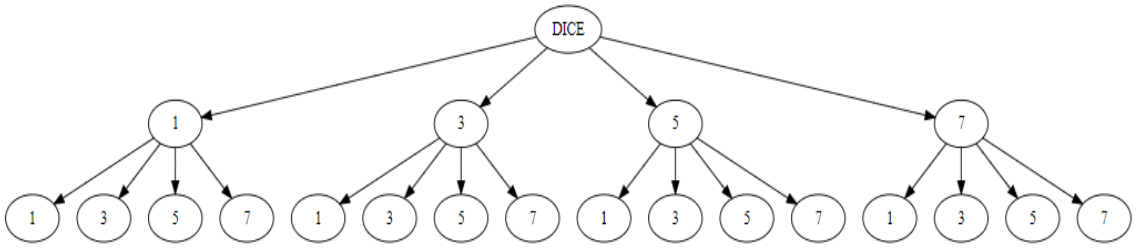
學生： 有兩個符合條件的結果，共一百個可能結果，也就是 $\frac{1}{50}$。

老師： 太好了！正確答案是 D。

例題三

說明：本題主要概念是藉由樹狀圖推論機率的實例。

（英文）The tree diagram shows a sample space of rolling a fair *n*-sided dice twice. The number shown in the oval represents the number shown on the dice.



(a) How many faces does the dice have?

(b) Find the probability of getting a sum equal to 10 when rolling the dice twice.

(c) Find the probability of getting a product as a multiple of 5 when rolling the dice twice.

（中文）樹狀圖上顯示你投擲一個公正的 *n*-面骰子兩次的樣本空間。橢圓裡的數字代表骰子面上的點數。試回答下列問題：

(a) 這個骰子有幾個面？

(b) 點數合等於十的機率為何？

(c) 點數乘積是五的倍數的機率為何？

Teacher: We can figure out the number of sides from the first layer of the tree diagram. In the first layer, there are four ovals. The outcomes include 1, 3, 5, and 7. So there are 4 sides.

What this tree diagram shows is not a classical dice. It has only 4 faces, and the outcome on each face is a different odd number. Are there any questions?

Student: No.

Teacher: To find the probability of the second and third questions, we have to find the size of the sample space first. What is the size of the sample space?

Student: I think the size is sixteen because there are 16 ovals in the last layer.

Teacher: Very good. Now, let's list the possible pair of results so that their sum is 10. For example, three is paired with seven. Are there any other possible pairs?

Student: I know. Five is paired with five, and seven is paired with three.

Teacher: After figuring out the possible pairs, we can find the possibility of having a sum as ten in two rolls. Does anyone know the answer?

Student: There are three possible pairs in all. So the probability is three out of sixteen.

Teacher: Correct. We can apply similar logic to find the probability of the third question. To have the product as a multiple of five, the outcome of one dice has to be five. What are the possible pairs?

Student: Five can be paired with 1, 3, 5, and 7. That is all.

Teacher: If the first roll shows a 1 and the second roll shows 5, does it satisfy the condition?

Student: Yes. I think (1, 5), (3, 5), and (7, 5) are also possible outcomes.

Teacher: In all, how many outcomes have the product as a multiple of 5?

Student: There are seven outcomes. So the probability is seven over sixteen.

Teacher: Very good.

老師： 我們可以從樹狀圖的第一層推斷出面的數量。第一層有四個橢圓形，結果包括 1、3、5 和 7。因此這個骰子有 4 個面。
我們可以看出這不是傳統的骰子，只有 4 個面，每個面的結果都是不同的奇數。到這邊有任何問題嗎？

學生： 沒有。

老師： 要找出第(2)和第(3)小題的機率，我們必須先找出樣本空間的大小。樣本空間是多少呢？

學生： 我認為是 16，因為最下面那一層有 16 個橢圓形。

老師： 非常好。現在，讓我們列出點數總和等於 10 的可能數對。例如，3 跟 7 配對。還有其他可能的數對嗎？

學生： 我知道。5 跟 5 配對、7 跟 3 配對。

老師： 找到所有可能的結果後，我們就可以求擲兩次骰子，點數合等於十的機率。有人知道答案嗎？

學生： 總共有三種可能。因此機率是$\frac{3}{16}$。

老師： 沒錯。接著，我們以此類推來找出第(3)小題的機率。要使點數乘積是五的倍數，其中一個骰子必須骰到 5。有哪些可能的數對呢？

學生： 5 可以與 1、3、5 和 7 配對，就這些了。

老師： 如果第一次擲出的是 1，第二次擲出的是 5，是否滿足條件？

學生： 對！也滿足條件。那麼(1,5)、(3,5)和(7,5)也是可能的結果。

老師： 那這樣總共有多少結果是點數乘積為五的倍數呢？

學生： 有七個結果。因此機率是 $\frac{7}{16}$。

老師： 非常好。

單元五　角柱與圓柱
Prisms and Cylinders

國立新竹科學園區實驗高級中等學校　吳珮蓁老師

■ 前言 Introduction

學生認識角柱與圓柱的分類與命名，認識角柱體積公式，藉由展開圖了解柱體的構成要素與表面積，最後利用習得的知識解決角柱及圓柱相關的生活實例。

■ 詞彙 Vocabulary

單字	中文	單字	中文
edge	邊	polyhedron	多面體（單數）
vertex	頂點	polyhedrons/polyhedra	多面體（複數）
face	面	prism	角柱
parallel	平行的	pyramid	角錐
volume	體積	rectangular prism	四角柱、長方體（柱）
lateral area	側面積	pentagonal prism	五角柱
surface area	表面積	hexagonal prism	六角柱
square centimeter	平方公分	right cylinder	直圓柱

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ _____ is formed by _____.

例句：A rectangular prism **is formed by** six rectangles.

長方體是由六個長方形組成的。

❷ _____ is composed of _____.

例句：A hexagonal prism **is composed of** two hexagonal bases and six rectangular faces.

六角柱是由兩個六角形的底和六個長方形的面組成的。

❸ Find the volume of _____.

例句：**Find the volume of** this cylinder.

找這個圓柱體的體積。

❹ Notice that _____.

例句：**Notice that** the bases are congruent polygons.

請注意底部是全等的多邊形。

❺ Suppose _____.

例句：**Suppose** the bases are regular hexagons with 4 cm edges, find the base area.

假設底面是邊長 4 公分的正六邊形，求底面積。

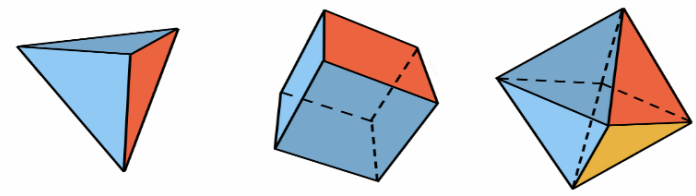
■ 問題講解 Explanation of Problems

☙ 說明 ☙

[Classifying Solids]

In this section, we will learn to name prisms and cylinders by their properties, and find the volume and surface area of these solids.

First, let's start with polyhedrons. A polyhedron is a solid bounded by polygons, like the following solids. These polygons are called faces.

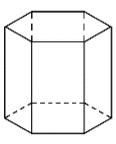


Take this tissue box as an example. This solid is bounded by 6 rectangles. These rectangles are faces. An edge of a polyhedron is a line segment formed by the intersections of two faces. In this tissue box, there are 12 edges. The place where edges meet is the vertex. There are 8 vertices in this tissue box.

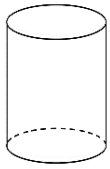
If there are two congruent polygons in parallel planes, the solid is a prism. If there is only one base, the solid is a pyramid and it will look like an Egyptian pyramid. We focus on prisms in this section. In the next section, we will talk more about pyramids.

Both prisms and pyramids are named by their bases. The bases of the tissue box are two congruent rectangles in parallel planes, so the tissue box is called a rectangular prism.

Let's look at this diagram. Is it a prism?



Yes, the two bases are congruent hexagons in parallel planes. This is a hexagonal prism. A hexagonal prism is composed of two hexagonal bases and six rectangular sides. The segment joining the two bases and perpendicular to both is the altitude of the prism. The length is called the "height."



This solid is a cylinder since its two bases are congruent circles in parallel planes.
Two circular bases are joined by a curved surface.

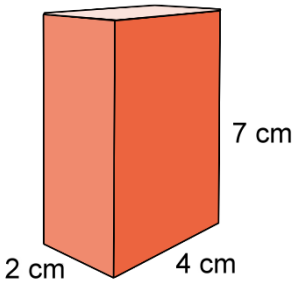
[The Volume of Solids]

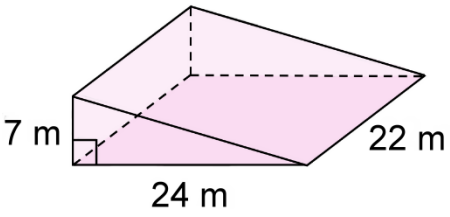
The volume is the amount of space that an object occupies or that is enclosed within a container. This definition comes from an English dictionary. We can find the volume of the prism by the formula:

$$\text{volume} = \text{base area} \times \text{height}$$

You must be familiar with this formula since you learned it in elementary school.

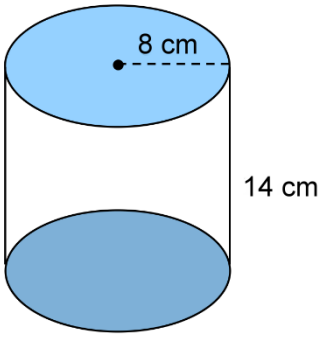
Let's do some examples to review. The first solid is a rectangular prism. Its base is a rectangle 2 cm by 4 cm, and the height is 7 cm.
The base area is $8\ \text{cm}^2$, so the volume is $8\ \text{cm}^2 \times 7\ \text{cm} = 56\ \text{cm}^3$.





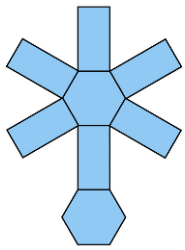
What kind of prism is it? What shape is its base? Is the rectangle lying on the ground its base? Don't be fooled by the solid lying down. Not every face lying on the ground is a base. The bases of a prism are two congruent polygons in parallel planes. In the picture, the rectangles are not congruent in parallel planes, but there are two congruent right triangles.
The base of this prism is a right triangle, and the height of the prism is 22 m.
The base area is $\frac{7\ \text{m} \times 24\ \text{m}}{2} = 84\ \text{m}^2$, and the volume is $84\ \text{m}^2 \times 22\ \text{m} = 1848\ \text{m}^3$.

The formula of volume also works for cylinders. The base of the cylinder is a circle, with the radius 8 cm. Its base area is $8^2\pi = 64\pi$. The height is 14 cm, so the volume of the cylinder is $64\pi \times 14 = 896\pi$ cm^3



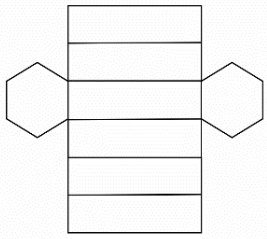
[Surface Area of Solids]

The surface area of a prism is the sum of the areas of its lateral faces and two bases. Take a regular hexagonal prism as an example. The bases are regular hexagons.



This is the net of a regular hexagonal prism. A net shows the faces of 3D objects when it is opened flat. Cut out along the edges, fold along the connected edges, and you can form a hexagonal prism. These rectangles are lateral faces. You can find the area of one rectangle, then times it by 6 and you will get the lateral area.

There are multiple nets of a hexagonal prism. See this one. All the rectangles are connected, and they become a bigger rectangle. The length of this big rectangle is the sum of six sides, which is the perimeter of the hexagon. The width of this big rectangle is the height of the prism. The lateral area can be calculated by multiplying the perimeter of the base by the height of the prism. Both ways lead you to the same answer.



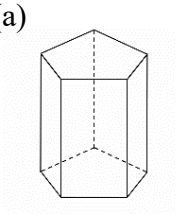
The surface area of a cylinder is equal to the sum of the flat areas and the curved area. The flat areas are two circles. The curved part is the lateral face. How do you find the lateral area? Take out a piece of paper. How do you make a cylinder with this piece of paper? Just roll it until the edges of the paper meet each other. You can make this rectangle into the curved part of the cylinder. So the lateral area of the cylinder is this rectangle. The length of the rectangle is the circumference of the circle $2\pi r$; the width of the rectangle is the height of the cylinder, h. The lateral area equals the circumference times the height, $2\pi rh$

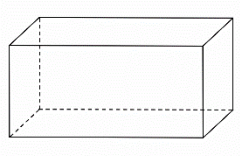
☙運算問題的講解 ❧

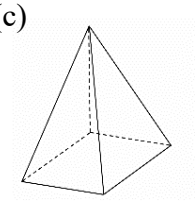
例題一

說明：讓學生辨別多面體及練習命名。

（英文）Decide whether each solid is a polyhedron or not. If it is a polyhedron, name it.
（中文）以下立體圖形何者是多面體。若是多面體，請問為何種多面體？

(a) 

(b) 

(c) 

(d) 

Teacher: What is a polyhedron?
Student: A solid formed by polygons.
Teacher: Can you tell which solid is a polyhedron and name it?
Student: (a) is a prism. The bases are congruent pentagons. They look parallel.
Teacher: Try to name it.
Student: Pentagon prism?
Teacher: Pentagonal prism.
Student: (b) is a rectangular prism.
(c) is a pyramid.
Teacher: Is (c) a polyhedron?
Student: Yes. I see triangles and a square. They are polygons.
Teacher: Is (d) a polyhedron?
Student: I do not see any polygons. So I guess not.
Teacher: Correct. It is not a polyhedron. It is a cone.

老師： 什麼是多面體？
學生： 多面體是由多邊形構成的立體圖形。
老師： 能否判斷出哪個立體圖形是一個多面體嗎？若是，請問為何種多面體？
學生： (a) 是角柱。底面是全等的五邊形，也看起來是互相平行。
老師： 請命名這個圖形。
學生： 五角柱？

老師： 沒錯，叫做五角柱。

學生： (b) 是長方體。(c) 是三角錐。

老師： (c) 是多面體嗎？

學生： 是，我看到三角形和一個正方形，它們都是多邊形。

老師： 那 (d)是一個多面體嗎？

學生： 我找不到多邊形，所以我認為不是。

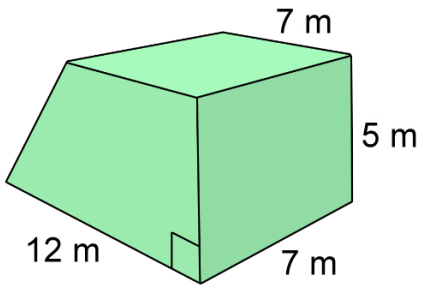
老師： 沒錯，它不是多面體，而是圓錐。

例題二

說明：讓學生練習辨識體積中的底面積，並利用公式求得體積。

（英文）Find the volume of the following prism.

（中文）找出下列柱體的體積。



Teacher: First, find its base. Which shape is the base?

Student: I know. The rectangle is the base.

Teacher: Which rectangle? I see multiple rectangles there.

Student: The one lying on the ground, 7 m by 12 m.

Teacher: Do you see the other congruent rectangle in parallel planes?

Student: There is another rectangle on the top. I think they are parallel.

Teacher: The bases of a prism need to be two congruent polygons in parallel planes. Can you tell their dimensions?

Student: The bottom one is 7 m by 12 m, and the top one is 7 m by…7 m.

Teacher: Are they congruent?

Student: No, they are not the same.

Teacher: I am glad that you can tell the difference. Do you see any congruent polygons in parallel planes?

Student: Trapezoids. I can see one trapezoid from this side. The other trapezoid is on the other side.

Teacher: Good. The trapezoid is the base, so please find its base area.

Student: $(7+12)\times 5\times\frac{1}{2}=47.5\ \text{m}^2$

Teacher: What is the height of the prism?

Student: 5. The height is standing upward.

Teacher: The height of the prism is perpendicular to the base. Is the 5 m-long segment perpendicular to the trapezoid?

Student: No. The 7 m-long segment is perpendicular to the base.

Teacher: Correct. Now can you find the volume?

Student: $47.5\ \text{m}^2\times 7\ \text{m}=332.5\ \text{m}^3$. This lying down prism is too tricky. I got confused several times.

Teacher: Indeed. Students are easily confused with this kind of prism. You have to be clear about the definitions. You can find the base and the height based on their definitions, not just from feelings or the image.

老師： 首先，辨別他的底面是什麼形狀？

學生： 我知道，底面是長方形。

老師： 是哪一個長方形？我看到很多長方形。

學生： 貼在地面上的那個，7 公尺×12 公尺。

老師： 那你們有看到其他平行，且全等於這個貼在地面上的長方形嗎？

學生： 上面有另一個長方形，我覺得也平行。

老師： 柱體的底面必須是兩個平行的全等多邊形。這兩個長方形的尺寸是多少？

學生： 下面的那個是 7 公尺×12 公尺，上面的那個是 7 公尺×…7 公尺。

老師： 有全等嗎？

學生： 沒有耶。

老師： 很棒，分辨出差別了。再找看看有沒有平行的全等多邊形。

學生： 有看到梯形。我可以從側邊看到梯形；另一個梯形則在另一邊。

老師： 很好。所以梯形是這個柱體的底。請算出它的底面積。

學生： $(7+12)\times 5\times\frac{1}{2}=47.5$ 平方公尺。

老師： 高是多少？

學生： 5 公尺。高度是垂直向上的。

老師： 一個柱體的高會跟底垂直。5 公尺的線段跟梯形有垂直嗎？

學生：沒有，7 公尺的線段才跟底面垂直。

老師：沒錯，現在會算體積了嗎？

學生：47.5 平方公尺×7 公尺=332.5 立方公尺。橫躺的柱體好麻煩，搞混好多次。

老師：確實，同學們很容易被躺下的柱體騙到。大家要記清楚柱體的定義，找到底面和高，不能只靠感覺或只看題目的圖。

∞ 應用問題 / 會考素養題 ∞

例題一

說明：利用日常生活可見的容器，引導學生學習不同柱體的側面積和體積。

（英文）A popcorn booth serves popcorn in a cylindrical container and a regular hexagonal container. The radius of the cylindrical container is 6 cm, and the height is 15 cm. Each side of the base of the hexagonal container is 6 cm, and the height is 15 cm.

How much wrapping paper is needed to cover the lateral face of each container?

Which container needs more wrapping paper?

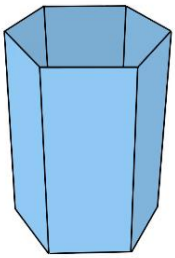
Which container can hold more popcorn?

（中文）爆米花攤把爆米花裝在兩個不同容器中，圓柱體容器和正六角柱容器。圓柱體容器半徑 6 公分，高 15 公分；正六角柱容器的底面每邊長 6 公分，柱體高 15 公分。

每個容器需要多少包裝紙才可以包覆側面？

哪個容器需要較多的包裝紙？

哪個容器可以裝較多的爆米花？



Teacher: This question is asking about the lateral area of each solid. The cylindrical container is a cylinder. The lateral area is the curved part. How do you evaluate the lateral area?

Student: The rectangle. $2\pi r$ times h.

Teacher: Please plug in the numbers, and give me the answer.

Student: 2π times 6 times 15 equals 180 π square centimeters.

Teacher: Okay. As you can see, the regular hexagonal container is a hexagonal prism. The base of this prism is a regular hexagon, with six equal sides and equal angles. The

lateral area is the sum of the six rectangles. What is the area of one rectangle?

Student1: Base times height. 6 times 15 equals 90 square centimeters.

Student2: The lateral area is 90 times 6, 540 square centimeters.

Teacher: Which container needs more wrapping paper? Actually, it is asking which container has a greater lateral face. 180 π and 540, which one is greater?

Student: Can we use a calculator?

Teacher: Well, can you estimate?

Student: Okay. π is 3.14…. 180 times 3 equals 540. 180 times 3 point-something must be greater than 540. I think 180 π is greater. The cylinder is greater.

Teacher: Great job! See, you can do it without a calculator. Let's move on to the next question. Which container can hold more popcorn? The space for popcorn is the volume. It is asking which container has the greater volume. Please compare the volume for each container.

Student 1: The volume of the cylinder is $\pi r^2 \times$ h. π times 36 times 15 equals 540 π.

Student 2: I forgot how to find the volume of a regular hexagon.

Teacher: The volume is base area times height. You have to find the area of this regular hexagon. The regular hexagon can be split into six equilateral triangles, with 6 as the length of the side. Can you find the area of an equilateral triangle?

Student 1: $\frac{\sqrt{3}}{4} \times s^2$. So, it is $9\sqrt{3}$.

Student 2: $9\sqrt{3}$ times 6 equals $54\sqrt{3}$.

Teacher: $54\sqrt{3}$ is the base area. Next, you have to multiply by 15 to get the volume.

Student: No. Is there an easier way?

Teacher: These two solids have the same height. To compare the volume, we can only compare the base area. Do you know why?

Student: The volume is "base times height". Their base area will both time 15. We just need to compare the base area.

Teacher: Correct. Just compare 36π and $54\sqrt{3}$, which is greater?

Student: Can we use a calculator now?

Teacher: Estimate first, and use a calculator to check your answer.

Student 1: 36 times 3…54 times 1.7…I think 36π is greater.

Student 2: I used a calculator. The base area of the cylinder is 113.04. The base area of the prism is 93.528. The cylinder has a greater base area.

Teacher: Yes, the cylinder can hold more popcorn.

老師： 本題其實是要求柱體的側面面積。圓柱形容器是圓柱體，它的側面面積是彎曲的部分。如何計算側面積？

學生： 側面是矩形，$2\pi r \times h$。

老師： 代入數字算出答案。

學生： $2\pi \times 6 \times 15 = 180\,\pi$平方公分。

老師： 很好。接著可以看到，正六邊形容器是六角柱，底是正六邊形，有六個相等的邊和相等的角。側面積是六個矩形的和。每個矩形的面積是多少？

學生： 底×高。$6 \times 15 = 90$ 平方公分。

學生： 側面面積是$90 \times 6 = 540$ 平方公分。

老師： 哪個容器需要更多包裝紙？實際上問的是哪個容器的側面面積比較大。$180\,\pi$和 540，哪個比較大？

學生： 我們可以用計算機嗎？

老師： 要不要試著估算看看？

學生： 好的。π是 3.14...，$180 \times 3 = 540$；180×3 點多一定比 540 大。180π，也就是圓柱體比較大。

老師： 很好！你們都可以在沒有計算機的情況下算出來。繼續看下一題，哪個容器可以裝更多爆米花？裝爆米花的空間即是體積，所以這個問的是哪個容器體積較大。請比較每個容器的體積。

學生： 圓柱體的體積是 $\pi r^2 \times h$。$\pi \times 36 \times 15 = 540\pi$。

學生： 我忘記正六角柱體積怎麼算了。

老師： 體積是底面積乘以高。你必須找到這個正六邊形的面積。正六邊形可以分成六個邊長為 6 的等邊三角形。會算等邊三角形的面積嗎？

學生： 公式是 $\frac{\sqrt{3}}{4} \times s^2$，算出來等於 $9\sqrt{3}$。

學生： $9\sqrt{3} \times 6 = 54\sqrt{3}$。

老師： $54\sqrt{3}$ 是底面積。接下來，要乘以 15 才能得到體積。

學生： 有沒有更簡單的方法？。

老師： 兩個柱體的高相同。比較體積可以只比較底面積。知道為什麼嗎？

學生： 體積是「底面積乘以高」。兩個底面積都會乘以 15，因此只需要比較底面積。

老師： 對。只要比較 $36\,\pi$ 和 $54\sqrt{3}$，哪個更大？

學生： 現在可以用計算機嗎？

老師： 先估算一下，待會再用計算機驗算。

學生： 36×3 點多；54×1.7…，我覺得 36π 比較大。

學生： 我用了計算機。圓柱體的底面積為 113.04，六角柱的底面積為 93.528。圓柱體的底面積比較大。

老師： 對，因此圓柱體可以裝的爆米花較多。

例題二

說明：利用日常生活可見的滾筒油漆刷，學習圓柱體的側面積。

（英文）Jason is going to roll paint with a cylindrical paint roller. The diameter of the roller is 10cm, and the length of the roller is 27cm. He loads the roller with paint and starts to paint a wall. In the first stroke, he only rotates the cylindrical roller once round, how much area will be covered with paint?

（中文）傑森要使用圓筒滾輪漆油漆。滾輪的直徑是 10 公分，長度是 27 公分。他沾了油漆後要塗在牆壁上。在第一刷中，他只轉了滾筒一圈，請問有多少面積的牆壁沾滿了油漆。

Teacher: Jason rotates the cylinder roller once round. The area covered with paint is the lateral area of this cylindrical roller. How do you find the lateral area?

Student1: Circumference times height.

Student2: $2\pi r$ times height.

Teacher: Correct. You can use a calculator to find the answer.

Student1: $2 \times 5 \times \pi \times 27$ equals 270π.

Student2: 847.8 square centimeters.

老師： 傑森的滾輪滾一圈，塗上油漆的面積就是這個圓柱形滾輪的側面面積。那麼側面面積要怎麼算呢？

學生 1： 周長乘上高。

學生 2： $2\pi r \times$ 高

老師： 對，現在用計算機求解。

學生 1： $2 \times 5 \times \pi \times 27 = 270\pi$。

學生 2： 847.8 平方公分。

單元六　角錐與圓錐
Pyramids and Cones

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■ 前言 Introduction

學生認識角錐與圓錐的分類與命名，藉由展開圖認識錐體的構成要素與表面積，認識側面積上的高與錐體的高之差異，學習角錐與圓錐的側面積、表面積與體積公式，最後利用習得的知識解決角錐及圓錐相關的生活實例。

■ 詞彙 Vocabulary

單字	中文	單字	中文
pyramid	角錐	equilateral triangle	正三角形
cone	圓錐	isosceles triangle	等腰三角形
volume	體積	square pyramid	四角錐
surface area	表面積	pentagonal pyramid	五角錐
lateral area	側面積	triangular pyramid	三角錐
apex	頂點	hexagonal pyramid	六角錐
sector	扇形	slant height	斜高
central angle	圓心角	hypotenuse	直角三角形的斜邊

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ _____ is named by _____.

例句：The pyramid **is named by** the shape of its base.

角錐是以底面的形狀命名的。

❷ Round the answer to the nearest _____.

例句：**Round the answer to the nearest** tenth.

將答案四捨五入到十分位。

❸ _____ are congruent.

例句：All lateral faces of a regular pyramid **are congruent** isosceles triangles.

正角錐的所有側面都是全等的等腰三角形。

❹ _____ is perpendicular to _____.

例句：The altitude of the pyramid **is perpendicular to** the base.

角錐的高垂直於底面。

❺ Cut open _____.

例句：If you **cut open** a cone, you will have a sector and a circle. This is the net of a cone.

如果你切開一個圓錐，你會看到 1 個扇形與一個圓形。這是圓錐的展開圖。

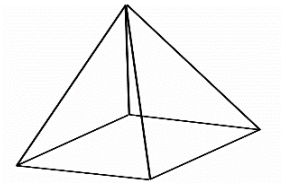
■ 問題講解 Explanation of Problems

☙ 說明 ☙

[Classify Solids]

In this section, we will learn about pyramids and cones.

Prisms have two congruent polygons in parallel planes, but a pyramid only has one base. When talking about pyramids, you probably think of the Egyptian pyramids, which are some of the most amazing man-made structures in history. They are the tombs of Egyptian kings and queens.

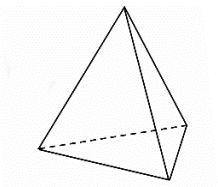


This is a pyramid, just like the Egyptian ones. A pyramid is formed by a polygon as the base, and several congruent isosceles triangles as the lateral faces. The intersections of two faces are called edges. The point where the edges meet is the vertex. The vertex at the top is the apex.

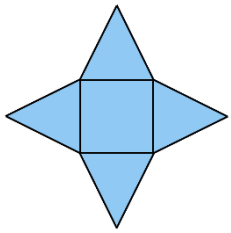
When the base is a regular polygon, it is a regular pyramid. Otherwise, it is an irregular pyramid. In this section, we focus on the regular pyramid only. The regular pyramid is a right pyramid, which stands vertically. All lateral edges are congruent, and all lateral faces are congruent isosceles triangles.

Do you remember how we name prisms? We name the prisms by the shape of the base. We do the same thing for pyramids. This pyramid has a square as the base and four isosceles triangles as the lateral faces, so it is named a square pyramid.

The pyramid below has an equilateral triangle as the base and three isosceles triangles as the lateral faces, so this pyramid is called a triangular pyramid. Additionally, these four triangles are congruent equilateral triangles.



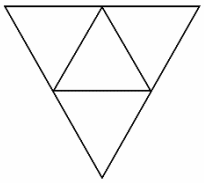
[Surface Area of Pyramids]



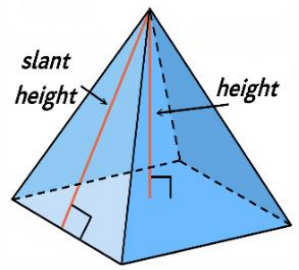
This is the net of a pyramid. The net shows the faces of 3D objects when it is opened flat. Cut out along the edges, and fold along the connected edges. Which type of pyramid will you get? The square is the base. You will get a square pyramid.

The square is the base, and these four congruent triangles are lateral surfaces. The surface area of the pyramid is the sum of the areas of these faces. You can find the area of one triangle and then times it by 4 and then you will get a lateral area.

This is the net of another pyramid. Which type of pyramid is it?



It is a triangular pyramid. The surface area of the pyramid is the sum of the areas of four congruent equilateral triangles. You can find the area of one equilateral triangle and then time it by 4 and then you will get the surface area.



The segment from the vertex perpendicular to the base is the altitude, and its length is called the **height**.

We already know the area of a triangle: $\frac{\text{base} \times \text{height}}{2}$.

However, we have two different kinds of height.

In order to separate the height of the pyramid and the height of the lateral triangle, we name the height of the triangle **slant height**. The diagram shows the difference between the two heights. The height of the pyramid is perpendicular to the center of the base, and the slant height is perpendicular to the base of the triangle. The slant height is the height of a lateral face, measuring from the apex to the midpoint of the edge. The area of a lateral triangle is $\frac{\text{base} \times \text{slant height}}{2}$.

[Volume of Pyramids]

The volume of a pyramid is one-third of the volume of a prism which has the same base and height as the pyramid. The volume of the pyramid formula is $\frac{\text{base area} \times \text{height}}{3}$.

To understand the relationship between pyramids and prisms, we can do an activity. Use some cardboard to make a pyramid container and a prism container with congruent heights and bases. For example, you can make a rectangular pyramid and rectangular prism or a pentagonal pyramid and pentagonal prism, as long as their bases and heights are congruent. We are going to fill the containers with sand. You can use other materials to make containers, as long as they are strong enough. We need a strong container to hold the sand. Following are the steps.

Fill the pyramid container with sand. Next, pour the sand from the pyramid container into the prism container. We can see the prism is about one-third full. Repeat the process two more times and the prism container will be full of sand.

Now, we have learned this formula with a hands-on activity. In the future, you can learn how to prove it with calculus.

[Cones]

The first image that comes to mind when we say the word "cone" could possibly be an ice cream cone. That is a cone! A cone has a circular base and a curved surface that meets at a point. The point is at the end of the cone, and it is called an apex or vertex. The curved surface is the lateral surface.

In a right cone, the axis of the cone (the line segment between the apex and the center of the base) is perpendicular to the base. If the axis is not perpendicular to the base, the cone is an oblique cone. In this section, we focus on the right cones.

[Surface Area and the Volume of Cones]

If you cut open a cone, you will get a sector and a circle. The area of the sector is the lateral area. The sum of the sector and the circle is the total surface area of the cone.

The sector is the portion of a circle. It is enclosed between two radii and the arc. The area of the sector is defined as $\pi r^2 \times \frac{\text{central angle}}{360}$, where the central angle is measured in degrees. πr^2 represents the area of the whole circle. The ratio $\frac{\text{central angle}}{360}$ tells us how much of the circle is covered by the sector.

The volume of a cone is one-third of the volume of a cylinder with the same base and same

height. The relationship between cone and cylinder is like the relationship between pyramid and prism. You can explore this relationship by making a cone and a cylinder with the same base and height, just like what we did earlier.

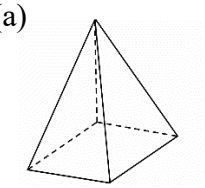
☙運算問題的講解 ❧

例題一

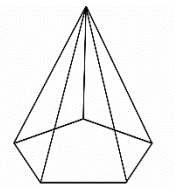
說明：讓學生練習命名錐體。

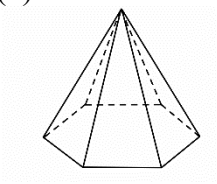
（英文）Decide whether the following objects are pyramids or cones. Name the objects.

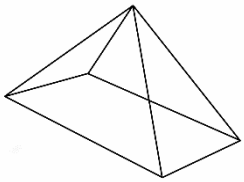
（中文）判斷以下為角錐或圓錐，並命名以下錐體。

(a) 

(b) 

(c) 

(d) 

(e) 

Teacher: Which objects are pyramids? Which objects are cones? Please explain.
Student: (b) is the only cone. The base is a circle. The others are pyramids.
Teacher: Correct. Tell me the shape of the base, and try to name it. Start with (a).
Student: The base is a square, so it is a square pyramid.
Teacher: Yes. It is a square pyramid. What about (e)?
Student: The base is a rectangle. Is it a “rectangle” pyramid?
Teacher: Rectangular pyramid. The first word “rectangular” is an adjective, and it is used to describe the pyramid. Now move on to (c).
Student: The base is a pentagon. Is it a “pentagon” pyramid?
Teacher: We call it a pentagonal pyramid. What about (d)?

Student: It is a hexagon. Is it a hexagonal pyramid?

Teacher: Yes. You got it.

老師： 解釋一下選項中哪些是角錐，哪些是圓錐？

學生： (b)是唯一的圓錐。底面是圓形。其他的都是角錐。

老師： 沒錯。跟我說底面的形狀，再對圖形命名。請從(a)開始。

學生： 底面是正方形，所以是正四角錐。

老師： 沒錯，是四角錐。那(e)呢？

學生： 底是長方形。是長方形錐嗎？

老師： 是長方錐。第一個字“長方形的”是一個形容詞，用來形容角錐，現在換(c)。

學生： 底是五邊形。所以是五角形錐？

老師： 是五角錐。那(d)呢？

學生： 底是六邊形。為六角錐

老師： 答對了。

例題二

說明：利用角錐的體積求高。

（英文）The base of a right pyramid is a square with sides 50 cm long. If the volume of the pyramid is 10000 cm^3, then find its height.

（中文）一直角錐的底是邊長為 50 公分的正方形。若角錐的體積是 10000 cm^3，角錐的高為何？

Teacher: How do you find the volume of a pyramid?

Student: Base area times height times $\frac{1}{3}$ (one third).

Teacher: What is the base area?

Student: The area of the square is 50 times 50, 2500.

Teacher: We don't know the height. We can use "h" for the height. What is the volume in terms of "h"?

Student: $\frac{2500\times h}{3}$.

Teacher: Good. With the given information of 10000, can you write an equation?

Student: $\frac{2500\times h}{3} = 10000$.

Teacher: Multiply both sides by 3. You will get $2500 \times h = 30000$. Find the value for h.

Student: h is 12.

老師： 角錐的體積要怎麼算？

學生： 底面積×高×$\frac{1}{3}$。

老師： 底面積是多少呢？

學生： 正方形的面積是50×50，為 2500。

老師： 我們不知道高度。可以用 h 表示高度。用 h 表示的話，體積的公式是什麼？

學生： $\frac{2500 \times h}{3}$

老師： 很好。題目給定體積是 10000 cm^3，能列出方程式嗎？

學生： $\frac{2500 \times h}{3} = 10000$

老師： 兩邊×3，會得到 $2500 \times h = 30000$。h 是多少呢？

學生： $h = 12$。

例題三

說明：求圓錐體的體積。

（英文）Find the volume of a cone with a height of 8 and a base radius of 5.
（中文）一圓錐高 8，底部半徑 5，求其體積。

Teacher: What is the base area?

Student: The radius is 5. The area is 25π.

Teacher: The volume of a cone is the base area times height times $\frac{1}{3}$. Can you find the answer?

Student: $25\pi \times 8 \times \frac{1}{3}$.

Teacher: Use the calculator and round the answer to the nearest tenth.

Student:　209.3.

老師：　先求圓錐的底面積。

學生：　半徑是 5。面積是 25π。

老師：　錐體的體積是底面積×高×$\frac{1}{3}$。能算出答案嗎？

學生：　$25\pi \times 8 \times \frac{1}{3}$。

老師：　用計算機算並四捨五入至小數點第一位。

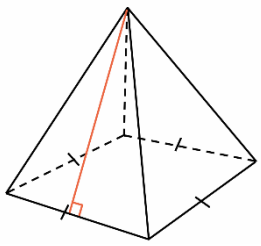
學生：　答案是 209.3。

∞ 應用問題 / 會考素養題 ∞

例題一

說明：利用日常生活的蛋糕模型讓學生練習角錐體積與側面積，並利用畢氏定理求出斜高。

（英文）Edwin plans to make a pyramid cake for Mother's Day. He needs to build a square pyramid cake tin, with base side 10cm and height 12cm.



How much cake batter does he need to fill up the tin?

After the cake is baked, Edwin will take out the cake from the tin and decorate the lateral faces of the cake with chocolate sauce. How much of the cake's area will be covered with chocolate sauce?

（中文）為了母親節，Edwin 要做一個金字塔形狀的蛋糕。他要先製造一個四角錐的蛋糕模型，底面邊長為 10 公分，高 12 公分。他需要多少蛋糕麵糊才能填滿蛋糕模型？蛋糕烤熟後，Edwin 把蛋糕取出後用巧克力醬裝飾蛋糕的側面，請問多少面積的蛋糕會覆蓋巧克力醬？

Teacher:　Yummy question! I like chocolate cake. Let's start with the tin. How much cake

batter does he need to fill up the container? What does the question ask for?

Student: Volume?

Teacher: Yes. Volume is the amount of space that is enclosed in a container. The amount of batter is the volume of the pyramid. How do we find the volume?

Student: Base area times height times $\frac{1}{3}$.

Teacher: The base is a square. The area is 100. What is the next step?

Student: $100 \times 12 \times \frac{1}{3}$, the answer is 400 cm^3.

Teacher: Okay. It's time to decorate the cake. How much area will be covered with chocolate? What does the question ask for?

Student: Surface area.

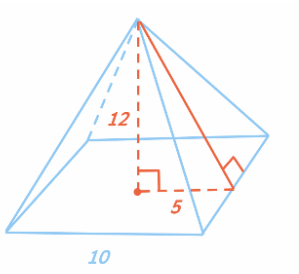
Teacher: Well, Edwin only decorates the lateral faces. The question is asking for the lateral areas. Only four triangles are included, no square.

Student: But I don't have the height of the triangle. How can I find the area?

Teacher: You should be more specific. What you don't have is the "slant height."

Student: Yes, I don't have the slant height. But, how can I find it?

Teacher: First, draw a pyramid and its height. I use a dashed line for the height because you can't see it from the outside. The height of the pyramid is perpendicular to the square. The lateral face is an isosceles triangle. Plot the midpoint of the base. Connect the midpoint and apex with a solid line because you can see it from the outside. The segment joining the apex and midpoint is perpendicular to the base. Connect the end of the height and the midpoint. These three segments form a right triangle.



Let's focus on this right triangle. 5 and 12 are two legs. The hypotenuse is the slant height. Now you can use the Pythagorean theorem to find its length.

Student: I got it. 5, 12, and 13 are Pythagorean triples. Therefore, the slant height is 13.

Teacher: I think you can figure out the lateral area.

Student1: The area of one triangle is $\frac{10\times13}{2}$.

Student2: The lateral area is $\frac{10\times13}{2} \times 4 = 260$ cm^2.

Teacher: Great. That is how much cake will be covered with chocolate sauce.

老師： 這題看起來好好吃！我很愛巧克力蛋糕。
先從蛋糕模型開始。需要多少麵糊才能填滿模型？題目要求什麼呢？

學生： 體積？

老師： 沒錯，體積就是容器所含的空間。麵糊的量是這個角錐的體積。體積怎麼算？

學生： 底面積×高×$\frac{1}{3}$ 。

老師： 底面是一個正方形，面積是 100，接下來呢？

學生： $100 \times 12 \times \frac{1}{3}$，答案是 400 平方公分。

老師： 很好，那現在來裝飾蛋糕。多少面積的蛋糕會被巧克力醬蓋到？題目要求什麼？

學生： 表面積。

老師： 是的，Edwin 裝飾的部分只有側面。題目要求的是側面面積，因此只有側邊的四個三角形，正方形不算。

學生： 但我不知道三角形的高，那要怎麼找到面積？

老師： 應該說明確一點，你沒有的是「斜高」。

學生： 對，我不知道斜高。那要怎麼找？

老師： 首先，畫角錐還有它的高，我用虛線表示高度，因為你從外面看不到它。角錐的高垂直於正方形，側面是一個等腰三角形。接著標出三角形底邊的中點，用實線連接中點和頂點，因為你可以從外面看到它，所以用實線表示。最後，將高的終點與三角形底邊的中點連起來。這樣，三條線就會構成一個直角三角形。
現在來自仔細看這個直角三角形，5 和 12 是它的兩股，斜邊是斜高。
現在可以使用畢式定理找到斜高的長度。

學生： 我知道了，5, 12, 13 是一個畢式數組，所以斜高為是 13。

老師： 那現在大家可以算出側面面積啦。

學生： 三角形的面積是 $\frac{10\times13}{2}$。

學生： 側面積是 $\frac{10\times13}{2} \times 4 = 260\ \text{cm}^2$。

老師： 很好！這就是巧克力醬會覆蓋蛋糕的面積。

例題二

說明：利用圓錐展開圖的底圓與扇形關係求出扇形的圓心角，並畫出展開圖，求扇形面積。

（英文）Edwin serves jelly in a plastic cone container in his café. The radius of the cone container is 3 cm, and the slant height is 8 cm.

(1) Graph the net of the cone and indicate the measurement of the central angle.

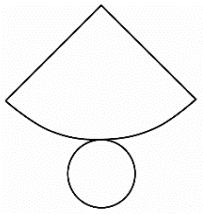
(2) How much wrapping paper does he need to fully cover the curved surface of his container?

（中文）Edwin 在他的咖啡店裡用圓錐形狀的容器裝果凍，此圓錐狀容器的半徑是 3 公分，斜高是 8 公分。

(1) 畫出此容器展開圖，並標示圓心角角度。

(2) Edwin 需要多少包裝紙才完全覆蓋容器的曲面？

Teacher: You will have a sector and a circle when cutting open a cone. The length of the arc is the circumference of the circle.



We need the central angle to find the length of the arc.

Let's assume the central angle is x degrees. The slant height is the radius of the sector. $2\pi \times 8$ is the circumference of the whole circle, but we only need part of it. $\frac{x}{360}$ tells us how much of the circle is needed. $2\pi \times 8 \times \frac{x}{360}$ is the length of the arc.

Now it's your turn. How do you find the circumference of the circle?

Student: The radius of the circle is $3cm$, circumference is $2 \times 3 \times \pi = 6\pi$.

Teacher: We can combine the information and write an equation: $2\pi \times 8 \times \frac{x}{360} = 6\pi$. Next, solve for x.

Student: Cross out π on both sides, and multiply both sides by 360.

I have $2 \times 8 \times x = 6 \times 360$. x is 135.

Teacher: Yes. Move on to the next question. How much wrapping paper does Edwin need to cover the curved surface?

Student: Does the question ask for the lateral area?

Teacher: Yes. Because he only mentions the "curved" part, we just need the area of the sector.

$\frac{x}{360}$ indicates the portion of the circle, and we will use $\pi r^2 \times \frac{x}{360}$ to find the area of the sector. Plug in r and x.

Student: $\pi \times 8^2 \times \frac{135}{360} = 24\pi$.

Teacher: That is the lateral area. Use 3.14 for π. What is the answer?

Student: 75.36 cm^2.

Teacher: Correct.

老師：當我們把圓錐展開，會得到一個扇形和一個圓形。弧長即為圓形的周長，需要藉由中心角來計算。

假設中心角是 x 度，斜高是扇形的半徑。$2\pi \times 8$ 是整個圓的周長，但我們只需要一部分，也就是 $\frac{x}{360}$，$2\pi \times 8 \times \frac{x}{360}$ 就是弧長的長度。

現在輪到你了，圓的周長怎麼算？

學生：圓的半徑是 3 公分，周長是 $2 \times 3 \times \pi = 6\pi$。

老師：好的，現在寫出等式：$2\pi \times 8 \times \frac{x}{360} = 6\pi$，求 x。

學生：兩側刪去 π，然後× 360，得到 $2 \times 8 \times x = 6 \times 360$，$x = 135$。

老師：很好，接下來看第(2)小題。Edwin 需要多少包裝紙才完全覆蓋容器的曲面？

學生：這個問題問的是側面面積嗎？

老師：沒錯，因為題目只提到了「曲面」，我們只需要扇形的面積。$\frac{x}{360}$ 表示圓的一部分，我們用$\pi r^2 \times \frac{x}{360}$ 來計算扇形的面積。代入 r 和 x。

學生：$\pi \times 8^2 \times \frac{135}{360} = 24\pi$。

老師：是，這就是側面面積。用 3.14 代替 π。答案是多少？

學生：是 75.36。

老師：答對了。

國內外參考資源 More to Explore

國家教育研究院樂詞網 查詢學科詞彙 https://terms.naer.edu.tw/search/	
教育雲：教育媒體影音 為教育部委辦計畫雙語教學影片 https://video.cloud.edu.tw/video/co_search.php?s=%E9%9B%99%E8%AA%9E	
Oak Teacher Hub 國外教學及影音資源，除了數學領域還有其他科目 https://teachers.thenational.academy/	
CK-12 國外教學及影音資源，除了數學領域還有自然領域 https://www.ck12.org/student/	
Twinkl 國外教學及影音資源，除了數學領域還有其他科目，多為小學及學齡前內容 https://www.twinkl.com.tw/	

Khan Academy 可汗學院，有分年級數學教學影片及問題的討論。 https://www.khanacademy.org/	
Open Textbook (Math) 國外數學開放式教學資源 http://content.nroc.org/DevelopmentalMath.HTML5/Common/toc/toc_en.html	
MATH is FUN 國外教學資源，還有數學相關的小遊戲 https://www.mathsisfun.com/index.htm	
PhET: Interactive Simulations 國外教學資源，互動式電腦模擬。除了數學領域，還有自然科。 https://phet.colorado.edu/	
Eddie Woo YouTube Channel 國外數學教學影音 https://www.youtube.com/c/misterwootube	

國立臺灣師範大學數學系陳界山教授網站	
國高中數學雙語教學相關教材 https://math.ntnu.edu.tw/~jschen/index.php?menu=Teaching_Worksheets	
2023 年第四屆科學與科普專業英文(ESP)能力大賽	
科學專業英文相關教材，除了數學領域，還有其他領域。 https://sites.google.com/view/ntseccompetition/%E5%B0%88%E6%A5%AD%E8%8B%B1%E6%96%87%E5%AD%B8%E7%BF%92%E8%B3%87%E6%BA%90/%E7%9B%B8%E9%97%9C%E6%95%99%E6%9D%90?authuser=0	

國中數學領域雙語教學資源手冊：英語授課用語

[九年級下學期]

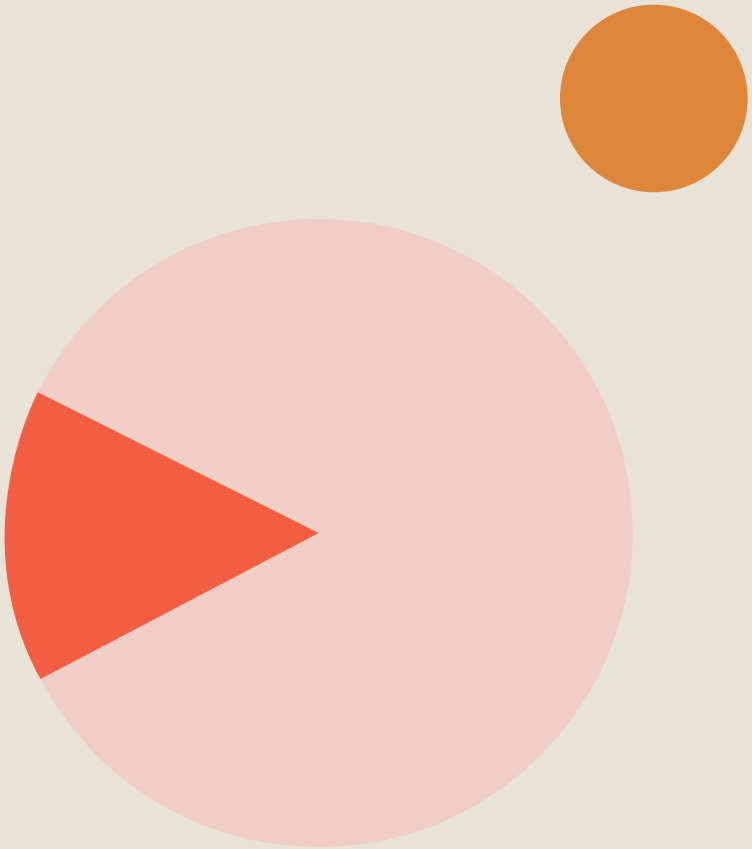
A Reference Handbook for Junior High School Bilingual Teachers in the Domain of Mathematics: Instructional Language in English

[9th grade 2nd semester]

- 研編單位：國立臺灣師範大學雙語教學研究中心

  國立彰化師範大學雙語教學研究中心
- 指導單位：教育部師資培育及藝術教育司
- 撰稿：盧昶元、邱奕銘、周慧蓮、吳珮蓁
- 學科諮詢：(單元一～單元二) 張淑珠

  (單元三～單元六) 鄭章華
- 語言諮詢：(單元三～單元六) 李壹明
- 綜合規劃：王宏均、曾松德
- 編輯排版：吳依靜
- 封面封底：JUPE Design



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