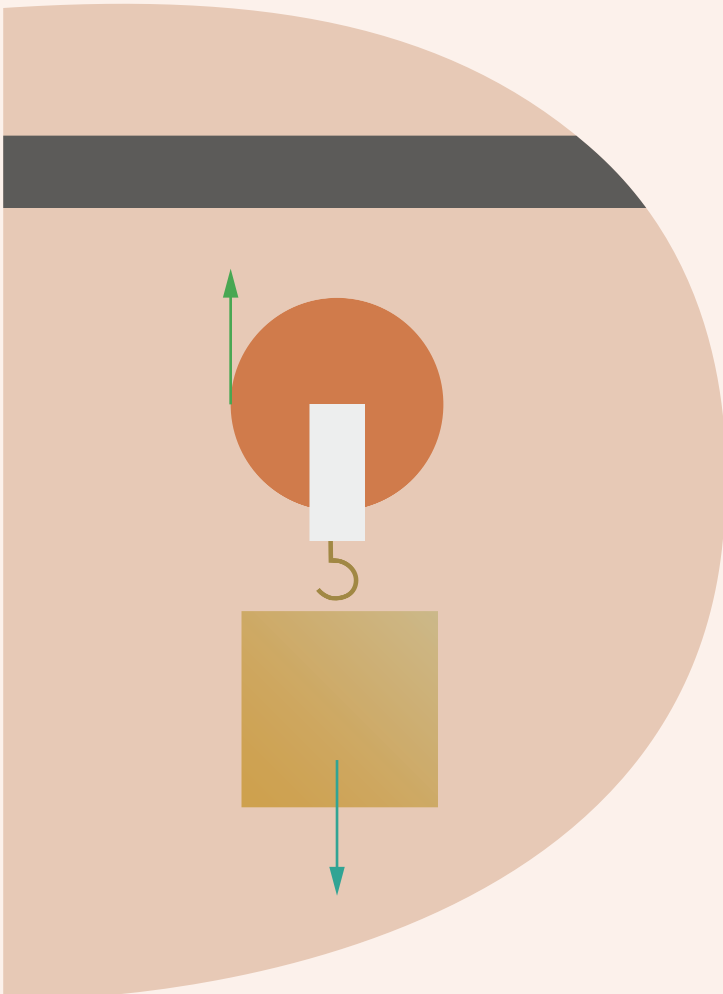
高中自然領域

雙語教學資源手冊
物理科 英語授課用語

A Reference Handbook for Senior High School Bilingual Teachers in the Domain of Natural Sciences (Physics): Instructional Language in English

〔高中選修(II)〕



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★ 主題一 動量與角動量 ★
Momentum and Angular Momentum

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■ 前言 Introduction

本章介紹「動量」與「衝量」，來討論物體運動的狀態及改變，並以牛頓第二運動定律推導出「衝量－動量定理」。接著說明物體不受外力作用或合力為零時，其動量必守恆。針對動量守恆的系統，我們會利用質心的運動，來推導系統在過程中的運動現象。最後介紹角動量與力矩之關係，並探討角動量守恆定律之限制條件與應用。

1-1 動量與衝量
Momentum and Impulse

■ 前言 Introduction

本節將定義「動量」與「衝量」，透過動量變化來定義物體所受到的衝量，並將兩者與先前所學的「力」概念連結。同時熟悉物體受到定力或非定力下時，衝量的計算。最後，利用動量時變率描述牛頓第二運動定律，進而推出衝量等於動量變化的意義。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
momentum	動量	impulse - momentum theorem	衝量—動量定理
the change of momentum	動量變化	fixed force, constant force	定力
initial momentum	初動量	the time rate of change of momentum	動量時變率 動量的時間變化率
final momentum	末動量	quantity of motion	運動的量
impulse	衝量	airbag	安全氣囊

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ _________ the product of ___________ and __________.

例句：Momentum is defined as **the product of** mass **and** velocity.

動量是由質量和速度的乘積來定義。

❷ The change in _____ over a period of time is ______ under____.

例句：**The change in** momentum of an object **over a period of time is** equal to the area **under** the force-time graph.

物體在一時段內的動量變化，即為受力與時間軸，所涵蓋的面積。

❸ _________ impressed on __________ since _________.

例句：Airbags may decrease the net force **impressed on** the passengers in the car, **since** it increases the duration of the impact for passengers.

安全氣囊會在緊急剎車時，因為它讓乘客受撞擊的時間增長，所以可減少受力大小。

■ 問題講解 Explanation of Problems

學習目標

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to know that:

一、認識動量與衝量的定義。

Knowing the definitions of momentum and impulse.

二、根據牛頓第二運動定律，推出衝量等於動量變化，並能以向量計算。

Derive the relation of impulse equal to the change of momentum based on Newton’s Second Law, using vectors to manipulate the quantities.

꧁ 例題講解 ꧂

例題一

說明：熟悉衝量與動量變化的向量關係。

Understand that impulse and momentum are vector quantities, and impulse is the change in momentum.

As depicted in the figure below, a baseball with a mass of m is pitched horizontally to a batter at a speed of v. The ball turns straight upward and moves at a speed of v after being struck by the bat. Assuming the ball's horizontal motion is in the +x direction and the vertical motion is in the +y direction, what is the direction and magnitude of the impulse delivered to the baseball by the bat?

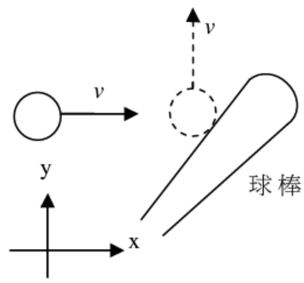
(A) 2mv, moving in the +*y* direction.

(B) mv, at an angle of 45 degrees to +*x*.

(C) mv, at an angle of 135 degrees to +*x*.

(D) $\sqrt{2}mv$, at an angle of 45 degrees to +*x*.

(E) $\sqrt{2}mv$, at an angle of 135 degree to +*x*.



如下圖所示，一質量為 m 的棒球以速度 v 水平飛向擊球手，擊球手揮棒擊球，使球以速度 v 鉛垂向上飛出，設水平飛行方向為+x，鉛垂向上飛出方向為+y，則球所受到衝量的量值及方向為下列何者？

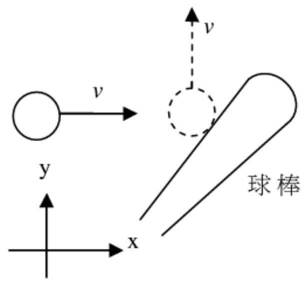
(A) 2mv，向+*y* 方向。

(B) mv，與+*x* 方向成 45° 。

(C) mv，與+*x* 方向成 135° 。

(D) $\sqrt{2}mv$，與+*x* 方向成 45° 。

(E) $\sqrt{2}mv$，與+*x* 方向成 135° 。

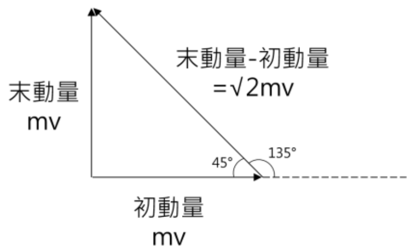


（98 年指考 13）

解題 Solution：

Impulse applied to an object is equal to the change in its momentum. The following illustration shows that initial momentum of the baseball is mv and in the +x direction. The baseball's final momentum is mv and in the +y direction. Therefore, the impulse can be determined by having the initial momentum subtracted from the final momentum $\sqrt{[mv^2 + mv^2]} = \sqrt{2}mv$ at an angle of 135 degree to +x.

物體的衝量等於動量變化。如下圖所示，棒球的初始動量為 mv，向+x 方向；末動量為 mv，向+y 方向。衝量為末動量與初動量兩向量相減：$\sqrt{[mv^2 + mv^2]} = \sqrt{2}mv$，與+x 方向成 135°。



Teacher: Do you still remember the impulse-momentum theorem?

Student: Yes, the theorem states that impulse is equal to the change in momentum.

Teacher: Good job. The impulse received by an object causes a change in its momentum. That is why impulse and momentum are closely related to each other. Now, how do we calculate an object's momentum changes over a certain amount of time?

Student: We can subtract that object's initial momentum from its final momentum.

Teacher: That is correct! Be aware of the direction of momentum. Because momentum is a vector, it has a direction. Please follow the instruction to draw the directions for the initial momentum and final momentum of the baseball asked. This will help us to determine the impulse delivered to the ball.

Student: Subtracting the baseball's initial momentum from its final momentum $\sqrt{[mv^2 + mv^2]} = \sqrt{2}mv$.

Teacher: What's its direction?

Student: It is at an angle of 135 degrees to +x.

老師： 還記得衝量—動量定理嗎？

學生： 記得，衝量會等於動量變化。

老師： 很好，物體所受衝量是因，動量變化是結果。我們可以透過此定理，來連結衝量及動量變化。那我們如何計算動量變化呢？

學生： 利用末動量減去初動量。

老師： 沒錯！不過動量是向量，須注意方向性。請依照題目指示分別畫出初向量及末向量，並計算衝量。

學生： 末動量與初動量相減的量值為$\sqrt{[mv^2 + mv^2]} = \sqrt{2}mv$。

老師： 請問它的方向為何？

學生： 與+x 方向成 135°。

例題二

說明：根據牛頓第二運動定律($\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$)，及動量($\vec{p} = m\vec{v}$)與衝量($\vec{J} = \vec{F}\Delta t$)的定義，求出物體的受力大小。

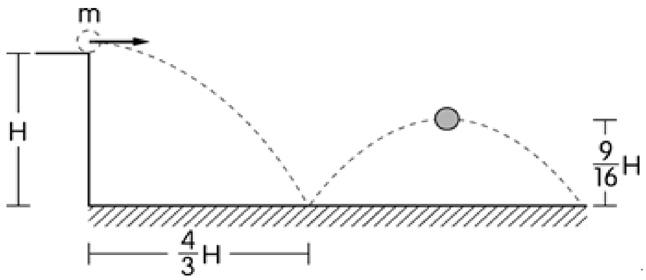
Students can figure out the magnitude of a force exerted on an object using Newton's second law of motion ($\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$) and the formulas for momentum ($\vec{p} = m\vec{v}$) and impulse ($\vec{J} = \vec{F}\Delta t$).

As shown in the figure below, a ball with a mass of m, or a point mass, is released from a point H above the ground. This ball is thrown horizontally. The travel distance of the ball before its first rebound is $\frac{4H}{3}$ and the rebound height is $\frac{9H}{16}$. Assuming the surface of the ground is smooth and polished, air resistance is negligible, the time for the ball in contact with the ground is t, and the acceleration gained by the ball due to gravity is g, what is the magnitude of the average force exerted on the ball in the vertical direction by the ground?

(A)$\frac{m\sqrt{2gH}}{4t}$ (B) $\frac{7m\sqrt{2gH}}{16t}$ (C) $\frac{25m\sqrt{2gH}}{16t}$ (D) $\frac{5m\sqrt{2gH}}{4t}$ **(E)** $\frac{7m\sqrt{2gH}}{4t}$.

如圖所示，一質量為 m 可視為質點的小球從離地 H 處水平射出，第一次落地時的水平位移為 $\frac{4H}{3}$，反彈高度為 $\frac{9H}{16}$。若地板為光滑，且空氣阻力可以忽略，而小球與地板接觸的時間為 t，重力加速度為 g。求第一次落地碰撞期間，小球在鉛直方向所受到的平均作用力之量值為何？

(A) $\frac{m\sqrt{2gH}}{4t}$　(B) $\frac{7m\sqrt{2gH}}{16t}$　(C) $\frac{25m\sqrt{2gH}}{16t}$　(D) $\frac{5m\sqrt{2gH}}{4t}$　**(E)** $\frac{7m\sqrt{2gH}}{4t}$。



（102 年指考 9）

解題 Solution：

When the ball is thrown horizontally, it is in free-fall in the vertical direction$V_{1y} = \sqrt{2gH}$ downward. When the ball rebounds, the acceleration gained by the ball is still g in the downward direction: the topmost point $v_y^2 = 0^2 = v_{1y}'^2 - 2g \cdot (\frac{9H}{16})$

$\Rightarrow v_{1y}' = \frac{3}{4}\sqrt{2gH}$ upward. $F \cdot t = m〔\frac{3}{4}\sqrt{2gH} - (-\sqrt{2gH})〕$

$\Rightarrow F = \frac{7m\sqrt{2gH}}{4t}$.

平拋過程，在鉛直方向作自由落體運動：$V_{1y} = \sqrt{2gH}$，向下。反彈後，在鉛直方位作鉛直上拋，加速度依然是 g 向下：最高點 $v_y^2 = 0^2 = v_{1y}'^2 - 2g \cdot (\frac{9H}{16})$

$\Rightarrow v_{1y}' = \frac{3}{4}\sqrt{2gH}$，向上。$F \cdot t = m〔\frac{3}{4}\sqrt{2gH} - (-\sqrt{2gH})〕$

$\Rightarrow F = \frac{7m\sqrt{2gH}}{4t}$。

Teacher: What happens when the ball first strikes the ground?

Student: It rebounds.

Teacher: It is correct. Then, why does the rebound happen? What occurs of the ball during the time of hitting the ground?

Student: The ground exerts a force upwards on the ball. Thus, changes the direction of the ball's momentum.

Teacher: That is great! When the ball is in contact with the ground, the ground delivers an impulse to the ball and changes the ball's momentum.

Teacher: The magnitude of the force exerted on this ball can be determined using the impulse-momentum theorem. Does anyone remember this theorem?

Student: I do! Impulse is equal to the change in momentum produced by the force $\vec{J} = \vec{F}\Delta t = \Delta p$.

Teacher: That's right. But, what does impulse have to do with the average force asked by the question?

Student: Impulse is the product of the average force and the time duration that this force acts F · t.

Teacher: As a result, we need to find out how much the ball's momentum changes during the collision.

Student: Oh, I kind of get it, but how do I know?

Teacher: Do you still remember the definition of momentum?

Student: Yes, I do. Momentum is the product of the mass of a moving object and its speed in a particular direction $\vec{p} = m\vec{v}$.

Teacher: That is correct. Which element in this equation remains unchanged throughout?

Student: The ball's mass m! So, the only thing we need to figure out is what the velocities of the ball are before and after it hits the ground.

Teacher: That is great! Before that, we still need to know by what type of force the ball is being pulled before it rebounds.

Student: It's gravity, and this tells us the velocity before the ball strikes the ground is $V_{1y} = \sqrt{2gH}$ (downward).

Teacher: Great job! How about the velocity 'v_{1y}' after the ball hits the ground?

Student: The ball is still being pulled by gravity. According to the motion equation for constant acceleration, $v_y^2 = 0^2 = v_{1y}'^2 - 2g.\ (\frac{9H}{16})$ .Thus $v_{1y}' = \frac{3}{4}\sqrt{2gH}$ (upward).

Teacher: That is right. Now, we have the velocities of the ball before and after it strikes the ground. We can solve for the change in the ball's momentum.

Student: I see it now. $F.\ t = m〔\frac{3}{4}\sqrt{2gH} - (-\sqrt{2gH}\ )〕$ The answer is $F = \frac{7m\sqrt{2gH}}{4t}$

老師：小球第一次落地時，發生了什麼事？
學生：小球反彈了。
老師：沒錯！那為什麼小球會反彈？小球與地板接觸的時間 t 內發生了什麼事？
學生：小球受到了一個向上的力，改變了小球的運動方向。
老師：很好~小球在接觸地板過程中，受到了地板提供的衝量，改變了小球的動量。
老師：我們可以利用衝量—動量定理得到小球的受力大小，有人知道衝量—動量定理嗎？
學生：我知道，衝量等於動量變化，$\vec{J}=\vec{F}\Delta t=\Delta p$。
老師：是的，那麼衝量與題目所求的平均作用力有什麼關係呢？
學生：衝量等於物體之平均受力與受力時間的乘積，F．t。
老師：所以我們只要找出小球的動量變化，即可得到答案了。
學生：哦~我懂了！不過，要怎麼知道小球的動量變化呢？
老師：還記得動量的定義吧？
學生：嗯嗯，動量是物體質量與速度的乘積，$\vec{p}=m\vec{v}$。
老師：是的，其中哪一項在題目中的狀況是不變的呢？
學生：小球質量！所以只要知道小球與地板接觸前後的速度變化就可以了！
老師：很好~求得小球與地板接觸前的速度，我們需要知道小球從水平射出到第一次落地之間，受到什麼力作用呢？
學生：小球只受重力作用。所以小球與地板接觸前的速度$V_{1y}=\sqrt{2gH}$ (向下)。
老師：很棒！那小球與地板接觸後的速度 v_{1y}'呢？
學生：小球一樣只受重力作用。由等加速度公式 $v_y^2=0^2=v_{1y}'^2-2g\cdot(\frac{9H}{16})$可得 $v_{1y}'=\frac{3}{4}\sqrt{2gH}$ (向上)。
老師：沒錯，現在我們有小球與地板接觸前後的速度，就可以求出動量變化了。
學生：我知道了。$F\cdot t=m〔\frac{3}{4}\sqrt{2gH}-(-\sqrt{2gH})〕$答案是 $F=\frac{7m\sqrt{2gH}}{4t}$。

1-2 動量守恆定律
Law of Conservation of Momentum

■ 前言 Introduction

前一節提到「物體所受的合力等於動量對時間的變化率」($\vec{F}=\lim_{\Delta t\to 0}\frac{\vec{J}}{\Delta t}=\lim_{\Delta t\to 0}\frac{\Delta\vec{p}}{\Delta t}$)，當物體不受外力作用或合力為零時，物體的動量並無改變，過程中系統之動量必守恆。此定律通常用在系統有明顯內力作用的過程，如:碰撞、爆炸，根據牛頓第三運動定律，內力會彼此抵消，故系統的總動量維持守恆。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
law of conservation of momentum	動量守恆定律	external force	外力
vector	向量	internal force	內力
projectile motion	拋體運動	explosion	爆炸 (n.)
horizontal component	水平分量	collision	碰撞 (n.)
vertical component	鉛直分量	embed	嵌入 (v.)
point mass, particle	質點		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ **In ___________, if __________, _______ will _________.**

例句：**In** the absence of any external force, **if** the initial momentum is not zero, the object **will** keep on moving with initial speed in a straight line.

在沒有任何外力的情況下，若初始動量不為零，則物體會繼續以初始速率作直線運動。

❷ **When _____ acting on ______, the momentum of the _____ is ______.**

例句：**When** the total external force **acting on** a system of a particle is zero, **the total momentum of the** system **is** conserved.

當質點系統所受外力總合為零時，系統之總動量守恆。

❸ **_______ of the system consisting of _________ and _________.**

例句：After the bullet embeds into the block and moves with it, the total momentum **of the system consisting of** the block **and** the bullet is conserved.

當子彈嵌入木塊並一起運動後，木塊與子彈所形成的系統，總動量守恆。

■ 問題講解 Explanation of Problems

☙ 學習目標 ❧

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to know that:

一、了解內力不會改變系統之總動量。

Understand that internal forces do not change the total momentum of a system.

二、熟悉動量守恆定律的條件及用法。

Be familiar with the conditions of the law of conservation of momentum and how to apply it.

例題講解

例題一

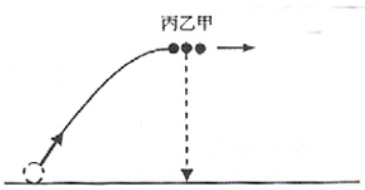
說明：熟悉動量守恆定律的概念，並透過系統總動量守恆，求得各質點之速度。

Be familiar with the concepts of conservation of momentum. Be able to calculate the speed of a point mass within a system using the total momentum of a system of masses.

A point mass is projected upwards at an angle to the right from the ground. When it reaches its maximum height, it explodes into three equal point masses, A(甲), B(乙), and C(丙). As shown in the figure below, point mass B(乙) is in free-fall acceleration from rest after the explosion, while point mass C(丙) follows the original path and returns to the projectile point. Assuming the air resistance is negligible, what is the approximate ratio of the velocities between point masses A(甲) and C(丙) at the moment of explosion?

(A) 1/2　　(B) 1　　(C) 2　　(D) 3　　**(E) 4**

一個質點自水平地面朝右上方斜向拋射，在最高點時，突然爆裂為質量相等的甲、乙、丙三質點，如下圖所示。爆裂之後乙自靜止作自由落體運動，丙循原路徑回落到原拋射點。若忽略空氣阻力，則爆裂瞬間甲與丙速率的比值約為何？



(A) 1/2　　(B) 1　　(C) 2　　(D) 3　　**(E) 4**

（99 年指考 10）

解題 Solution：

When a point mass is projected at an angle and reaches its maximum height, the vertical velocity of that point mass equals 0 m/s, but it is still moving horizontally at a speed of v. On the other hand, point mass C(丙) follows the original path and returns to the projectile point of projection after explosion, so C's(丙的) velocity is v but in the opposite direction from the original one. In addition, the explosion contains only internal force, where no horizontal external forces involved. All three masses are solely pulled by gravity in the vertical direction. As a result, the total horizontal momentum of this system before and after the explosion is conserved.

Let the mass of the original projectile point be m and the velocity of point mass A be v.

The entire motion of the three point masses in the system can be written mathematically as $mv = \frac{m}{3} \times v' + \frac{m}{3} \times 0 + \frac{m}{3} \times (-v)$ thus $v' = 4v$. The ratio of the (horizontal) velocities between A and C at the moment of explosion is $\frac{v'}{v} = \frac{4v}{v} = 4$.

質點在斜拋抵達最高點時，鉛直方向之速度=0，只剩下水平速度v，而丙在爆炸後沿原路徑回到拋射點，故丙的速度為v，但與原方向相反。又爆炸屬於內力，系統在過程中，僅受到鉛直之重力作用，沒有水平之外力，所以爆炸前後的系統在水平之總動量守恆。

設原質點質量為m甲質點速度為 v'，則可得$mv = \frac{m}{3} \times v' + \frac{m}{3} \times 0 + \frac{m}{3} \times (-v)$，因此$v' = 4v$，故爆裂瞬間甲與丙速率的比值為$\frac{v'}{v} = \frac{4v}{v} = 4$。

Teacher: Recall what we have learnt before. When a point mass reaches its maximum height, in what direction this point mass is still moving?

Student: When a point mass is thrown at an angle to its maximum height, that means that the vertical velocity of this point mass is zero, while its horizontal velocity retains the momentum.

Teacher: That is right. At this highest point, the point mass explodes. Tell me, does an explosion contain an internal force or external one?

Student: Explosion contains only internal force, and it means that this system of the three point masses is not influenced by external force.

Teacher: Great job! Will the total momentum of the system before and after the explosion change?

Student: No, the horizontal momentum of the system won't change due to explosion, as the system is only subject to gravity. There is no horizontal external force exerted on the system. So, the total horizontal momentum of this system before and after the explosion is conserved.

Teacher: That is correct. As mentioned earlier, the horizontal velocity of this system remains constant at the apex of its trajectory. Assuming that horizontal velocity at the moment of explosion is v, the mass of the point mass is m, can you determine the total momentum of the system before the explosion?

Student: The total momentum of the system before the explosion is equal to the mass (m) multiplied the velocity (v) of a point mass $m\vec{v}$.

Teacher: That is great! We know this point mass splits into three equal masses, A, B, and C after the explosion. The question also tells us that point mass B is in free-fall acceleration from rest after the explosion, while point mass C follows the original path along the projectile trajectory. What is the momentum of these two point masses?

Student: Because the masses of point masses A, B, and C, are equal, their masses are $\frac{m}{3}$. Point mass B is is at rest after the explosion, so its initial velocity is zero and its momentum is $\frac{m}{3} \times 0 = 0$. Point mass C follows the original path and returns to the projectile point, so its velocity is $-v$ and its momentum is $\frac{m}{3} \times (-v') = \frac{-mv}{3}$.

Teacher: That is correct. Assuming the velocity of point mass A is v', what is the total momentum of the system that comprises all the three point masses after the explosion?

Student: Because the momentum of point mass A is $\frac{m}{3} \times v'$, the total momentum of the system after the explosion is the sum of the momentum of the three point masses, A, B, and C $\frac{m}{3} \times v' + \frac{m}{3} \times 0 + \frac{m}{3} \times (-v)$.

Teacher: That is right. As mentioned earlier, the total momentum of this system before and after the explosion is conserved in the horizontal direction. Now, we have these two equations and can solve for Point Mass A's velocity and the ratio between its velocity and C’s.

Student: I see it now. Because the total momentum of this system is conserved, from $mv = \frac{m}{3} \times v' + \frac{m}{3} \times 0 + \frac{m}{3} \times (-v)$, we could find the velocity of point A: $v' = 4v$. As a result, the ratio between the velocities of points A to C at the moment of explosion is $\frac{v'}{v} = \frac{4v}{v} = 4$.

老師： 我們之前學過，當質點到達斜拋最高點時，會剩什麼方向的速度？

學生： 當質點到達斜拋最高點時，因為鉛直方向的速度是零，所以只會剩下水平方向的速度。

老師： 沒錯。而此時質點發生爆炸，那麼爆炸是屬於內力還是外力？

學生： 爆炸是屬於內力，系統不受水平之外力。

老師： 很好！所以爆炸前後的系統總動量會受到影響嗎？

學生： 不會，因為系統在爆炸過程中，僅受到鉛直之重力作用，沒有水平之外力，所以爆炸前後的系統水平總動量守恆。

老師： 是的。那我們前面有提到質點在最高點時只剩水平速度，若設此速度為 v，設質點的質量為 m，那麼系統爆炸前的總動量是多少呢？

學生： 爆炸前的系統總動量為質點質量與速度的乘積，也就是 $m\vec{v}$。

老師： 很棒！而質點在爆炸後，變成質量相等的甲、乙、丙三質點。乙自靜止作自由落體運動，而丙循原路徑回落到原拋射點，那乙和丙的動量分別是多少呢？

學生： 因為甲、乙、丙三質點質量相等，所以三者質量皆為$\frac{m}{3}$。乙自靜止作自由落體運動，速度為 0，所以動量是$\frac{m}{3} \times 0 = 0$；丙循原路徑回落到原拋射點，所以速度是$-v$，因此動量為$\frac{m}{3} \times (-v') = \frac{-mv}{3}$。

老師： 沒錯。那如果令甲質點的速度為v'，則爆炸後的系統總動量是多少呢？

學生： 甲質點的動量為$\frac{m}{3} \times v'$，爆炸後的系統總動量為甲、乙、丙三質點的動量總和，所以是$\frac{m}{3} \times v' + \frac{m}{3} \times 0 + \frac{m}{3} \times (-v)$。

老師： 是的。那我們前面有提過爆炸前後，系統水平總動量守恆，現在我們有爆炸前跟爆炸後的系統水平總動量，就能求出甲的速度，與甲與丙速率的比值了。

學生： 我知道了，因為系統總動量守恆，所以$mv = \frac{m}{3} \times v' + \frac{m}{3} \times 0 + \frac{m}{3} \times (-v)$，算出甲速度$v' = 4v$，因此爆裂瞬間甲與丙速率的比值為$\frac{v'}{v} = \frac{4v}{v} = 4$。

例題二

說明：了解接球前後，系統不受任何水平之外力，所以水平之總動量守恆。進而推導出A、B 兩人的速度，及相對速度。

Understand that, before and after a ball is caught, because there is no external force in the horizontal direction of a system, the total horizontal momentum of the system is conserved. Therefore, the velocities and relative velocities of the two catchers, A and B can be obtained.

A and B are the two catchers wearing ice skates. Now, they are standing still on the ice and facing each other. A throws a basketball to B. Assuming the masses of A and B as m_A and m_B, respectively. The mass of the basketball is m, and its horizontal flight velocity is v. What is the magnitude of A's relative velocity with respect to B after the basketball is thrown.

(A)$mv(\frac{1}{m_A}-\frac{1}{m_B+m})$

(B) $\boldsymbol{mv(\frac{1}{m_A}+\frac{1}{m_B+m})}$

(C) $mv(\frac{1}{m_A}-\frac{1}{m_B})$

(D) $mv(\frac{1}{m_A}+\frac{1}{m_B})$

(E) $mv(\frac{1}{m_A+m_B})$

A、B 兩人各穿著冰刀，面對面靜止站在冰上，今 A 把手中籃球拋傳給 B 接住。設兩人的質量各為m_A及m_B，籃球的質量為m，而籃球傳出的水平速度為v，則當籃球傳過之後，A、B 兩人相對速度之量值為何？

(A)$mv(\frac{1}{m_A}-\frac{1}{m_B+m})$

(B) $\boldsymbol{mv(\frac{1}{m_A}+\frac{1}{m_B+m})}$

(C) $mv(\frac{1}{m_A}-\frac{1}{m_B})$

(D) $mv(\frac{1}{m_A}+\frac{1}{m_B})$

(E) $mv(\frac{1}{m_A+m_B})$

（67 日大 5）

解題 Solution：

During the passing and catching of a basketball, there are only action forces and reaction forces between A and B with equal but opposite direction to each other. In the system that comprises A, B, and the ball, the internal forces between A and B can be canceled. No external forces in the horizontal direction act on this system, so its total horizontal momentum is conserved.

When A throws the basketball forward with an initial velocity of v, the basketball gains forward momentum. At the same time, an opposite force is acting on A, pushing him backward. This causes A to experience a change in velocity $-v_1$ (the direction is negative). Let this velocity be represented by the equation between the basketball and A can use this math equation to represent $0 = -m_A v_1 + mv$. Simplifying this equation can solve for A's velocity $-v_1 = \frac{-mv}{m_A}$. In a similar manner, when B catches the basketball, the momentum of both objects is changed in an equal magnitude (the direction is positive). Because the total momentum of this system that comprise A, B, and the ball remains constant, $mv = (m_B + m)v_2$. Simplifying this equation $v_2 = \frac{mv}{m_B+m}$.

As a result, the magnitude of the relative velocity of A with respect to B is $v_2 - (-v_1) = \frac{mv}{m_B+m} + \frac{mv}{m_A} = mv(\frac{1}{m_B+m} + \frac{1}{m_A})$

.

由於接球前後，因為只有 AB 兩人之間傳球的作用力，對於[A+B+球]的系統，屬於作用力與反作用力(內力)，會相互抵銷，系統不受任何水平之外力，所以系統水平之總動量守恆。當 A 以速度v將籃球向前拋出時，籃球獲得向前之動量，A 同時也會受到一向後的動量變化(方向為負)。設此速度為$-v_1$，則可得出$0 = -m_A v_1 + mv$，將其整理後得到 A 的速度$-v_1 = \frac{-mv}{m_A}$。當 B 接到球後，B 和球同時也會得到相等的動量變化(方向為正)，因接球前後，[A+B+球]的系統總動量不變，因此$mv = (m_B + m)v_2$，將其整理後得到$v_2 = \frac{mv}{m_B+m}$。故相對速度量值為$v_2 - (-v_1) = \frac{mv}{m_B+m} + \frac{mv}{m_A} = mv(\frac{1}{m_B+m} + \frac{1}{m_A})$。

Teacher: Let's analyze the forces acting on the basketball before and after it is passed. When A passes the ball, what forces act on it?

Student: A exerts a force on the ball in the direction of B.

Teacher: How about after B catches it?

Student: B exerts a force acting on the basketball that moves in A's direction.

Teacher: That is great! Now, how do we find the velocities of A and B respectively?

Student: This is so complicated! I don't know how to do it.

Teacher: We can avoid calculating these complicated forces using a simple method. That is, we can group the basketball and the two persons into one system and only discuss the external forces acting on it.

Student: Oh! So, I can treat A, B, and the basketball as a system that comprises them. This way, the action forces between two individuals' ball passing and catching can be seen as internal forces. The total momentum remains constant unless an external force is applied.

Teacher: That is right. There is no force in the horizontal direction acting on this system, so its total horizontal momentum is conserved. As a result, what we need to do now is to figure out the total momentum of these three objects before and after the ball is thrown. Tell me, what is the total momentum of the system before A passes the ball?

Student: A begins throwing the ball from rest. This means that the total momentum of the system (A and the basketball) before the throwing is zero.

Teacher: That is right. After the throwing, the force exerted by A pushes the basketball forwards. At this time, the ball's velocity is v and in what direction is A's velocity?

Student: Because the total momentum of this system is conserved, A's velocity after the throw should be in the backward direction. His velocity is negative.

Teacher: Yes. A's velocity is $-v_1$, and we also know other three elements: 1) the total amount of the initial momentum of this system is zero, 2) A's mass is m_A, and 3) v is the velocity of the basketball and m is its mass. Now, we can solve for $-v_1$.

Student: I get it now! $0 = -m_A v_1 + mv$ A's velocity is $-v_1 = \frac{-mv}{m_A}$.

Teacher: That is correct. In a similar manner, we can treat B and the thrown basketball as another system. Before B catches the ball, what is the total momentum of this system that comprises B and the thrown basketball.

Student: Assuming the basketball's velocity is v, with B' stationary status before B catches the ball, the total momentum of this system is mv.

Teacher: That is right. When B catches the ball, their velocities are in the backward, positive direction v_2. What is the total momentum of the system after B catches the ball?

Student: The total amount of the momentum of this system is $(m_B + m)v_2$.

Teacher: Good job. We have determined the total momentum of the system before and after B catches the ball. Now, we can solve for v_2.

Student: I get it. $mv = (m_B + m)v_2$ thus$v_2 = \frac{mv}{m_B+m}$.

Teacher: That is correct. Now we can determine the magnitude of the relative velocity of A with respect to B after the basketball is thrown.

Student: Oh I see it now. In this case, $v_2 - (-v_1) = \frac{mv}{m_B+m} + \frac{mv}{m_A} = mv(\frac{1}{m_B+m} + \frac{1}{m_A})$, so the magnitude of the relative velocity of A with respect to B $mv(\frac{1}{m_B+m} + \frac{1}{m_A})$.

老師： 我們來分析籃球拋傳的前後的受力情形。A 把球傳出去時，球的受力如何？

學生： A 對球施加指向 B 方向的力。

老師： 那 B 接到球之後呢？

學生： B 給球一個往 A 方向的力。

老師： 很好~那我們要如何找到 A、B 兩人的速度呢？

學生： 球與人之間受力情況很複雜耶，我不知道該如何繼續。

老師： 我們可以使用一個簡單的方法，避免計算這些複雜的力。劃分出適當的系統，只討論系統的外力。

學生： 我懂了！先把 A、B 和籃球，視為一個系統，兩人之間傳球的作用力就可以視為內力，只要系統不受外力，動量就會守恆。

老師： 沒錯，本題的系統在水平方向不受力，所以系統水平之總動量守恆。因此，我們需要了解拋球前後，A、B、和籃球的總動量，在 A 拋出球前的系統總動量是多少呢？

學生： 因為 A 是由靜止狀態將球拋出，所以系統總動量是零。

老師： 沒錯。那麼在拋球後，籃球受到一向前的速度v，此時 A 的速度方向應該是指向哪個方向呢？

學生： 因為系統總動量守恆，所以 A 的速度方向是向後的，速度為負。

老師： 很好！假設 A 的速度是$-v_1$，現在我們知道系統前的總動量為零，也知道 A 的質量 m_A，以及籃球的速度v跟質量m，就可以求出$-v_1$了。

學生： 我知道了。$0 = -m_A v_1 + mv$，得到 A 的速度$-v_1 = \frac{-mv}{m_A}$。

老師： 是的。再來我們也可以將 B 和拋出後的籃球視為另一個系統，在 B 接到球前的系統總動量是多少呢？

學生： 籃球的速度是v，而 B 靜止，所以接球前的系統總動量是mv。

老師：沒錯。那當接球後，B 和球同時也會受到一向後、方向為正的速度，如果設此速度為v_2，那麼接球後的系統總動量應該會是多少呢？

學生：接球後的系統總動量是$(m_B + m)v_2$。

老師：很棒，現在我們知道接球前後的系統總動量，就能求出v_2了。

學生：我懂了，$mv = (m_B + m)v_2$，所以$v_2 = \frac{mv}{m_B+m}$。

老師：是的，那現在知道$-v_1$和v_2是多少之後，就可以求出在籃球傳過之後，A、B 兩人相對速度之量值是多少了。

學生：喔～我知道了，所以$v_2 - (-v_1) = \frac{mv}{m_B+m} + \frac{mv}{m_A} = mv(\frac{1}{m_B+m} + \frac{1}{m_A})$，因此 A、B 兩人的相對速度量值是$mv(\frac{1}{m_B+m} + \frac{1}{m_A})$。

1-3 質心運動與系統總動量
Motion of Center of Mass and Total Momentum of a System

■ 前言 Introduction

本小節從物體的重心位置來引入質心位置，同時將牛頓定律由單一質點延伸到多質點或有體積的物體，討論其質心加速度與質心速度，再利用質心速度引入系統總動量，來解決物體運動(動力學)的問題。根據牛頓第三運動定律，得知多質點系統中的內力，不會影響質心的速度，只有外力才能改變質心的運動狀態，因此可由質心的運動狀態，得知系統中每個質點的運動情形。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
center of gravity	重心	motion of center of mass	質心運動
gravitational torque	重力力矩	equation of motion of center of mass	質心運動方程式
resultant force	合力	trajectory	軌跡
center of mass	質心	parabola	拋物線
total momentum of system	系統總動量	interaction	交互作用

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ **The _____ is the average _______ of all the particles in the system, ______ according to _______.**

例句：**The** center of mass **is the average** position **of all the particles in the system**, weighted **according to** their masses.

質心為系統中所有質點質量加權後的平均位置。

❷ **Since the total momentum of system is conserved, after _________, the _________________________.**

例句：**Since the total momentum of the system is conserved**, **after** the explosion, **the** momentums of the small fragments will be the same as before.

因為系統總動量守恆，在爆炸後，小碎片的動量相加會與原來相同。

■ 問題講解 Explanation of Problems

學習目標

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to know that:

一、能了解爆炸是系統內力，故可由動量守恆，求出破片之運動。

Be able to understand that explosion involves only internal force, so can find the motion of a fragment through the conservation of momentum.

二、熟知質心位置、各質點位置與質量間的關係。

Be able to determine the position of the center of mass by means of the positions and the masses of each point mass.

ᘛ 例題講解 ᘚ

例題一

說明：了解系統之內力不會影響系統質心的速度，並透過系統質心與其餘各質點的位置，找出特定質點的位置。

Understand that internal force does not influence the speed of the center of mass. Thus, the position of a specific point mass can be determined via the center of mass and positions of other point masses.

A bomb free-fall from 600-meter high, the gravitational acceleration is 9.8 m/sec^2. The bomb explodes into two pieces vertically during the fall, the two pieces have equal mass and separate upwards and downwards. If the air resistance is ignored, 10 seconds after the bomb falls down, one of the two pieces hits the ground. How high the other piece is from the ground?

(A) 110 meters

(B) 220 meters

(C) 280 meters

(D) 490 meters

(E) 0 meter

一炸彈自 600 米的高空自由落下，重力加速度為 9.8 米/秒^2，於中途爆裂成兩個等重的碎片，在垂直線上分上、下炸開。如空氣阻力可以不計，炸彈下落後 10 秒時有一破片擊中地面，則此時另一破片距地面之高度為何？

(A) 110 米

(B) 220 米

(C) 280 米

(D) 490 米

(E) 0 米

（70 日大 5）

解題 Solution：

Explosion is internal force. The total momentum of the system is conserved without any external force, so the center of mass of the system keep constant velocity motion. It is known that the center of mass falls on the ground 10 seconds after the explosion. $h = ½ \times 9.8 \times 10^2 = 490$ m. At this moment, the distance between the center of mass and the ground is: $600 - 490 = 110$ m. Meanwhile, one piece hits the ground. Due to the two pieces having the same weight, the center of mass is in the middle of the line that connects the two pieces. Therefore, it is known that the other piece is $110 \times 2 = 220$ meters high above the ground.

爆炸為系統內力，在系統不受外力作用之下，系統的總動量會守恆，因此系統質心會繼續沿原來軌跡等速直線運動。故可知質心 10 秒後下落 $h = ½ \times 9.8 \times 10^2 = 490$ m，此時質心距離地面 $600 - 490 = 110$ m，且同時有一破片擊中地面。由於兩破片等重，因此質心會位於兩破片連線的中心位置，所以可知另一破片距地面之高度為 $110 \times 2 = 220$ m。

Teacher: Does the bomb experience any force when it explodes?

Student: Yes. The power of explosion makes the bomb into pieces and the pieces fly away.

Teacher: Exactly. Can you tell me the motions of the two pieces after explosion?

Student: It's too hard to calculate.

Teacher: It is hard, indeed. If you cannot tell the motion of each piece, you can take the pieces as one system.

Student: Oh, this would be much easier. The bomb receives the power of the explosion… but I don't know how powerful the force is.

Teacher: Don't wonder how powerful it is. Try to think about the force, is it external or internal force to the bomb?

Student: It's the internal force of the system, so it doesn't influence the total momentum of the system!

Teacher: Great! According to Newton's third law of motion, action and reaction, the forces offset if they happen to the same system. Therefore, the internal force of the system must be offset.

Teacher: Let's see if there is any external force that influences the motion of the system.

Student: The bomb free falls, it is only influenced by gravity.

Teacher: Good. Its horizontal external force is zero. According to Newton's second law of motion, to a system with system points mass or an object, $\sum \vec{F} = m\overrightarrow{a_{Gm}}$.
Because the external force = mg(↓). the horizontal acceleration of the center of mass = 0.

Teacher: Horizontally, the center of mass goes forward remaining at the same speed; vertically, it does uniform acceleration due to gravity.

Student: Well, then vertically, we can use the formula of uniform acceleration. The center of mass falls in 10 seconds, h = ½ × 9.8×10^2 = 490 m.

Teacher: Good. What's next?

Student: The center of mass would be 490 meters high. One piece of bomb hits the ground, so the other piece would be 490 meters high above.

Teacher: Wait, the center of mass falls 490 meters from 600 meters high, so…?

Student: Oh, the center of mass is 600 – 490 = 110 meters high above the ground.

Teacher: Yes, the center of mass is 110 meters high above the ground, and the other piece hits the ground, where is the other piece?

Student: It is higher than 110 meters, but I don't know where it is exactly.

Teacher: The question says that the two pieces are same weight, so the center of mass would be in the middle of the connection of the two pieces.

Student: I see. The other piece would q 110 x 2 = 220 meters high above the ground.

老師： 炸彈爆炸時，炸彈有受力嗎？

學生： 有~爆炸的威力，會使炸彈分裂形成碎片，碎片受力會飛出去。

老師： 是的，那你能我告訴炸彈爆炸後兩片碎片分別的運動狀態嗎？

學生： 太難了！根本算不出來。

老師： 的確很難，如果無法分析碎片個別的狀況，那我們可以將所有炸彈碎片視為一個系統。

學生： 哦~這樣就簡單多了。炸彈受到爆炸的力量……可是我不知道這個力有多大阿。

老師： 別急著找出力的大小，先想想這個力量，對整個炸彈系統來說是外力還是內力？

學生： 是系統的內力！所以它不會影響系統的總動量！

老師： 太棒了！根據牛頓第三運動定律，作用力與反作用力，若同時在系統作用，則可以抵銷，因此系統內力必然互相抵銷。

老師： 那我們來找找有任何外力，會影響系統的運動狀態嗎？

學生： 炸彈自由落下，它只受到重力作用。

老師： 很好~所以水平的外力=0，根據牛頓第二定律，對多質點系統或有體積的物體而言，$\sum\vec{F} = m\overrightarrow{a_{Gm}}$，因為外力=mg(↓)，質心的水平加速度=0。

老師： 所以，在水平方向質心維持原有的速度前進，而鉛直方向則因受重力，而做等加速度運動。

學生： 哦~那鉛直方向，就可以用等加速度運動公式，質心 10 秒後下落

$h = ½ \times 9.8\times10^2 = 490$ m

老師： 很好，接下來呢？

學生： 質心會在高 490 m 的位置，有一破片擊中地面，所以另一破片的位置會高於 490 m。

老師： 等等，質心由 600 米的高空落下 490 米，所以？

學生： 哦~質心離地面 600 − 490 = 110 m。

老師： 是的，此時質心離地面 110 公尺，且有一破片擊中地面，那另一破片的位置在哪？

學生： 會高於 110 公尺，不過我不知道確切會在哪。

老師： 題目告訴我們兩破片等重，因此質心會位於兩破片連線的中心位置。

學生： 我懂了，另一破片會距離地面 110 × 2 = 220 m。

例題二

說明：了解爆炸可視為內力，不受外力時，利用 $\sum \vec{F} = m\overrightarrow{a_{Gm}}$ 得知質心可維持動量守恆，再根據質心之定義，由各質點位置和質量分布，求出碎片質量。

Students understand that explosion can be taken as internal force, when there is no external force, use the formula $\sum \vec{F} = m\overrightarrow{a_{Gm}}$ to know that the center of mass could keep the momentum conserved. Also, according to the definition of the center of mass, students are able to find the mass of the pieces through the position and distribution of point mass.

An object, which has 8.0 kilograms mass, explodes from 30 meters high above the ground. The object splits into two pieces from the explosion and the pieces fly away along the vertical direction. After 2.0 seconds of the explosion, one of the pieces just hits the ground, and the other is 16 meters high above the ground. If the air resistance and the lost mass from the explosion are ignored, how many kilograms is the mass of the piece that hits the ground first? (The gravitational acceleration is set at 10 m/sec^2)

一個質量為 8.0 公斤的物體在距地面高度 30 公尺處由靜止發生爆炸，爆炸瞬間分裂為兩碎片，且同時沿鉛直方向飛離。在爆炸後 2.0 秒時，其中一碎片恰落地，而另一碎片尚離地面 16 公尺高。若空氣阻力與物體因爆炸而損失的質量均可不計，則爆炸後先落地的碎片之質量為多少公斤？（取重力加速度為 10 公尺/秒 ^{2}）

(A) 7.0

(B) 6.0

(C) 5.0

(D) 4.0

(E) 3.0

（107 指考 18）

解題 Solution：

An explosion is seen as the internal force of a system. It can be inferred that the external force the object receives is only gravity through the formula $\sum \vec{F} = m\overrightarrow{a_{Gm}}$. As a result, the horizontal total momentum conserves, and the vertical total momentum keeps free fall. The falling distance of the center of mass 2.0 seconds after explosion is $y = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 2^2 = 20$ m.

So, the height of the center of mass is h=30-20=10 m. Let's assume the mass of the piece that hits the ground first is m kilograms, and from the relationships of the position the center of mass, mass, and distance, we can get the equation $10 = \frac{m\times 0+(8-m)\times 16}{8}$. Finally, $m = 3$kg.

將爆炸視為系統之內力，利用$\sum \vec{F} = m\overrightarrow{a_{Gm}}$可推出物體所受外力僅有重力，因此，系統的水平總動量守恆，而鉛直方向則保持自由落體。故爆炸後 2.0 秒質心落下距離為$y = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times 2^2 = 20$ m。因此質心離地高度 h=30-20=10 m。假設先落地的碎片之質量為 m公斤，由質心位置、質量、距離的關係可得$10 = \frac{m\times 0+(8-m)\times 16}{8}$，因此可知 $m = 3$ kg。

Teacher: We've mentioned before, does an explosion involve internal force or external force of a system?

Student: The horizontal component involves internal force only. So, the horizontal total momentum of the center of mass is conserved.

Teacher: Correct. According to Newton's third law of motion, action and reaction is the internal force of the system, it offsets.

Teacher: Therefore, the system only includes gravity, which is downwards, and the center of mass of the system moves with constant acceleration motion downwards: a = g. Then how high is the center of mass at 2.0 seconds after explosion?

Student: The center of mass becomes free-fall from motionless after explosion, so the distance the center of mass falls at 2.0 seconds after explosion is $y = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times 2^2 = 20$ m.

The height of the center of mass is h=30-20=10 m.

Teacher: Great. One of the two pieces just hits the ground at the moment. Let's assume the mass is m. The other piece is 16 meters high from the ground. How do you know the mass of the piece that hits the ground first?

Student: Well, I don't know.

Teacher: Do you remember the relationship between point mass and the position of the center of mass in a system?

Student: The mean of the position of the point mass of the system plus the mass of the point mass equals the center of mass.

Teacher: Yes. And the formula of the position of the center of mass is $\vec{r_c} = \frac{m_1\vec{r_1}+m_2\vec{r_2}+\cdots+m_N\vec{r_N}}{m_1+m_2+\cdots+m_N} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r_i}$. Now we know the mass of the center of mass and its position, which is $10 = \frac{m\times0+(8-m)\times16}{8}$, as well as the positions of the two point mass, we can get the mass of the point mass.

Student: I get it, $10 = \frac{m\times0+(8-m)\times16}{8}$, so $m = 3$ kg. Therefore, the mass of the piece which hits the ground first is 3 kg.

老師： 我們之前有提過，爆炸是系統的內力還是外力呢？

學生： 是系統的內力，所以質心的水平總動量，維持守恆。

老師： 沒錯，根據牛頓第三運動定律，作用力與反作用力若是系統內力，則會互相抵銷。

老師： 因此，系統僅包含向下的重力，所以系統質心會呈現向下等加速度運動:a=g。那麼當爆炸後 2.0 秒時，質心的高度會是多少呢？

學生： 爆炸後，質心由靜止做自由落體運動，所以質心在爆炸後 2.0 秒時，下落了$y = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times 2^2 = 20$ m，質心離地面高度 h 是 h=30-20=10 m。

老師： 很好，此時兩碎片的其中一片恰好落地，假設其質量為m，而另一個碎片離地高度 16 公尺，那要怎麼知道先落地碎片的質量呢？

學生： 嗯...我不知道耶。

老師： 還記得系統中的質點跟質心位置的關係嗎？

學生： 系統中的質點位置和質點質量加權後取平均值，會等於質心的位置。

老師： 沒錯，又質心位置的公式是$\vec{r_c} = \frac{m_1\vec{r_1}+m_2\vec{r_2}+\cdots+m_N\vec{r_N}}{m_1+m_2+\cdots+m_N} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r_i}$，那我們現在知道質心的質量跟位置$10 = \frac{m\times0+(8-m)\times16}{8}$，還有兩質點的位置，我們就可以求出質點的質量了。

學生： 我懂了，$10 = \frac{m\times0+(8-m)\times16}{8}$，所以$m = 3$ kg，因此先落地碎片的質量是3 kg。

1-4 角動量與力矩
Angular Momentum and Torque

■ 前言 Introduction

透過生活中省力及費力的實例，定義力矩是受力及乘以力臂，並以單擺為例，分析力矩的方向。接著介紹角動量與力矩的關係，也比較平移及轉動的各種物理量及定義。最後討論角動量守恆的條件與應用，由理論驗證克卜勒第二行星運動定律。力矩及角動量皆是轉動相關的物理量，都需使用向量外積來推導。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
point of application	施力點	circular motion	圓周運動
torque	力矩	normal force	法向力
pivot point	支點	tangential force	切線力
arm of force / moment arm	力臂	radius of rotation	旋轉半徑
outer product / cross product	外積	angular momentum	角動量
right hand rule	右手定則	angular displacement	角位移
rotational motion	旋轉運動	angular velocity	角速度
counterclockwise rotation	逆時針旋轉	law of angular momentum conservation	角動量守恆定律
clockwise rotation	順時針旋轉		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ **According to _______, the direction of _________ is ____________.**

例句：**According to** the right hand rule, **the direction of** the angular momentum **is** pointing up out of the page.

根據右手定則，角動量的方向為出紙面方向。

❷ **________ rotate clockwise/counterclockwise around ___________.**

例句：The object **rotates counterclockwise around** the reference point.

物體繞著參考點逆時針旋轉。

■ 問題講解 Explanation of Problems

☙ 學習目標 ❧

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to:

一、了解力矩的定義，及力矩與角動量變化的關係。

Understand the definition of torque and the relationship between torque and change of angular momentum.

二、熟悉力矩對物體運動狀態改變的影響。

Be familiar with how torque affects changes in an object's state of motion.

例題講解

例題一

說明：能判斷力臂大小來計算力矩，並利用力矩與角動量的關係解題。

Students are able to calculate the torque through the power of the arm of force and solve questions utilizing the relationship of the torque and angular momentum.

One point mass does uniform circular motion around O on a horizontal plane, and the rate is v, as shown in the picture. A(甲), B(乙), C(丙), D(丁), and E(戊) are all on the circumference. Consider point D to measure the angular momentum of the point mass, at which point the value of the time rate of change of the angular momentum of this point mass is the largest?

(A) A (B) B **(C) C** (D) D (E) E

一質點以 O 為圓心在一水平面上作等速率圓周運動，其速率為 v，如圖所示。甲、乙、丙、丁、戊皆在圓周上，如果以丁點為參考點測量質點的角動量，則該質點角動量時間變化率的量值在圖中哪一處最大？

(A) 甲 (B) 乙 **(C) 丙** (D) 丁 (E) 戊

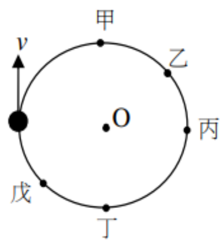


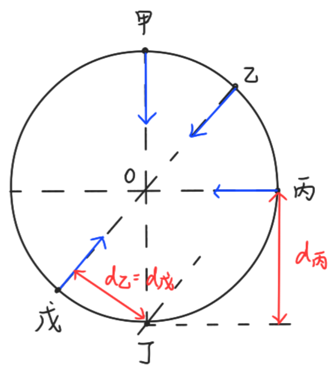
圖 6

（99 年指考 8）

解題 Solution：

The point mass does uniform circular motion means that the point mass receives a fixed centripetal force. The torque τ equals to the time rate of change of the angular momentum L. $\tau=\frac{\Delta L}{\Delta t}$. The torque τ can be represented as the product of force (F) and the moment arm(d): $\tau=F\times d$. Therefore, the time rate of change of the angular momentum is proportional to the force of arm (d): $\frac{\Delta L}{\Delta t}\propto d$. From the picture, it is known that $d_C>d_B=d_E>d_A=0$, and the value of the time rate of change of the angular momentum is largest at point C.

質點作等速率圓周運動表示質點受一固定向心力F作用。力矩τ為角動量 L 的時變率，$\tau = \frac{\Delta L}{\Delta t}$，力矩$\tau$又可以力$F$與力臂$d$的乘積表示，$\tau = F \times d$。因此，角動量時變率正比於力臂 d，$\frac{\Delta L}{\Delta t} \propto d$。由圖知： $d_{丙} > d_{乙} = d_{戊} > d_{甲} = 0$，角動量時間變化率量值在「丙點」最大。



Teacher: What does the time rate of change of angular momentum mean in the question?

Student: Torque.

Teacher: Exactly. According to $\tau = \frac{\Delta L}{\Delta t}$, torque can affect the change of angular momentum. Except for that, what is the source of torque?

Student: Torque is defined as force times the moment arm of the force, $\tau = F \times d$.

Teacher: Great, so $\tau = \frac{\Delta L}{\Delta t} = F \times d$. We can find the position that has the largest time rate of change of angular momentum from the power of the arm of force.

Student: But I don’t know how much force the point mass receives at each position.

Teacher: The question says that the point mass does uniform circular motion around O. In that case, do you know how much force does the point mass receive at each position?

Student: Well, the resultant force equals centripetal force, so the speed of the object does not change.

Teacher: Yes, and the force keeps towards the center of circle. We just need to know the arm of force considering point D.

Student: Oh, I see.

Teacher: Please come to the black board to draw the arm of force of point A, B, C, and D.

Teacher: Good. With this picture, we can tell the length of the arm of force. Can anybody tell me the relationships between their measurements?

Student: d_C is the largest, followed by $d_B$and d_E, then d_A. $d_C > d_B = d_E > d_A = 0$.

Teacher: That's right. The time rate of changing angular momentum is the largest at point C.

老師： 題目問的角動量時間變化率是什麼呀？

學生： 力矩！

老師： 沒錯，根據$\tau = \frac{\Delta L}{\Delta t}$，力矩可造成角動量變化的效果，除此之外，力矩的來源是什麼？

學生： 力矩來自外力乘上力臂，$\tau = F \times d$。

老師： 很好~所以$\tau = \frac{\Delta L}{\Delta t} = F \times d$，我們可以從力臂的大小，找出角動量時間變化率最大的位置。

學生： 可是我不知道質點在每個位置的受力大小耶。

老師： 題目告訴我們，質點以 O 為圓心在做等速率圓周運動，這樣你可以知道質點在每個位置的受力的大小嗎？

學生： 哦，質點在各處所受的合力等於向心力，所以大小是固定的。

老師： 是的，而且持續向著圓心，所以我們只要了解以丁點為參考點的力臂就可以了。

學生： 喔喔，我懂了。

老師： 請同學到黑板畫畫看甲、乙、丙和丁的力臂。

老師： 很好~由這張圖我們就可以判斷力臂大小，誰可以告訴我它們之間的大小關係呢？

學生： $d_{丙}$最大，再來是$d_{乙}$跟$d_{戊}$，最後是$d_{甲}$。$d_{丙} > d_{乙} = d_{戊} > d_{甲} = 0$。

老師： 沒錯，角動量的時間變化率量值，在「丙點」最大。

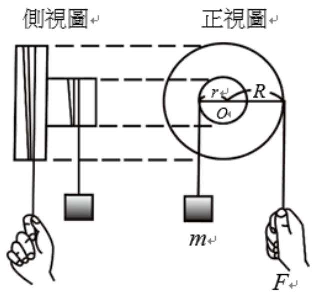
例題二

說明：由物體加速度，推出所受合力，再由受力推算力矩($\vec{\tau} = \vec{r} \times \vec{F}$)。
Infer the net force through acceleration of the object and further calculate the torque utilizing the determined force ($\vec{\tau} = \vec{r} \times \vec{F}$).

As shown in the diagram (side view on the left, front view on the right), the outer wheel radius R is twice the inner axle radius r. When a constant force F is applied to the rope on the outer wheel, it causes an object with mass m to move upward with an acceleration of 0.2g, where g is the gravitational acceleration. If air resistance, rope weight and friction can all be ignored, what is the magnitude of the torque generated at point O by the tension in the rope suspending the object?

如圖所示的輪軸（左邊為側視圖，右邊為正視圖），外輪半徑 R 為內軸半徑 r 的 2 倍。當施定力 F 於外輪上的細繩時，可使質量為 m 的物體以加速度 0.2 g 向上運動，g 為重力加速度。若空氣阻力、繩子的重量與摩擦力皆可忽略不計，則懸掛物體的繩子張力對軸心 O 點所產生的力矩之量值為下列何者？

(A) 0
(B) 0.6 mgR
(C) 1.2 mgR
(D) 1.6 mgR
(E) 2.0 mgR



（105 年指考 10）

解題 Solution：

Infer the resultant force through the known acceleration of the object and further calculate the tension of the rope T. Finally, calculate the torque of the tension utilizing the tension and its moment arm.

由已知物體之加速度推出合力，再由合力推出細繩張力 T，最後再由張力與力臂推出張力之力矩。使質量為 m 的物體以加速度 0.2 g 向上運動，因此$T - mg = m \times 0.2g$，得$T = 1.2mg$。則懸掛物體的繩子張力 T 對軸心 O 點所產生的力矩量值為$T \times r = 1.2mg \times \frac{R}{2} = 0.6mgR$。

Teacher: What is the definition of torque?

Student: The cross product of the vectors of the moment arm and the force.

Teacher: Yes. So, we need to know the tension of rope T and its moment arm. Think about it, what is the force exerted on the object with mass m?

Student: It receives rope tension T upwards and gravity mg downwards.

Teacher: What about the motion of the object?

Student: It goes upwards with 0.2 g acceleration.

Teacher: When the object receives rope tension T upwards and gravity mg downwards, it goes upwards with 0.2 g acceleration. How can we represent T?

Student: Because the acceleration of the object is upward, $T - mg = m \times 0.2g$. And we can get $T = 1.2mg$.

Teacher: Great. How much is the arm of force?

Student: It's the distance between point O and the line that rope tension works, which is r.

Teacher: That's right. Now we know the forces and their moment arms. We can find the torque of each force.

Student: It's easy! The value of torque is $T \times r = 1.2mg \times \frac{R}{2} = 0.6mgR$.

老師： 力矩的定義是什麼呢？

學生： 力臂與力的外積。

老師： 是的，所以我們要先找出繩張力 T 和力臂是多少。想一想，質量為 m 的物體受到哪些力的作用呢？

學生： 受到方向向上的繩張力 T 以及方向向下的重力 mg。

老師： 那物體的運動狀態如何呢？

學生： 它以加速度 0.2 g 向上運動。

老師： 所以物體受到向上的繩張力 T，以及向下的重力 mg 作用後，以加速度 0.2 g 向上運動，那麼我們可以怎麼表示 T 呢？

學生： 因為物體的加速度向上，所以$T - mg = m \times 0.2g$，得到 $T = 1.2mg$。

老師： 師：很好，那麼力臂是多少呢？

學生： 是軸心 O 點到繩張力作用線的距離，也就是 r。

老師： 沒錯，我們現在找出了力臂與力，就能求出力矩的量值了。

學生： 很簡單～力矩的量值就是 $T \times r = 1.2mg \times \frac{R}{2} = 0.6mgR$。

★ 主題二 功與能量 ★
Work and Energy

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■ 前言 Introduction

本章從「功」的定義，也透過 F-X 圖來計算作功的大小，並引入「功能定理」定理，且定義「功率」。接著定義保守力，並介紹重力位能及彈性位能兩種保守力的實例，符合「力學能守恆」。同時介紹重力位能的數學式，以計算物體的力學能。

2-1 功與動能
Work and Kinetic Energy

■ 前言 Introduction

本節定義「功」為外力 與位移 的純量積(內積) ($W = \vec{F} \cdot \vec{d}$)，因此作用力與位移的平行分量相乘，等於作用力對物體所作的功，兩者若垂直則不作功。並針對生活中作正功、作負功及不作功分別舉例說明。

也可以透過「作用力-位移」(F-X)之關係圖，來算出外力所作的功，此法可以解決非定力作功的問題，例如：拉長彈簧所需的功。最後介紹「功能定理」，並與「衝量—動量定理」相比較。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
energy	能量	inner product (scalar product)	內積(純量積)
principle of conservation of energy	能量守恆定律	external force	外力
work	功	chemical energy	化學能
kinetic energy	動能	positive / negative work	正/負功
potential energy	位能	work (-) energy theorem	功能定理
mechanical energy	力學能		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ __________ is exerted on _____________ over ____________.

例句：When an external force **is exerted on** an object **over** a certain distance, work has been done on the object.

當外力作用於物體並持續一段距離時，則該外力有對物體作功。

❷ The work done by ____________ on __________.

例句：The work done by an external force on an object is equal to the force multiplied by the distance parallel to which it is applied.

一個外力，作用在物體上所做的功，等於力乘以它所移動的平行距離。

■ 問題講解 Explanation of Problems

☙ 學習目標 ❧

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to know that:

一、了解動能的意義。

To understand the meaning of kinetic energy.

二、能正確運用功能定理。

Can correctly apply the work - energy theorem.

例題講解

例題一

說明：認識動能的定義，並可正確使用動能與速率的關係。

Students can understand the definition of kinetic energy and use its relationship with velocity correctly.

If a sphere of mass 720 kg has a kinetic energy of 3,600 kilo Joules, the speed of the sphere must be ___ m/s.

(A)10^2 (B) 10^3 (C) 10^4 (D) 10^5 (E) 10^6

若質量為 720 公斤的球體，具有 3600 千焦耳的動能，則此球體的速率約為多少公尺/秒？

(A)10^2 (B) 10^3 (C) 10^4 (D) 10^5 (E) 10^6

（111 學測 5）

解題 Solution：

Given the equation for kinetic energy $K = \frac{1}{2}mv^2$ where the mass of the object and the magnitude of its kinetic energy are known, we can solve for its velocity.

動能為$K = \frac{1}{2}mv^2$，已知物體質量及具有的動能，即可推知物體速率。$K = \frac{1}{2}mv^2 \Rightarrow 3600 \times 10^3 = \frac{1}{2} \times 720 \times v^2$。因此，速率$v = 100\ m/s$。

Teacher: Now, we have a sphere which weights 720 kg. It has a kinetic energy of 3,600 kilo Joules (KJ). What is kinetic energy related to?

Student: An object's kinetic energy is related to its velocity.

Teacher: That's great. Velocity means how fast or slow an object moves. How about this - Is kinetic energy only related to velocity?

Student: I guess so.

Teacher: Let's consider this example - if an elephant and a ping-pong ball are moving towards us at the same velocity, which of them has more kinetic energy?

Student: An elephant has more kinetic energy.

Teacher: That's correct! In this case, an object's kinetic energy is also related to what?

Student: Its mass.

Teacher: Correct. We can write kinetic energy as an equation $K = \frac{1}{2}mv^2$. Now, we are given with an object's mass and kinetic energy. We can solve for that object's velocity.

Student: Oh! I see it now! Plug in the numbers $3600 \times 10^3 = \frac{1}{2} \times 720 \times v^2$. Therefore, its velocity is $v = 100\ m/s$.

Teacher: Good job!

老師： 一個 720 公斤的球體，具有 3600 千焦耳的動能。請問，動能跟物體的什麼有關係呢？

學生： 跟物體移動的快慢有關係。

老師： 很好～物體移動的快慢就是物體的速率。那麼動能只跟速率有關係嗎？

學生： 應該是吧！

老師： 今天有一頭大象和一顆乒乓球用同樣的速率朝著我們前進，何者擁有較多動能呢？

學生： 大象擁有比較多的動能。

老師： 沒錯！所以動能還跟物體的什麼有關？

學生： 物體的質量。

老師： 是的！動能為 $K = \frac{1}{2}mv^2$，我們已知物體質量及具有的動能，就可以知道物體速率了。

學生： 哦！我懂了。把質量跟動能代入公式，$3600 \times 10^3 = \frac{1}{2} \times 720 \times v^2$。所以速率 $v = 100\ m/s$。

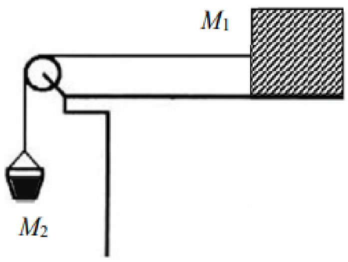
老師： 太棒啦！

例題二

說明：應用「功能定理」解決問題。

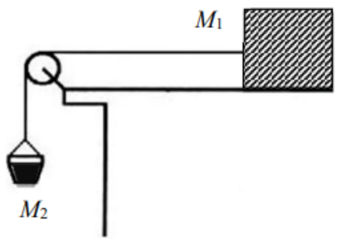
Students can apply work energy theorem to solve problems.

As shown below, a bucket is tied to a rope of a fixed length running through a frictionless pulley and attached to a wooden block of mass M_1 on a horizontal table. Sand is gradually added to the bucket until the system begins to move. At this time, the mass of the bucket with the added sand is M_2. There is no rolling motion with the block moving in the horizontal direction. The kinetic friction between the table and the block remains constant f, and the acceleration of the system is g. The magnitude and direction of the tension acting at any point in the rope is T.



When the block, initially at rest, is displaced to the left and the length of this displacement is s. The wooden block's kinetic energy is K_1, and the total kinetic energy of the bucket with the added sand is K_2. Which of the following equations are true? Select TWO that apply.

如圖所示，吊桶以長度不變的細繩繞過光滑定滑輪，連接到水平桌面上質量為 M_1 的木塊。今緩慢將微量細砂加入吊桶，直到吊桶與細砂的總質量為 M_2、木塊開始滑動為止。已知木塊水平移動且不轉動、受到的動摩擦力量值固定為 f，重力加速度量值為 g，且繩上各點的張力恆沿著繩子方向作用、量值均為 T。



當木塊由靜止開始，向左滑動的距離為 s 時，木塊的動能為 K_1，吊桶與細砂的總動能為 K_2，則下列等式哪些正確？（應選 2 項）

(A) $K_1=(T-f)s$　　(B) $K_1=(M_2g+T)s$　　(C) $K_2=(M_2g+T-f)s$

(D) $K_1+K_2=(M_2g-f)s$　　(E) $K_1+K_2=(M_2g-f-T)s$

（來源：110 學測 61）

解題 Solution：

The bucket and the wooden block are connected with the rope, so the magnitude of their displacements must be the same. Given the work-energy theorem $W = F \cdot S = \Delta K$ where F refers to the net force on an object, the net force on M_1 is T f and that on M_2 is M_2g T, so $K_1=F \cdot S=(T-f)s$；$K_2=F \cdot S=(M_2g-f)s$.

Adding M_1 and M_2 gives $K_1+K_2=(M_2g-f)s$.

木塊與吊桶以細繩相接，因此，兩者位移量相同。由功能定理$W = F \cdot S = \Delta K$，其中的 F 為物體所受之合力。M_1 所受合力$=T-f$，M_2 所受合力$=M_2g-T$，故 $K_1=F \cdot S=(T-f)s$；$K_2=F \cdot S=(M_2g-f)s$。

將 K_1 及 K_2 相加，得 $K_1+K_2=(M_2g-f)s$。

Teacher: When the wooden block moves from rest through a distance, there is a change in its kinetic energy. The work-energy theorem states that the work done by an external force on an object is equal to the change in its kinetic energy. So, we should figure out the work first.

Student: I know this one! Work is equal to force times displacement when the entire force is parallel to it. $W = F \cdot S = \Delta K$.

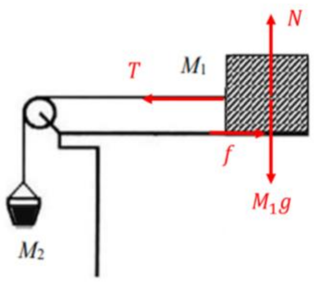
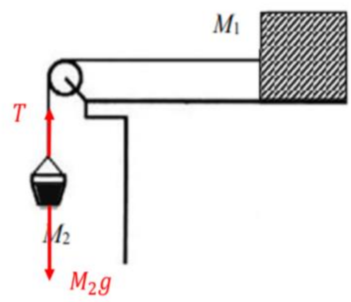
Teacher: Very good. Now, what does F in the work-energy theorem ($W = F \cdot S = \Delta K$) refer to?

Student: An external force.

Teacher: More specifically, F refers to the sum of all forces acting on an object. We call it net force.

Teacher: To calculate the kinetic energy of M_1 and M_2, we need to draw their respective free-body diagrams. I need two volunteers.

Student:

	
The forces applied on the wooden block	The forces applied on the bucket with the added sand

Teacher: Very good. Now, please calculate the work done by the external force on the wooden block.

Student: The gravity M1g is equal to the normal force in the positive direction on the wooden block N. The two forces cancel each other out. Therefore, the net force that gives total work done on M1 should be $W = F \cdot S$=(T−f)s.

Teacher: That's right! The wooden block is initially at rest, so its kinetic energy is zero. When the block starts to move by a force s, the change in the block's kinetic energy must be $K_1-0= K_1$. How do we express it in a mathematical equation using the work-energy theorem $W = \Delta K$?

Student: K1=(T−f)s.

Teacher: That's right. Following the same steps, we can solve for the kinetic energy of the bucket and sand K2.

Student: K2=(T−f)s

Teacher: Great! So, what will be the total kinetic energy of M1 and M2?

Student: The total kinetic energy of M1 and M2 combined is K1+K2=(M2g−f)s.

Teacher: Correct! So, what are the answers?

Student: (A) (D).

老師： 木塊從靜止到滑動，其中有動能變化。功能定理告訴我們，外力對物體作功會等於物體的動能變化。所以我們應該要先算出外力對物體作的功。

學生： 我知道，功等於力乘上沿著力方向的位移。$W = F \cdot S = \Delta K$。

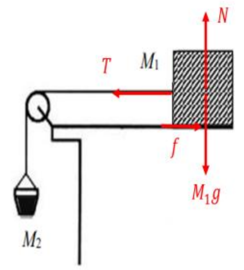
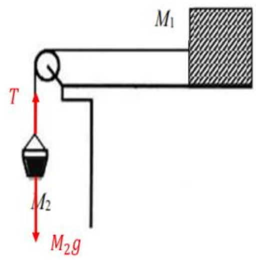
老師： 很好，那麼功能定理($W = F \cdot S = \Delta K$)中的F，指的是甚麼力？

學生： 外力。

老師： 更精確的說法，F指的是物體所受的外力總和，也就是合力。

老師： 為了算出 M_1、M_2 的動能，我們需分別畫出這兩個物體所受的力圖，請兩位同學上台來畫。

學生：

	
木塊受力	吊桶和細砂受力

老師：很好！請你把外力對木塊所作的功算出來。

學生：重力 M_1g 和桌子給木塊的正向力 N 相等，可以抵消，所以 M_1 所受的合力作功$W = F \cdot S$=(T−f)s。

老師：太棒了！木塊原本是靜止的，動能為零，所以當木塊受力滑動 s 後，木塊的動能變化就是 $K_1-0=K_1$。根據功能定理，$W = \Delta K$，我們可以怎麼列式呢？

學生：$K_1=(T-f)s$。

老師：是的，我們可以用相同的方法分析吊桶和細砂的動能 K_2。

學生：$K_2=(T-f)s$。

老師：很好~那麼 M_1 及 M_2 的總動能，會是多少呢？

學生：總動能為 $K_1+K_2=(M_2g-f)s$。

老師：沒錯！那麼答案應該是甚麼呢？

學生：(A)(D)。

2-2　功率 Power

■ 前言 Introduction

本節介紹功率，功率是「作功的效率」，效率與時間有關，因此定義平均功率為功對作功時間的比值。若作功時間極短，則可算出瞬時功率。另外，功率也可以表示能量轉移的快慢。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
Efficiency	效率	Energy conversion /transformation	能量轉換
Power	功率	consume	消耗
average power	平均功率	one kilowatt hour	一度(電)/1 千瓦·小時
instantaneous power	瞬時功率	horsepower (hp)	馬力
Watt	瓦(瓦特)		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ **__________ not only __________ but also __________.**

例句：Power is **not only** the rate at which external forces do work **but also** the rate at which energy is transferred.

功率不僅是外力作功的效率，也是能量轉移的快慢。

❷ **From the formula __________, it can be seen that __________**
Therefore, if __________, the power is __________.

例句：**From the formula** $P = \vec{F} \cdot \vec{v} = Fv\cos\theta$, **it can be seen that** power is the scalar product of external force and velocity. **Therefore, if** the two are perpendicular to each other, **the power** is zero.

由公式$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$可以看出，功率是外力與速度的純量積，因此若是兩者方向互相垂直時，功率為零。

❸ **__________ is the rate at which __________.**

例句：Power **is the rate at which** energy is transformed.

功率是能量轉換率。

■ 問題講解 Explanation of Problems

☙ 學習目標 ❧

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to know that:

一、了解功率、平均功率與瞬時功率的定義。

To understand the definitions of power, average power, and instantaneous power.

二、能藉由功率來計算功或能量轉換。

To be able to calculate the work or energy transferred with the power.

❧ 例題講解 ☙

例題一

說明：能透過功率和時間，求得所需能量。

Students are able to determine the required energy using power and time.

Ming needs to walk 1.2 km to get to school from home. When he walks at a velocity of 1 m/s, his body burns 2000 Joules of energy per minute off him. If he walks at this constant velocity from home to school, approximately how much energy will he use??

(A) 2×10^3 Joules (B) 4×10^3 Joules (C) 2×10^4 Joules **(D) 4×10^4 Joules**

小明從家裡到學校需走 1.2 公里的路。當他走路的速率為 1 m/s 時，身體消耗的能量為每分鐘 2000 焦耳。若他以此等速率從家裡走到學校，則大約消耗多少能量？

(A) 2×10^3 焦耳 (B) 4×10^3 焦耳 (C) 2×10^4 焦耳 **(D) 4×10^4 焦耳**

（92 學測補考 30）

解題 Solution：

When a person walks at a constant speed of 1 m/s, a 1.2 km route takes that person 1200/1=1200 s =20 min to complete. With this condition in mind, the question says that Ming's body releases 2,000 Joules of energy per minute under this rate. Therefore, the energy consumed by Ming when he walks to school from home is around $2000\times20=40000=4\times10^4$ J.

當走路速率為 1 m/s 時，1.2 公里的路程需花費 1200/1=1200 s =20 min。

又身體每分鐘消耗 2000 焦耳的能量，因此可知小明從家裡走到學校，大約需消耗

$2000\times20=40000=4\times10^4$ J。

Teacher: Ming uses 2,000 Joules of energy per minute, which means his power output is 2,000 Joules per minute. We also know that he walks at a speed of 1 m/s. Then, how do we know the energy Ming consumes when he walks to school from home?

Student: Given the equation E=P × t where t refers to the time taken by Ming to walk to school from home. Therefore, solving for t and multiplying it by the energy Ming consumes per minute should give us the answer to this question.

Teacher: That's right! This question says that the distance from Ming's home to school is 1.2km. How long it takes him to get to school?

Student: 1200/1=1200 s =20 min, it takes Ming 20 minutes to get to school.

Teacher: That is correct! Now, we plug in these numbers and solve for energy.

Student: I know this one! $2000\times20=40000=4\times10^4$ J. When Ming walks to school from home, he consumes 4×10^4 Joules of energy.

Teacher: Great job! Remember, the SI unit of time for energy and time should be consistent in your calculation.

老師：小明每分鐘消耗 2000 焦耳的能量，表示功率是 2000 焦耳/分，也知道他的走路速率是 1 m/s ，那要怎麼知道他從家裡到學校總共花了多少能量呢？

學生：因為 E=P × t，所以要先算出他從家裡到學校總共花了多少時間，再乘上他每分鐘消耗的能量。

老師：沒錯！已知小明的家距離學校 1.2 公里，請問他要花多少時間呢？

學生：1200/1=1200 s =20 min，小明要花 20 分鐘。

老師：答對了！現在我們有時間，再乘上功率，就能求出能量了！

學生：我知道！$2000\times20=40000=4\times10^4$ J，小明從家裡走到學校要消耗 4×10^4 焦耳的能量。

老師：太棒啦！計算時要注意到功率及時間，在時間單位的一致性。

例題二

說明：理解功率、摩擦力與速率三者間的關係。

Students can understand how energy relates to friction and velocity.

A truck with a mass of 2000 kg climbs a road slope with an incline angle of 7° at a constant speed without wheel slippage. Assume that air resistance and mechanical energy loss due to components can be neglected. Given that the gravitational acceleration is 10 m/s^2 , sin 7°= 0.12, and the engine output power is fixed at 80000 W. What is the approximate speed of the truck in kilometers per hour?

(A) 60　(B) 80　(C) 90
(D) 100　**(E) 120**

一輛小貨車的質量為 2000 公斤，在車輪不打滑的情況下，以等速度爬一坡度為 7° 的公路斜坡，若空氣阻力與機件引起的力學能損失皆可忽略，取重力加速度為 10 m/s^2，sin 7°= 0.12，而引擎輸出功率固定為 80000 瓦，則小貨車的速率約為多少公里/小時？

(A) 60　(B) 80　(C) 90
(D) 100　**(E) 120**

（101 指考 9）

解題 Solution：

Solving this question needs to determine the output rate of the engine first to solve for the truck's final velocity. The work-energy theorem is needed as it states work done is equal to the change in kinetic energy. As a result, the free-body diagram is to be drawn for the analysis of all forces acting on the truck.

As shown below, when the truck is moving on a hill, there are three forces acting on the truck: friction acts upward f and the force of gravity acts downward mg along the hill. There is also normal force N acting perpendicular to the hill. In addition to the force of gravity, it can be divided into two components: A force acting perpendicular to the hill $mg\cos\theta$ and a force acting parallel to the hill $mg\cos\theta$. From the question, it is safe to say that the net force acting on the truck in the vertical direction is zero（N= $mg\cos\theta$）.

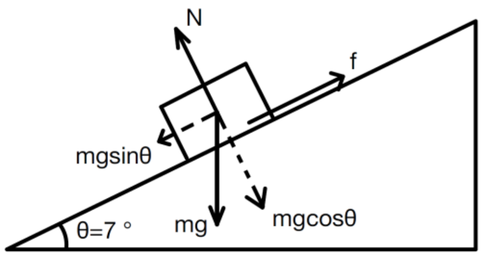
This indicates that the net force acting on the truck in the horizontal direction is $F = mgsin\theta = 2000 \times 10 \times 0.12 = 2400\ kg \cdot m/s^2$.

With the output rate of the truck's engine constantly at 80,000 watts, plugging in the numbers from above into the equation for power P=Fv gives the truck's velocity $v = \frac{P}{F} = \frac{80000}{2400}\ m/s$ =33.3 $/s\ = 120\ km/hr$.

此題要從引擎之輸出功率，推算貨車之末速。需要用到功能定理:合力作功=動能變化。因此，需先分析貨車所受的力圖。

當貨車於斜坡上行駛，貨車受到斜坡之向上摩擦力 f、向下之重力 mg，及正向力 N 的作用（如示意圖所示）。由於重力 mg 可劃分為垂直於斜坡的$mg\cos\theta$與平行於斜坡的$mg\cos\theta$兩力。又小貨車在垂直斜坡方向所受到的合力為零(N= $mg\cos\theta$)，因此可知小貨車在平行斜坡方向受到的合力：$F = mg\cos\theta = 2000 \times 10 \times 0.12 = 2400\ \text{kg} \cdot {}^{m}/_{s^2}$。

已知引擎輸出功率固定為 80000 瓦，因此由功率的公式 $P = Fv$ 可知小貨車的速率為 $v = \frac{P}{F} = \frac{80000}{2400}\ m/s$ =33.3 $m/s\ = 120\ km/hr$。



Teacher: When this truck is moving up a hill at an angle with a 7° incline, what types of forces will be acting on the truck?

Student: There are three forces acting on the truck: the force of gravity mg, normal force N, and friction from the plane f.

Teacher: Therefore, $F = mg\cos\theta = 2000 \times 10 \times 0.12 = 2400\ \text{kg} \cdot {}^{m}/_{s^2}$.

Teacher: Now, with this value and the truck's output rate of 80,000 watts, how do we solve for its velocity?

Student: Hm…no idea.

Teacher: Do you still remember the equation to represent the relationship between power, external force, and velocity?

Student: Oh, yes! $P = \vec{F} \cdot \vec{v} = Fv\cos\theta$. Power is the scalar product of external force and velocity.

Teacher: Excellent! Now you know how to calculate velocity from power and external force.

Student: Easy Peasy.

$v = \frac{P}{F} = \frac{80000}{2400}\ m/s = 33.3\ /s\ = 120\ km/hr$. The truck's velocity is 120 km/hr.

Teacher: There you go.

老師：當小貨車在爬坡度為 7° 的斜坡時，會受到哪些力的作用呢？

學生：小貨車會受到重力 mg、正向力 N 和地板給予的摩擦力 f 的作用。

老師：因此 $F = mg\cos\theta = 2000 \times 10 \times 0.12 = 2400\ \text{kg}\cdot{}^{m}/_{s^2}$。

老師：很好，現在我們有摩擦力，也知道小貨車的引擎功率是 80000 瓦，那要怎麼求小貨車的速率呢？

學生：嗯…我不知道。

老師：還記得功率和外力、速度的關係嗎？

學生：我想起來了！$P = \vec{F}\cdot\vec{v} = Fv\cos\theta$，功率是合外和速度的純量積。

老師：很棒！那麼就能知道如何用功率與外力來表達速率了。

學生：很簡單，$v = \frac{P}{F} = \frac{80000}{2400}\ m/s = 33.3\ m/s\ = 120\ km/hr$，所以小貨車的速率是 120 公里/小時。

老師：答對了！

2-3 力學能守恆：地表附近的重力位能
Conservation of Mechanical Energy: Gravitational Potential Energy Near the Surface of The Earth

■ 前言 Introduction

介紹物體在地表附近的重力位能及其數學式，比較位能變化與重力作功的關係。講解力學能守恆，並區分保守力與非保守力作功之不同，最後討論力學能守恆的應用—單擺。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
potential energy	位能	mechanical energy	力學能
gravitational potential energy	重力位能	conservation of mechanical energy	力學能守恆
elastic potential energy	彈性位能	conservative force	保守力
zero-potential surface	零位能面	non-conservative force	非保守力
air resistance	空氣阻力	Pendulum	擺錘
work-energy theorem	功能定理	vertical circular motion	鉛直圓周運動

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ _______ is independent of _______.

例句：The work done by a conservative force on an object **is independent of** the object's path, only depending on the initial and the final positions.

保守力對物體作功，與物體的移動路徑無關，只與起點與終點位置有關。

❷ When ___________, ____________.

例句：**When** gravity does positive work, the gravitational potential energy of an object decreases and kinetic energy increases due to the conservation of mechanical energy.

當重力作正功時，物體的重力位能減小，且動能增加，因為力學能守恆。

❸ __________ is subject to ____________.

例句：If a system **is** only **subject to** conservative forces, then the mechanical energy of the object is conserved.

若系統僅受保守力作用，則物體的力學能守恆。

■ 問題講解 Explanation of Problems

☙ 學習目標 ❧

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to know that:

一、理解位能、動能與力學能三者的關聯。

To understand the relationship between potential energy, kinetic energy, and mechanical energy.

二、了解保守力與非保守力的差別，並能判斷系統是否滿足力學能守恆。

To understand the difference between the conservative and non-conservative force, and be able to determine whether a system satisfies the conservation of mechanical energy.

൶ 例題講解 ൸

例題一

說明： 理解力學能的定義，並區分各情境所牽涉之能量轉換形式。

Students can understand the definition of mechanical energy and distinguish one setting from another where involves energy transformations.

Which of the following processes involve mechanical energy transformation? Check all that apply:

(A) The process of a meteorite falling onto the moon's surface.

(B) The periodic oscillation of a spring-mass system.

(C) The process of burning coal to generate heat.

(D) The process of solar power generation.

(E) The process of hydroelectric power generation.

下列哪些過程中有涉及到力學能的能量轉換？

(A) 隕石墜落到月球表面的過程。

(B) 彈簧與物體組合成的系統做週期性振盪運動。

(C) 燃燒煤產生熱的過程。

(D) 太陽能發電的過程。

(E) 水力發電的過程。

（110 學測 57）

解題 Solution：

(A) As a meteorite is falling to the surface of the moon, the gravitational potential energy between them decreases. This displacement is converted into meteorite’s kinetic energy. Therefore, this process must involve mechanical energy transformation.

(B) During the periodic oscillation of the spring-mass system, the elastic potential energy between them changes as the spring is being compressed and extended repeatedly. This periodic change in the energy leads to the changes in the kinetic energy. Therefore, this process must involve mechanical energy transformation.

(C) Burning coal to produce heat is a process of converting chemical energy to thermal energy. This process does not involve mechanical energy transformation.

(D) Using sunlight to generate solar energy is a process of converting solar energy to electrical energy. This process does not involve machinal energy transformation.

(E) Using moving water to generate hydro-energy is a process of decreasing the gravitational potential energy between Earth and water. This process converts that energy into the water's kinetic energy, which rotates the hydro turbines and moves the rotor to produce electricity. As a result, it must involve mechanical energy transformation as this energy consists of kinetic and potential energy.

(A) 隕石墜落到月球表面過程中，隕石與月球間的重力位能減少，轉換成隕石的動能，因此有涉及到力學能的轉換。

(B) 彈簧與物體作週期振盪過程中，彈簧與物體間的彈性位能不斷隨彈簧形變量改變而變化，此能量變化也造成動能不斷的變化，因此有涉及到力學能的轉換。

(C) 燃燒煤產生熱的過程，能量型式由化學能轉換成熱能，沒有牽涉到力學能轉換。

(D) 太陽能發電的過程，能量型式由光能轉換成電能，沒有牽涉到力學能轉換。

(E) 水力發電的過程中，水與地球間的重力位能減少，轉換成水的動能，以推動渦輪機帶動線圈，再將能量轉換成電能，因此有涉及力學能轉換，包含動能及位能。

Teacher: What is mechanical energy?

Student: Mechanical energy is the sum of kinetic energy and potential energy.

Teacher: That's right. From this concept, it is safe to say that a process is associated with mechanical energy transformation as long as there is a change in an object's kinetic or potential energy. Now, think about option A. When a meteorite is falling to the surface of the moon, is this process associated with mechanical energy transformation?

Student: A falling meteorite indicates a decrease in its potential energy, which is transformed into kinetic energy. So option A is associated with mechanical energy transformation.

Teacher: That is correct. How about option B? The motion of the spring-mass system oscillating.

Student: The motion of the spring-mass system oscillating is related to elastic potential and kinetic energies. As a result, option B is associated with mechanical energy transformation too.

Teacher: Very good. What is this with option C, the coal burning process that produces heat?

Student: The energy from burning coal is chemical energy. The heat released is thermal. This process of energy transformation is from chemical energy to thermal one. It is not associated with mechanical energy transformation.

Teacher: That's right. How about option D, the process of generating solar power?

Student: Solar panels receive sunlight and convert it into electricity. This process of energy transformation is from light energy into electricity. It is not associated with mechanical energy transformation.

Teacher: That is correct. Lastly, what is this with option E, the process of generating hydropower?

Student: Uhm…. I am not sure about it.

Teacher: Well, for example, hydropower can be generated when water flows from high ground to low ground. The force of moving water rotates the hydro generators for electricity.

Student: Oh I see! When water flows from high to low ground, the decreased potential energy of the running water is converted into kinetic energy. The hydro generators transform it into electricity. As a result, the process of generating hydropower must be associated with mechanical power transformation.

Teacher: There you go!

老師： 還記得甚麼是力學能嗎？

學生： 力學能就是動能與位能的總和。

老師： 沒錯，所以在能量轉換的過程中，和動能、位能改變有關係的，就表示有涉及到力學能的能量轉換。那麼，想想看(A)選項，隕石墜落到月球表面的過程，和力學能有沒有關係呢？

學生： 當隕石墜落時，隕石的位能減少，轉變成動能，所以和力學能轉換有關。

老師： 對的，那麼(B)選項，彈簧與物體組合成的系統做週期性振盪運動呢？

學生： 彈簧與物體組合成的系統做週期性振盪運動，和彈性位能、動能的變化有關，所以也涉及到力學能的能量轉換。

老師： 很好，再來(C)選項，燃燒煤產生熱的過程呢？

學生： 煤燃燒產生化學能，釋放的熱是熱能，是從化學能轉變成熱能，與力學能無關。

老師： 很棒，那麼(D)選項，太陽能發電的過程呢？

學生： 太陽能板接收太陽光，再轉變成電力，是由光能轉換成電能，也與力學能無關。

老師： 沒錯，最後的(E)選項，水力發電的過程呢？

學生：　嗯…我不太清楚水力發電的過程。
老師：　舉例來說，像是水從高處到低處流，水流推動機器旋轉，帶動發電機發電，就是水力發電的一種喔！
學生：　喔～我了解了。當水從高處到低處時，水所減少的位能逐漸轉變為動能，再帶動發電機發電，產生電能，所以和力學能的轉換有關！
老師：　答對了！

例題二

說明：在鉛直圓周運動中，透過力學能守恆，探討物體所受的速度及力。
Students can use their understanding of conservation of mechanical energy to explore the velocity and forces on an object experiencing circular motion in a vertical plane.

A roller coaster is an amusement ride commonly found in theme parks. Its track is usually designed with loops as shown below. Assuming the loops are made a perfect circle, and the track is situated on a vertical plane, the roller coaster cart is rotating around in circular motion without power drive. The track provides the centripetal force as the roller coaster cart moves through the loop. When the friction resistance can be negligible and the acceleration due to gravity is g, what is the magnitude of the acceleration when the roller coaster cart reaches the lowest point of the loop?

雲霄飛車是一種常見於主題樂園中的遊樂設施，其軌道通常有如圖所示的迴圈。若考慮正圓的迴圈軌道，且軌道可視為在一鉛直面上，雲霄飛車的車廂在沒有動力驅動之下，沿著軌道內側繞行，且軌道只能提供向心力，摩擦阻力可忽略，重力加速度為 g，則當車廂可沿整個圓圈軌道繞行時，車廂在軌道最低點的加速度量值至少為何？

(A) 2 g　　(B) 3 g　　(C) 4 g　　**(D) 5 g**　　(E) 6 g

（107 指考 17）

解題 Solution：

Let the velocity of the roller coaster carts at the highest point be v1, the radius of the circular track be R. When the cart reaches the highest point, the net force acting on the cart = gravity (downward) + normal force (downward), and the net force applied = normal force (downward).

Therefore, the formal force must be $F_c = m\frac{{v_1}^2}{R} = N + mg \geq mg$, so $v_1 \geq \sqrt{gR}$.

During the run, all the external forces applied are conservative, and this condition satisfies the principle of conservation of energy. Let the velocity of the roller coaster cart at the lowest point be v_2. $\frac{1}{2}m{v_1}^2 + mg(2R) = \frac{1}{2}m{v_2}^2$, so $v_2 \geq \sqrt{5gR}$. When the roller coaster cart accelerates to the lowest point, the cart's acceleration value is $a_c = \frac{{v_2}^2}{R} \geq 5g$.

令車廂在軌道最高點時速率為v_1，軌道半徑為R。

當車廂在最頂端時，所受合力=重力(向下)+正向力(向下)，而其合力=向心力(向下)，所以向心力應為： $F_c = m\frac{{v_1}^2}{R} = N + mg \geq mg$，故$v_1 \geq \sqrt{gR}$。

又因過程中所受外力皆為保守力，故滿足力學能守恆。令在軌道最低點時速率為v_2，$\frac{1}{2}m{v_1}^2 + mg(2R) = \frac{1}{2}m{v_2}^2$，因此 $v_2 \geq \sqrt{5gR}$，故車廂在軌道最低點的加速度量值為 $a_c = \frac{{v_2}^2}{R} \geq 5g$。

Teacher: When the roller coaster cart travels to the highest point, what types of forces will act on the cart?

Student: There will be normal force from the track acting on it. The gravitational force should also be at play. Both are in the downward direction.

Teacher: That is right. At this time, the net force acting on the roller coaster cart equals centripetal force. Do you still remember how to represent it mathematically?

Student: Yes, I do! Let the velocity of the roller coaster at the highest point be v_1, then the centripetal force at this point can be written $F_c = m\frac{{v_1}^2}{R}$. Therefore, $F_c = m\frac{{v_1}^2}{R} = N + mg$.

Teacher: That's great. Since all external forces acting on it are conservative forces, this condition satisfies the principle of conservation of energy. What is mechanical energy?

Student: Mechanical energy is the sum of kinetic energy and potential energy.

Teacher: Correct. Let's think about this first: when the mass m, the track's radius R, and gravity mg remain constant, what is the magnitude of the normal force N that results in the minimum v_1?

Student: I know this one! When the normal force N is zero, v_1 must be the minimum. So, plugging in these gives$v_1 \geq \sqrt{gR}$.

Teacher: Very good. How about the value of $v_2$when the roller coaster cart travels to the lowest point? You can use the principle of conservation of mechanical energy and write down the equation. The lowest point can be seen as the zero point.

Student: When the roller coaster cart travels to the highest point, its mechanical energy is $\frac{1}{2}mv_1{}^2 + mg(2R)$; when the cart travels to the lowest point, its mechanical energy should be $\frac{1}{2}mv_2{}^2$. According to the principle of mechanical energy conservation, the lowest point of the cart's mechanical energy equals to its highest point $\frac{1}{2}mv_1{}^2 + mg(2R) = \frac{1}{2}mv_2{}^2$. It is already known that the minimum value ofv_1 is$\sqrt{gR}$, By rearranging the equation, we can obtain the minimum value of v_2 is$\sqrt{5gR}$.

Teacher: That's right! Finally, the question asks for the magnitude of the acceleration of the car at the lowest point of the track when it can travel around the entire circular track. Do you still remember the formula for centripetal acceleration?

Student: I know! The centripetal acceleration of the car at the lowest point is $a_c = \frac{v_2{}^2}{R}$, Since the minimum value of v_2 $\sqrt{5gR}$. The acceleration of the cart at the lowest point is 5g (upward) gives $a_c = \frac{v_2{}^2}{R} \geq 5g$.

Teacher: Great job!

老師： 當車廂在軌道最高點時，會受到哪些力的作用呢？
學生： 會受到軌道給予的正向力，還有重力的作用，兩者的方向都向下。
老師： 沒錯，此時車廂所受的合力等於向心力，還記得向心力要怎麼表示嗎？

學生： 記得，如果令車廂在最高點的速度為v_1，則在最高點的向心力$F_c = m\frac{{v_1}^2}{R}$，因此可以得到$F_c = m\frac{{v_1}^2}{R} = N + mg$。

老師： 很棒。因過程中所受外力皆為保守力，故滿足力學能守恆。力學能是甚麼呢？

學生： 力學能=動能+位能。

老師： 很好，先想想，在質量 m、軌道半徑R、重力mg固定不變的情況下，當正向力 N 為多少時，v_1會有最小值呢？

學生： 我知道！當正向力 N 為 0 的時候，v_1會有最小值，所以移項整理可以得到$v_1 \geq \sqrt{gR}$。

老師： 很好！那當車廂在最低點的時候，它的速率v_2會是多少呢？試試看，我們可以運用力學能守恆，並把最低點當作零點來列式。

學生： 當車廂在最高點時，力學能為$\frac{1}{2}m{v_1}^2 + mg(2R)$；在最低點時，力學能為$\frac{1}{2}m{v_2}^2$。根據力學能守恆，車廂在高點和最低點的力學能相等：$\frac{1}{2}m{v_1}^2 + mg(2R) = \frac{1}{2}m{v_2}^2$。已經知道$v_1$最小值是$\sqrt{gR}$，移項整理可以得到$v_2$最小值為$\sqrt{5gR}$。

老師： 沒錯！最後，題目問當車廂可沿整個圓圈軌道繞行時，車廂在軌道最低點的加速度量值，還記得向心加速度的公式為何嗎？

學生： 我記得！車廂在最低點的向心加速度$a_c = \frac{{v_2}^2}{R}$，已經知道 v_2 的最小值是$\sqrt{5gR}$，因此$a_c = \frac{{v_2}^2}{R} \geq 5g$，車廂在最低點的加速度應至少為 5g(向上)。

老師： 太棒啦！

2-4 力學能守恆：彈力位能
Conservation of Mechanical Energy: Elastic Potential Energy

■ 前言 Introduction

說明物體所受彈力與彈簧伸長量 x 成正比，因此，彈力位能與彈簧伸長量的平方成正比，且彈力為保守力。若物體只受保守力作用，則符合力學能守恆。若物體只受彈力作用，則物體會作簡諧運動，我們將透過力學能守恆探討簡諧運動。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
spring	彈簧	compress	壓縮
equilibrium point	平衡點	elastic force	彈性力
extension	伸長量	spring constant	彈性常數
elastic potential energy	彈性位能	shoot	彈射
compression	壓縮量	simple harmonic motion	簡諧運動
stretch	伸長		

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ __________ is proportional to __________.

例句：When an object is subjected to elastic force, its elastic potential energy **is proportional to** the square of the spring's extension amount.

當物體受彈性力作用時，其彈力位能與彈簧伸長量的平方成正比。

❷ __________ is stretched/compressed by __________.

例句：When a spring with a spring constant *k* **is stretched or compressed by** distance *x*, its elastic potential energy is $U = \frac{1}{2}kx^2$.

當彈性常數為 k 的彈簧被伸長或壓縮 x 時,其彈力位能為 $U = \frac{1}{2}kx^2$。

❸ ________ the only work done on ________ is by ________.

例句：When an object is connected to a spring and undergoes simple harmonic motion on a frictionless horizontal surface. **The only work done on** the object **is by** the elastic force of the spring, thus satisfies the conservation of mechanical energy.

物體連接彈簧，在光滑水平面上做簡諧運動時，因僅受彈力作功，故滿足力學能守恆。

■ 問題講解 Explanation of Problems

學習目標

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to:

一、理解彈性位能的定義。

To understand the definition of elastic potential energy.

二、了解簡諧運動滿足力學能守恆。

To understand that mechanical energy is conserved in simple harmonic motion.

例題講解

例題一

說明：理解動量守恆與力學能守恆定律。

To understand the principles of conservation of momentum and mechanical energy.

There is a spring-gun at rest on a smooth and horizontal ground shooting a steel ball which has 0.20 kg mass. The speed of the ball is 4.0 m/s relative to the ground horizontally. It is known that the energy is completely provided by the compressed ideal spring, and the force constant of the spring is $2.4 \text{x} 10^3$ N/m. The mass of the spring-gun without the steel ball was initially 1.0 kg. The entity of the spring-gun can slide freely on the ground. If all friction generated in the process is ignored, what was the length of the compressed spring at the beginning of the spring-gun shooting the steel ball?

(A) 3.6 cm　　**(B) 4.0 cm**　　(C) 4.6 cm　　(D) 5.0 cm　　(E) 5.6 cm

一靜止於光滑水平地面的彈簧槍，將一顆質量為 0.20 kg 的鋼珠以相對於地面為 4.0 m/s 的水平速度射出，已知這一發射過程的能量完全由壓縮的理想彈簧提供，彈簧的力常數為 2.4×10^3 N/m ，裝鋼珠前彈簧槍的質量為 1.0 kg。彈簧槍槍身可於地面上自由滑動，若忽略過程中的所有摩擦力，則彈簧槍在一開始要發射鋼珠時，彈簧被壓縮的長度為下列何者？

(A) 3.6 cm　　**(B) 4.0 cm**　　(C) 4.6 cm　　(D) 5.0 cm　　(E) 5.6 cm

（106 年指考 5）

解題 Solution：

Because before and after releasing the steel ball, the system of the spring-gun and the ball only receives elastic force (internal force), the momentum conserves.

$p_{gun} = p_{ball}$ => 1.0 × v_{gun} = 0.2 ×4.0. Therefore, v_{gun} = 0.8 m/s.

And before and after the shooting, it only receives elastic force (conservative force) horizontally, so the mechanical energy conserves (the loss of the elastic potential energy = the sum of the momentum of the spring-gun and steel ball):

$$\left|\Delta U_{彈簧}\right| = \left|\Delta K_{槍}\right| + \left|\Delta K_{珠}\right| \Rightarrow \frac{1}{2}\times(2.4\times10^3)\times x^2 = \frac{1}{2}\times1.0\times0.8^2 + \frac{1}{2}\times0.20\times4.0^2 \therefore$$

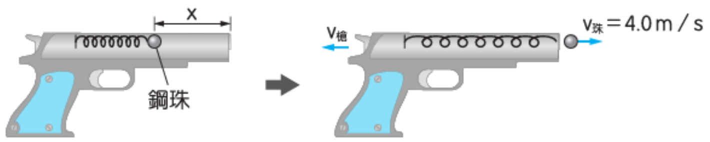
$$x = 4.0\times10^{-2}\ (m) = 4.0\ (cm)$$

由於彈簧槍與鋼珠系統釋放前後僅受彈力（內力）作用，故動量守恆$p_{槍} = p_{珠} \Longrightarrow$

$1.0\times v_{槍} = 0.2\times4.0$，故$v_{槍} = 0.8\ m/s$。而在彈射前後，水平方向僅受彈力（保守力）作功，故力學能守恆(損失之彈性位能=彈簧槍與鋼珠之動能和)：

$$\left|\Delta U_{彈簧}\right| = \left|\Delta K_{槍}\right| + \left|\Delta K_{珠}\right| \Rightarrow \frac{1}{2}\times(2.4\times10^3)\times x^2 = \frac{1}{2}\times1.0\times0.8^2 + \frac{1}{2}\times0.20\times4.0^2 \therefore$$

$$x = 4.0\times10^{-2}\ (m) = 4.0\ (cm)$$



Teacher:	If we see the spring-gun and the steel ball as a system, is the system affected by any external force during the process of the spring-gun at rest shooting the steel ball?
Student:	The system is not affected by any external force. The steel ball was shot because of the internal force, which is elastic force.
Teacher:	Good. Does the momentum of the system change after the spring-gun shoots the steel ball?
Student:	No. The system is not affected by any external force, so the momentum of the system does not change.
Teacher:	Yes. Now we know that the masses of the ball and the gun are 0.2 kg and 1 kg, respectively, and that the shooting speed of the ball is 4.0 m/s, then what would the speed of the spring-gun be after shooting the ball?

Student: The spring-gun was still before shooting the steel ball, so the momentum was 0. But after shooting, the momentum of the system becomes $1.0 \times v_{gun} = 0.2 \times 4.0$. Since the momentum remains still after shooting, we can infer that the speed of the spring-gun is $v_{gun} = 0.8$ m/s, backwards.

Teacher: Great! Do you remember what condition is required for conserving mechanical energy?

Student: Sure. If the system only receives conservative force, it makes mechanical energy conserve.

Teacher: Exactly. Does the system meet the condition of conserving mechanical energy?

Student: Yes, it does. During the shooting process, it is only acted upon by elastic force horizontally. Elastic force is conservative force, so the system meets the condition of conserving mechanical energy.

Teacher: Good. The conservation of mechanical energy of the system means that the elastic potential energy the spring loses equals the sum of the kinetic energy of the spring-gun and the steel ball. How would you write the equation?

Student: I know. Assume that the length of the compressed spring is x, the elastic potential energy is $U = ½ kx^2$. Since the elastic constant of the spring is 2.4×10^3 N/m, we get $½ \times (2.4 \times 10^3) \times x^2 = ½ \times 1.0 \times 0.8^2 + ½ \times 0.20 \times 4.0^2$.

Teacher: That's right. After organizing the equation, we know how much x is.

Student: It's easy. $x = 4.0 \times 10^{-2}$ (m) = 4.0 (cm). The spring is compressed by 4 cm.

Teacher: You're so good.

老師： 若將彈簧槍和鋼珠視為一個系統，從彈簧槍靜止到彈簧槍射出鋼珠的過程，系統有沒有受到外力的變化呢？

學生： 系統沒有受到外力的作用，鋼珠射出是受到內力—彈力的作用。

老師： 很好，那麼在彈簧槍射出鋼珠前後，系統的動量會改變嗎？

學生： 不會，因為系統不受外力作用，所以系統動量不變。

老師： 沒錯。現在已經知道鋼珠和槍的質量分別為 0.2 kg 和 1 kg，以及鋼珠射出的速度為 4.0 m／s ，那麼射出鋼珠後，彈簧槍的速度是多少呢？

學生： 彈簧槍射出鋼珠前是靜止的，因此動量是 0；射出後的系統動量則是$1.0\times v_{槍}+0.2\times 4.0$。因射出前後的系統動量相等，所以可以知道槍的速度 $v_{槍}=0.8\,m/s$，方向向後。

老師： 很棒！那還記得力學能守恆的條件嗎？

學生： 記得，系統僅受保守力作功，即滿足力學能守恆。

老師： 沒錯，那請問這個系統滿足力學能守恆嗎？

學生： 對，因為在鋼珠彈射前後，水平方向僅受彈力作功，彈力屬於保守力，因此系統滿足力學能守恆。

老師： 很好，系統的力學能守恆，也就表示彈簧損失的彈性位能等於彈簧槍與鋼珠之動能和，那要怎麼表示呢？。

學生： 我知道！假設彈簧被壓縮的長度為 x，那麼彈性位能$U=\frac{1}{2}kx^2$，且彈簧的彈性常數為 2.4×10^3 N／m，因此可得

$$\frac{1}{2}\times(2.4\times10^3)\times x^2=\frac{1}{2}\times1.0\times0.8^2+\frac{1}{2}\times0.20\times4.0^2$$ 。

老師： 沒錯，整理式子後就能知道 x 是多少了！

學生： 很簡單！移項整理可得$x=4.0\times10^{-2}\,(m)=4.0\,(cm)$，所以彈簧被壓縮 4 公分。

老師： 太棒了！

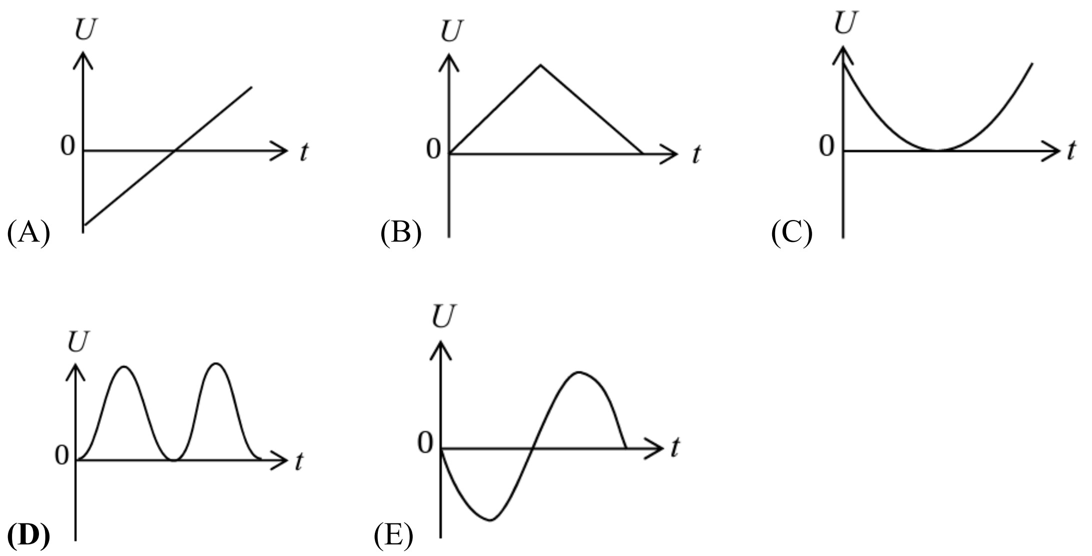
例題二

說明：分析簡諧運動中的彈性位能變化。

To analyze the changes of elastic potential energy in simple harmonic motion.

An ideal spring with negligible mass has one end fixed and the other end attached to a point mass. The system undergoes one-dimensional simple harmonic motion on a smooth horizontal surface. Which of the following graphs most likely represents the change in elastic potential energy (U) over time (t) during one period?

有一質量可忽略的理想彈簧一端固定，另一端繫有一質點，在光滑水平面上作一維簡諧運動，則在一個週期內，彈性位能 U 隨時間 t 的變化圖最可能為下列何者？



（103 年指考 5）

解題 Solution：

Based on the formula for elastic potential energy, $U = \frac{1}{2} kx^2$. When a point mass is at either end of its motion, the system has the maximum elastic potential energy. When the point mass is at the equilibrium position, the elastic potential energy is zero. One period of motion consists of the point mass moving from the equilibrium position to one end, returning through the equilibrium position to the other end, and finally returning to the equilibrium position. Option (D) is the graph that represents the relationship between the elastic potential energy of the system and time.

根據彈性位能 $U=\frac{1}{2}kx^2$。質點在兩端點時，系統擁有最大彈性位能；在平衡點時，系統的彈性位能則為零。若質點由平衡點出發，先到達一邊的端點，再回來經過平衡點，到達另外一邊的端點，最後回到平衡點，此為一個週期內的運動。則(D)是系統的彈性位能與時間之關係圖。

Teacher: How should we express the elastic potential energy of a point mass when it is under elastic force?

Student: Elastic potential energy, which is $U=\frac{1}{2}kx^2$, is proportional to squared x, which is the squared length of stretched spring.

Teacher: Yes. At which point the point mass is, the elastic potential energy would be the smallest?

Student: When the point mass is at the balanced point. Because the stretched length of the spring is 0, the elastic potential energy is the smallest, which is 0.

Teacher: Good. Then, at which point the point mass is, the elastic potential energy would be the largest?

Student: It is easy. When the point mass is at the end of the compressed spring, as well as the stretched spring. Because it has the largest stretched length *x*, the elastic potential energy would be the largest.

Teacher: Great. If the simple harmonic motion starts from the balanced point, what points would the point mass pass in order in a period?

Student: Starting from the balanced point, the point mass would reach one end of the spring and go back to the balanced point. Then, it reaches the other end and finally goes back to the balanced point.

Teacher: So, which chart represents the relationship between the elastic potential energy and time?

Student: The elastic potential energy goes to the maximum from 0 and goes back to 0 from the maximum. Later, it goes to the maximum and finally back to 0. Therefore, the answer is (D).

Teacher: Correct!

老師： 當一質點受到彈力作用時，他的彈力位能該如何表示呢？

學生： 彈性位能$U = \frac{1}{2}kx^2$，和彈簧伸長量x的平方成正比。

老師： 沒錯，那麼當質點在哪個位置時，彈性位能會最小呢？

學生： 當質點在平衡點的時候，因為此時伸長量x是 0，彈性位能會最小，等於 0。

老師： 很好。那當質點在哪個位置時，彈性位能會最大呢？

學生： 很簡單！當質點在壓縮跟伸長的兩端點的時候，因為此時伸長量 x 最大，彈性位能會最大。

老師： 很棒！如果從平衡點出發作簡諧運動，在一周期內依序會經過的位置是哪些呢？

學生： 質點從平衡點出發，會先到達其中一端的端點，再回到平衡點，而後到達另一邊的端點，最後再回到平衡點。

老師： 所以，他的彈性位能與時間之關係圖，應該是哪個呢？

學生： 彈性位能會從 0 到達最大值，再由最大值回到 0 之後，又回到最大值，最後回到 0，所以是(D)。

老師： 答對了！

2-5 力學能守恆：重力位能的一般形式
Conservation of Mechanical Energy: General Form of Gravitational Potential Energy

■ 前言 Introduction

介紹物體在遠離地球表面時的重力位能變化（稱為一般式），並利用重力位能一般式與力學能守恆，計算物體能脫離地球束縛的最低速度（稱為脫離速度）。最後透過人造衛星的例子，根據力學能，探討其動能與位能的變化。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
gravitational potential energy	重力位能	satellite	人造衛星
infinity	無窮遠	gravitational constant	重力常數
gravity	地球引力	mechanical energy	力學能
escape velocity	脫離速度	kinetic energy	動能

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ **__________ is defined as __________ at _______________.**

例句：The gravitational potential energy **is defined as** zero **at** infinity (where the gravitational potential energy is at its maximum), and thus the gravitational potential energy in the equation $U = -\frac{GMm}{r}$ is always negative.

由於將無窮遠處(重力位能最大)的重力位能定義為零，因此 $U = -\frac{GMm}{r}$ 式中的重力位能恆為負值。

❷ **___________ regardless of _______________.**

例句：For the Earth, the escape velocity for all the objects, **regardless of** their mass, is always approximately 11.2 kilometers per second.

對地球而言，所有物體不論質量大小，脫離速度均約為 11.2 公里/秒。

❸ **_________ is unable to ____________.**

例句：When the mechanical energy of an object is less than zero, it **is unable to** escape the gravitational force of the Earth.

當物體的力學能小於零時，無法脫離地球引力的束縛。

■ 問題講解 Explanation of Problems

學習目標

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to:

一、瞭解重力位能的定義。

To understand the definition of gravitational potential energy.

二、理解脫離速度的定義，以及人造衛星的重力位能、動能與力學能的關係式。

To understand the definition of escape velocity and the relationship between the gravitational potential energy, kinetic energy, and mechanical energy of satellites.

例題講解

例題一

說明：評量學生對於雙星系統中，各項物理量的理解。

To assess students' understanding of each physical quantity in the binary system.

There is a system consisting of planet A and B, which can be seen as two point masses. The masses are m and M, respectively (M > m). With gravity of the two stars, they do circular motion around the center of mass O with radius r and R. If the center of mass does not move, the two stars are infinitely far away from each other, and the total gravity potential energy is zero, which of the following statements are true? (G is gravitation constant.)

(A) The sum of the momentum of the two stars is zero.

(B) The kinetic energy of the two stars is equal.

(C) The cycle of the two stars circling around the center of mass O is equal.

(D) The total gravity potential energy of the two stars is -GmM (1/r +1/R).

(E) The relationship between the mass and radius of the two stars is mR = Mr.

一系統由可視為質點的甲、乙兩星球組成，其質量分別為 m 與 M (M > m)，在彼此間的重力作用下，分別以半徑 r 與 R 繞系統的質心 O 做圓周運動。若質心 O 靜止不動，兩星球相距無窮遠時，系統的總重力位能為零，則下列敘述，哪些正確？（G 為重力常數，亦即萬有引力常數）

(A) 兩星球的動量和為零。

(B) 兩星球的動能相等。

(C) 兩星球繞 O 運動的週期相等。

(D) 兩星球的總重力位能為 -GmM (1/r +1/R) 。

(E) 兩星球的質量與繞行半徑有 m R = M r 的關係。

（94 年指考 16）

解題 Solution：

In a binary system, the system is not affected by any external force, so the total momentum is conserved.

(A) Since the center of mass is initially rested, the total momentum of the system is zero. The momentum of the two stars is equal but in opposite directions.

(B) When the two stars have the same momentum, the kinetic energy is inversely proportional to the mass ($K = ½ mv^2 = p^2/2m$).

(C) The center of mass is fixed, so the relative position of the two stars does not change, and the cycle is the same.

(D) Potential energy can be calculated with the distance (R+r) directly.

(E) Balanced moment of force: mr = MR.

在雙星系統中，系統不受到任何外力，所以總動量守恆。

(A) 因為原本質心為靜止，故系統總動量為零，兩星球之動量大小相等，方向相反。

(B) 兩星球之動量相等時，動能與質量成反比($K = ½ mv^2 = p^2/2m$)。

(C) 因質心不動，故兩者相對位置維持不變，所以週期相同。

(D) 位能可直接以距離(R+r)計算。

(E) 力矩平衡：mr = MR。

Teacher: Do you remember what a binary system is?

Student: Yes. It is a system consisting of two stars which circle around a shared center of mass on an orbit.

Teacher: Great. What is the feature of the momentum of the two stars?

Student: They have the same momentum but in opposite directions.

Teacher: That's right. The binary system does not receive any external force, only the internal force of each other, so the momentum is conserved. Let's take a look at option (A). In a binary system, does the sum of the momentum of the two stars, which is the total momentum of the system, equal zero?

Student: Yes, it does. The system is not affected by any external force, so the total momentum of the system equals zero. The center of mass remains unmoved.

Teacher: Great. What about option (B)? Does the kinetic energy of the two stars equal? Think about it the relationship between kinetic energy $K = 1/2mv^2$ and momentum $P = mv$ when the momentum of the two is the same.

Student: No, it doesn't. Because $K = p^2/2m$, and when the momentum is the same, the kinetic energy is inversely proportional to the mass. The mass of the two is not equivalent, so the kinetic energy is not the same.

Teacher: Good! Now option (C). Does the cycle of the two stars circle around the center of mass O the same?

Student: Yes. The system is not affected by external force, and the center of mass of the system is fixed and on the connection of the two stars. The relative position of the two stars remains unchanged, so the circling cycle is the same.

Teacher: You're right. Next, option (D). Is the total gravity potential energy of the two stars $-GmM\ (1/r + 1/R)$? Remember, gravity potential energy is related to the distance between the two stars!

Student: No. The distance between the center of the mass of the two stars is (R+r), the total gravity potential energy should be $-GmM\ (1/r + 1/R)$.

Teacher: That's right. Finally, option (E). The relationship between the mass of the two stars and the circulation radius is $mR = Mr$, is that correct?

Student: No, it is wrong. The relationship should be $mr = MR$.

Teacher: Bingo!

老師： 還記得什麼是雙星系統嗎？

學生： 記得，是由兩顆恆星組成的系統，它們圍繞著共同的質心，在軌道上互繞。

老師： 很好，那這兩顆恆星的動量又有什麼特性呢？

學生： 兩者的動量大小相等，方向相反。

老師： 沒錯。因為雙星系統沒有受到任何外力，只有彼此間的內力作用，所以動量守恆。那麼(A)選項，雙星系統中，兩星球的動量和，也是系統的總動量，會等於 0 嗎？

學生： 會，因為系統不受外力作用，所以系統的總動量等於 0，他們的質心可維持靜止。

老師： 很棒。那麼(B)選項，兩者的動能會相等嗎？可以想想看，在動量相同的情況下，動能 $K = 1/2mv^2$ 和動量 $P = mv$ 間的關係。

學生： 不相等，因為 $K = p^2/2m$，在動量相等的情況下，動能和質量成反比。兩者的質量不同，所以動能不相等。

老師： 很好～那麼(C)選項，兩星球繞質心 O 運動的週期相等嗎？

學生： 相等，因為系統不受外力，系統質心不動且維持在兩星球連線上，因此兩星球的相對位置不變，故繞行週期相等。

老師： 是的！那麼(D)選項，兩星球的總重力位能是 $-GmM\ (1/r + 1/R)$ 嗎？要記得，重力位能和兩顆星球間的距離有關喔。

學生： 不對，因為兩顆星球質心間的距離是 $(R+r)$，所以它們的總重力位能應該是 $-GmM\ (1/r + 1/R)$。

老師： 沒錯！那麼最後一個，(E)選項，兩星球的質量與繞行半徑有 $mR = Mr$ 的關係，對嗎？

學生： 不對，兩星球的質量與繞行半徑應滿足 $mr = MR$ 的關係。

老師： 答對啦！

例題二

說明：理解物體動能與位能間的關聯。

To understand the relationship between kinetic energy and potential energy of an object.

The gravitational acceleration of a star with radius R and without atmosphere is g. If an object without propulsion is shot upwards vertically at initial speed $v=\sqrt{gR}$, what would the distance between the highest point and the surface of the star?

(A) $\frac{R}{4}$ (B) $\frac{R}{2}$ **(C) R** (D) $\frac{3R}{2}$ (E) 2R

一個半徑為 R、沒有大氣的星球，在其表面處的重力加速度為 g。若由該星球表面以 $v=\sqrt{gR}$ 的初速，垂直向上發射一個沒有推進力的物體，則此物體上升的最高點與星球表面的距離，為下列何者？

(A) $\frac{R}{4}$ (B) $\frac{R}{2}$ **(C) R** (D) $\frac{3R}{2}$ (E) 2R

（100 年指考 8）

解題 Solution：

During the process of the object rising, the loss of kinetic energy = the increase in potential energy. Name the mass of the object m, the mass of the star M, and the distance between the highest point that the object rises and the center of the star x, we get that $\frac{1}{2}m(\sqrt{gR})^2=(-\frac{GMm}{R+h})-(-\frac{GMm}{R})$. Apply this to the surface $g=\frac{GM}{R^2}$, $\frac{1}{2}m\frac{GM}{R}=\left(-\frac{GMm}{R+h}\right)-(-\frac{GMm}{R})$, and we get $-\frac{1}{2}\frac{GMm}{R}=(-\frac{GMm}{R+h})$.

So the highest point that the object rises is h = R away from the surface.

在物體上升的過程中，其減少的動能=增加的位能。

令物體質量為 m，星球質量為 M，物體上升最高點和球心的距離為 x，即可列式 $\frac{1}{2}m(\sqrt{gR})^2=(-\frac{GMm}{R+h})-(-\frac{GMm}{R})$，代入地表處的 $g=\frac{GM}{R^2}$，$\frac{1}{2}m\frac{GM}{R}=\left(-\frac{GMm}{R+h}\right)-(-\frac{GMm}{R})$，

得 $-\frac{1}{2}\frac{GMm}{R}=(-\frac{GMm}{R+h})$，因此上升最高點距離地表 h = R。

Teacher: How does the object receive force?

Student: The object receives gravity downwards and air resistance upwards.

Teacher: Well, the question says there is no atmosphere on this star.

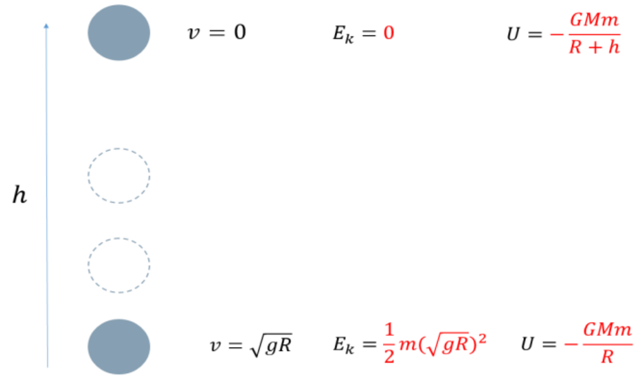
Student: Oh, then it only receives gravity downwards.

Teacher: Exactly. If we see this from the perspective of kinetic energy, is gravity conservative force or non-conservative force in motion of the object?

Student: It is conservative force.

Teacher: Good. If the object only receives conservative force, and the mechanical energy conserves during the motion, the loss of kinetic energy = the increase in potential energy. Please come to the stage and mark the energy of the object in different positions. Let's assume the mass of the planet is M.

Student:



Teacher: Amazing! The loss of kinetic energy is converted to potential energy. We can make the equation: $\frac{1}{2}m(\sqrt{gR})^2 = (-\frac{GMm}{R+h}) - (-\frac{GMm}{R})$.

Student: But what about gravitational acceleration g?

Teacher: Can anybody tell me how to express the gravitational acceleration g on this planet?

Student: Is it 9.8 m/s^2?

Teacher: That's the gravitational acceleration on the surface of Earth. The question says that the radius of this star is R, and we assume that the mass of the star is M, then how do you represent the gravitational acceleration g?

Student: I know. It is $g = GM/R^2$.

Teacher: Yes! That's correct. We can apply $g = GM/R^2$ to the equation. $\frac{1}{2}m\frac{GM}{R} = \left(-\frac{GMm}{R+h}\right) - (-\frac{GMm}{R})$.

Student:　Oh, I get it. The answer I get is that the distance between the highest point the object rises and the surface of the star h is R.

老師：　物體的受力情形是如何呢？

學生：　物體受到向下的重力，還有向上的空氣阻力。

老師：　哦？記得題目告訴我們這個星球上沒有大氣。

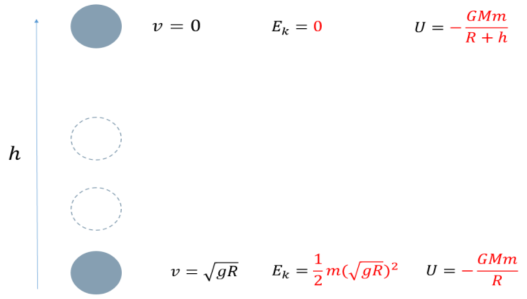
學生：　啊！只受向下的重力。

老師：　沒錯！如果我們以能量的觀點來討論，重力在物體運動中是屬於保守力還是非保守力呢？

學生：　是保守力。

老師：　很好~如果物體只受保守力作功，運動過程中力學能守恆，其減少的動能=增加的位能。請同學上台標出物體在不同位置的能量。我們假設星球質量為 M。

學生：



老師：　太棒了！減少的動能轉換成位能，我們可以列出式子$\frac{1}{2}m(\sqrt{gR})^2 = (-\frac{GMm}{R+h}) - (-\frac{GMm}{R})$

學生：　可是重力加速度 g 怎麼辦啊？

老師：　有人能告訴我在這個星球的重力加速度 g 表示方法嗎？

學生：　是$9.8m/s^2$嗎？

老師：　那是地球地表附近的加速度喔。題目告訴我們這個星球的半徑是 R ，我們又假設星球質量為 M ，那麼重力加速度 g 可以怎麼表示呢？

學生：　我知道！$g = \frac{GM}{R^2}$。

老師：　答對了！我們可以把$g = \frac{GM}{R^2}$代入式子。$\frac{1}{2}m\frac{GM}{R} = \left(-\frac{GMm}{R+h}\right) - (-\frac{GMm}{R})$。

學生：　哦哦，我算出來是物體上升的最高點與星球表面的距離 h 是 R 。

★ 主題三 牛頓運動定律的應用 ★
Application of Newton's Laws of Motion

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■ 前言 Introduction

力圖的繪製與分析，是判斷物體加速度的重要依據，本章除了介紹許多生活中常見的力之形式以外，還說明如何以向量，來分析物體的受力狀態，以及物體達到力平衡狀態的條件與現象。此外，我們也探討物體見之碰撞現象，並探討其動量與能量，在不同碰撞現象中的變化。

3-1　生活中常見的力
Force in Everyday Life

■ 前言 Introduction

本節介紹許多生活中常見的力，像是電力、重力、磁力等超距力，還有彈性力、正向力、摩擦力等接觸力。它們如何作用在物體上，為本章討論的重點。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
electric force	電力	friction	摩擦力
gravity	重力	contact force	接觸力
magnetic force	磁力	kinetic frictional force	動摩擦力
action at a distance	超距力	static frictional force	靜摩擦力
elastic force	彈力	perpendicular	垂直
normal force	正向力	vertical	鉛直

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ It is found by experiments that the _____ is proportional to the ____.

例句：**It is found by experiments that the** magnitude of the maximum static friction force **is proportional to the** magnitude of the normal force.

由實驗發現，最大靜摩擦力的量值，與正向力的量值成正比。

❷ ______ can be divided into two categories according to _____: ______ and _______.

例句：Forces **can be divided into two categories according to** the way they act: force at a distance **and** contact force.

「力」，依其作用的方式，可以分成兩類：超距力與接觸力。

❸ The ____ and _____ of the ground are _____ commonly seen in life.

例句：**The** normal force **and** frictional force **of the ground are** contact forces **commonly seen in life.**

地面的正向力與摩擦力等，都是生活中常見的接觸力。

■ 問題講解 Explanation of Problems

☙ 學習目標 ❧

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to know that:

一、熟悉合力矩的計算。

Be familiar with calculating the resultant torque.

二、掌握物體所受合力與加速度之關係。

Comprehend the relationship between the resultant force on an object and its acceleration.

❧ 例題講解 ☙

例題一

說明：熟悉力矩的計算。

Be familiar with the calculation of torque.

A thin rod with length of d and negligible mass has a fixed center point O. Two point masses, m and 2m, are placed at each end. The thin rod is orientated at an angle of θ from the vertical line as shown in Figure on the right. Assuming the acceleration of gravity as g, what is the magnitude of the total torque received by the rod about center point O?

(A) $3mgd\sin\theta$

(B) $2mgd\sin\theta$

(C) $mgd\sin\theta$

(D) $\boldsymbol{(1/2)mgd\sin\theta}$

(E) $(2/3)mgd\sin\theta$。

一長度為 d、質量可以略去的細桿，其中心點 O 固定，兩端各置有質量為 m 及 $2m$ 的質點；細桿與鉛直方向之夾角為 θ，如右圖所示。設重力加速度為 g，則重力對 O 點所產生的力矩之量值為何？

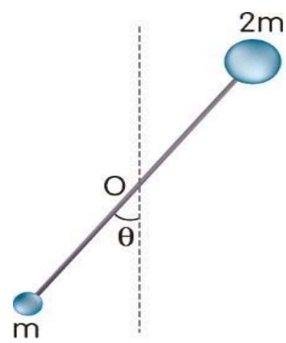
(A) $3mgd\sin\theta$

(B) $2mgd\sin\theta$

(C) $mgd\sin\theta$

(D) $\boldsymbol{(1/2)mgd\sin\theta}$

(E) $(2/3)mgd\sin\theta$。



（翰林版 110 下課本 (選修物理 II) 第二章學習評量 第 5 題）

解題 Solution：

Given the formula of torque, *i.e.*, torque around a point = force × the distance perpendicular to the force. The torque exerted by 2m about center point O is $2mg\times d/2\times\sin\theta$ in the clockwise direction. Similarly, the torque by m about is $mg\times d/2\times\sin\theta$ in the counterclockwise direction.

Thus, the resultant torque is $\tau=mg\times d/2\times\sin\theta-2mg\times d/2\times\sin\theta=-(1/2)mgd\sin\theta$.

The magnitude of it is $(1/2)mgd\sin\theta$, and the direction is clockwise.

由[力矩=力×力臂]可知，質量為 2m 的質點對中心 O 點所造成的力矩為 $2mg\times d/2\times\sin\theta$ 順時鐘方向。而質量 m 的質點所造的力矩為 $mg\times d/2\times\sin\theta$ 逆時鐘方向。

所以可得合力矩 $\tau=mg\times d/2\times\sin\theta-2mg\times d/2\times\sin\theta=-(1/2)mgd\sin\theta$，

即力矩量值為 $(1/2)mgd\sin\theta$，方向為順時針。

Teacher: Class, this question asks about the torque produced by gravity around point O of the thin rod. Can someone tell me where the axis is?

Student: The axis is at center point O and perpendicular to the paper.

Teacher: That is right. With the given axis, we can calculate the torque exerted by the two point masses, 2m and m, about the perpendicular axis at point O. Now, draw the forces acting on the rod and the moment arms on the diagram.

Student: Finished!

Teacher: Next, we determine the magnitude of the total torque. Please refer to your force diagrams and tell me the magnitudes of the moments exerted by point masses 2m and m about the axis.

Student: The torque exerted by point mass 2m about center point O is $2mg\times d/2\times\sin\theta$ in the clockwise direction. And, the torque exerted by point mass m about center point O is $mg\times d/2\times\sin\theta$ in the counterclockwise direction.

Teacher: That's correct. Since these two torques have opposite directions, we need to find the difference between them. We subtract the smaller value $mg\times d/2\times\sin\theta$ from the greater one $2mg\times d/2\times\sin\theta$. What is your answer in terms of the magnitude of the resultant torque and its direction?

Student: The magnitude of it is $(1/2)mgd\sin\theta$ and its direction is clockwise.

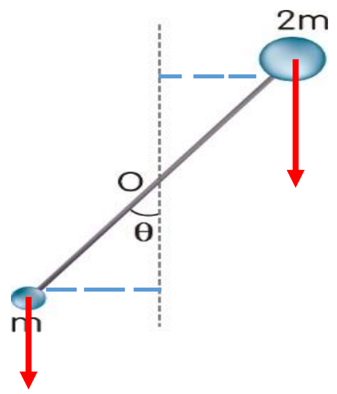
Teacher: Well done. However, we were only asked for the magnitude, so D is the answer to the question.

老師： 同學們，這題問重力對中心 O 點所產生的力矩，那麼我們要假設哪裡為轉軸呢？

學生： 中心 O 點，垂直於紙面的軸。

老師： 沒錯，接著我們要開始算質量為 2m 和質量為 m 的質點分別對通過 O 點的垂直軸所造成的力矩，那麼現在請同學們在此圖上畫出力和力臂吧!

學生： 畫好了！



老師： 那畫出力和力臂之後，我們可以計算力矩了。根據你們所畫出的圖，質量為 2m 和質量為 m 的質點，分別對轉軸造成的力矩為多少呢？

學生： 質量 2m 的質點所造的力矩為 $2mg\times d/2\times \sin\theta$ 順時鐘方向，而質量 m 的質點所造的力矩為 $mg\times d/2\times \sin\theta$ 逆時鐘方向。

老師： 沒錯，那因為兩個質點所造成的力矩方向不同所以兩力矩的值要相減，我們就以數值比較大的 $2mg\times d/2\times \sin\theta$，減去比較小的 $mg\times d/2\times \sin\theta$，最後得到的合力矩大小和方向為何呢?

學生： 大小為$(1/2)mgd\sin\theta$，方向為順時針。

老師： 沒錯，所以答案選(D)。

例題二

說明：學習摩擦力的相關計算。

Learning the relevant calculations of friction.

A horizontal conveyor belt is moving in the +x direction at a constant velocity v. A box with a mass of m is gently placed onto the moving conveyor belt at time t=0. At t=0, the box's velocity is u=0 in the horizontal direction, as shown in Figure. Assuming the coefficient of static friction between the box and the conveyor belt is μ_s, the coefficient of kinetic friction is μ_k, the acceleration of gravity is g, and the effect of air resistance can be negligible:

1. At t=0, what kind of friction is acting on the box?
2. At t=0, what is the magnitude of the box's acceleration?
3. When the speed of the box reaches v from 0, what is the travel distance of the box?
4. When the speed of the box reaches v from 0, what is the work done by the friction acting on the box?"

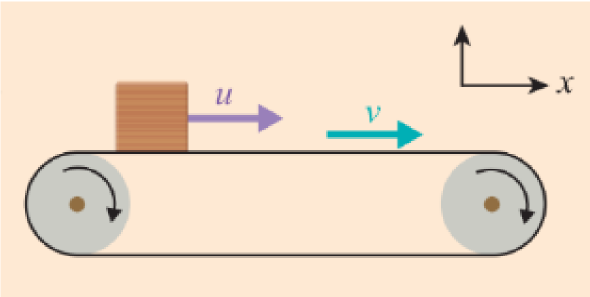
一水平輸送帶恆以等速度 v 沿 +x 方向移動，在時刻 t=0 時，將一質量為 m 的箱子以水平速度 u=0 輕放於輸送帶上，如附圖所示。若箱子與輸送帶之間的靜摩擦係數為 μ_s，動摩擦係數為 μ_k，重力加速度為 g，忽略空氣的影響，則：

1 在時刻 t=0 時，箱子受到哪一種摩擦力的作用？ **動摩擦力**

2 在時刻 t=0 時，箱子的加速度量值為何? $\mu_k g$

3 當箱子的速度由 0 達到 v 的期間，箱子的位移量值為何?$\frac{v^2}{2\mu_k g}$

4 當箱子的速度由 0 達到 v 的期間，摩擦力對箱子作功為何？ $\frac{1}{2}mv^2$



（龍騰版 110 下課本 (選修物理 II) 第三章 第 118 頁 範例 3-2）

解題 Solution：

1. At t=0, the conveyor belt is moving with a constant velocity v to the +x direction, and the box's velocity at this time is zero. Thus, the box has a relative velocity to the belt to the left. Therefore, the belt exerts a kinetic friction f_k on the box to the right, which accelerates the box until it reaches the same velocity as the belt.
2. Applying $f_k = \mu_k mg = ma$, we have a=$\mu_k g$.
3. Applying $v^2 = 0^2 + 2(\mu_k g)S$, we have S=$\mu_k g$.
4. Thus, W= F×S = ($\mu_k$mg) ×$\frac{v^2}{2\mu_k g}$ = $\frac{1}{2}mv^2$.

1. 由於 t=0 的時候輸送帶有速度 v 往+x 方向移動，而此時的箱子速度為零，故兩者之間有相對運動向左，故箱子受向右的動摩擦力f_k，使箱子加速，直到與輸送帶速度相同為止。
2. 由$f_k = \mu_k mg = ma$，可得 a=$\mu_k g$。

3. 由$v^2 = 0^2 + 2(\mu_k g)S$，可得 S=$\frac{v^2}{2\mu_k g}$。

4. 由 W= F×S = ($\mu_k$mg) ×$\frac{v^2}{2\mu_k g}$= $\frac{1}{2}mv^2$ 。

Teacher: Now, let's look at the first question. It asks for the type of friction acting on the box. In the current scenario, the box slides over the surface with respect to the conveyor belt, so static friction will not come into play. Instead, it is kinetic friction that acts on the box. What type of friction is this?

Student: It's kinetic friction as the box is initially at rest at t=0. While the conveyor belt moving with a constant velocity v, there is relative motion between the belt and the box.

Teacher: That's correct! We also know that we can ignore the effect of air resistance. This means that at t=0 only the kinetic friction exerted by the belt on the box. With this in mind, how do we determine the acceleration of the box?

Student: Applying the equation $\vec{F}=\sum m\vec{a}$, we have $f_k = \mu_k mg = ma$. Thus, a=μ_kg.

Teacher: Correct. Moving on to the third question. It asks us to determine the travel distance of the box when the box reaches the velocity of v from 0. With the value of the box's increasing acceleration and the change in its velocity, we should be able to answer this question. Can you tell me how to solve for box’s travel distance?

Student: We can substitute those values into this formula $v^2 = v_0^2 + 2a$S.

This gives us $v^2 = 0^2 + 2(\mu_k g)$S. Thus, S=$\frac{v^2}{2\mu_k g}$.

Teacher: Great job! Finally, the last option is about the work done of the friction acting on the box with time. According to the work-energy theorem, the resultant reaction is equal to the change in kinetic energy, not any other form of energy. In other words, the net force equals the frictional force. So, the work done by friction is$\frac{1}{2}mv^2$.

Student: I see it now. Thank you, Teacher!

老師： 我們先看第一題，它問箱子受什麼樣的摩擦力作用，物體受靜摩擦力時，代表箱子和接觸面之間沒有相互滑動，反之，則是受動摩擦力作用。那請問這題是哪一種摩擦力呢？

學生： 動摩擦力。因為箱子在 t=0 時的速度為零而輸送帶為 v，所以兩者之間有相對運動。

老師： 說得對！然後由於題目說忽略空氣的影響，所以在 t=0 時箱子僅受到動摩擦力的影響，那我們要怎麼計算箱子此時的加速度呢？

學生： 利用 $\vec{F}=\sum m\vec{a}$，$f_k=\mu_k mg=ma$，可以得 $a=\mu_k g$。

老師： 沒錯，接著第三題問，箱子的速度由 0 達到 v 的期間，箱子的位移為何？這題已知加速度和速度變化，我們如何求位移呢？

學生： 利用公式 $v^2=v_0^2+2aS$，將值帶入得 $v^2=0^2+2(\mu_k g)S$，最後可得 $S=\frac{v^2}{2\mu_k g}$。

老師： 很棒喔！最後一題問摩擦力對箱子的作功，根據「功能定理」，[合力作功=動能變化]，合力=摩擦力，所以摩擦力所作的功，是 $\frac{1}{2}mv^2$。

學生： 了解了，謝謝老師！

3-2　靜力平衡與應用
Static Force Equilibrium and Application

■ 前言 Introduction

本節介紹物體，如何維持靜止、或維持等速度平移，及等角速度轉動，稱為「力平衡」。其平衡的之條件包含所受的合力為零，且所受的合力矩也為零。

本章節所探討的問題，不僅是兩力之間的平衡，也涵蓋多力之間的平衡(向量和)，同時，還需滿足所受合力矩為零的條件，才能達到「力平衡」。因此，同時讓學生們練習物體是如何在多個力的作用下，保持平移的平衡，和轉動的平衡，為本節的核心目的。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
static force equilibrium	靜力平衡	fulcrum	支點
translational equilibrium	平移平衡	clockwise	順時針
rotational equilibrium	轉動平衡	counterclockwise	逆時針
resultant force	合力	component force	分力
resultant torque	合力矩	normal force	正向力

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ A ________ will start to ________ when ________.

例句：**A** stationary object **will start to** accelerate **when** it is acted on by external forces. Similarly, if a stationary object is subjected to a torque, it **will start to** accelerate rotationally.

靜止的物體受外力作用後，會開始加速前進，同樣地，若靜止的物體受力矩作用，則會開始加速旋轉。

❷ The conditions of _____ and _____ may not be _____ at the same time

例句：**The conditions of** translational equilibrium **and** rotational equilibrium **may not be** satisfied **at the same time.**

平移平衡與轉動平衡的條件，不一定能同時成立。

❸ ________ may be a ________, or it may be a ________.

例句：An identified system **may be a** complete object, **or it may be a** part of an object.

系統的選定，可能是一個完整的物體，也可能是某一物體的局部。

■ 問題講解 Explanation of Problems

☙ 學習目標 ❧

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to know that:

一、靜力平衡的判斷與計算。

Determination and calculation of static force equilibrium.

二、靜力平衡分析。

Static force equilibrium analysis.

꧁ 例題講解 ꧂

例題一

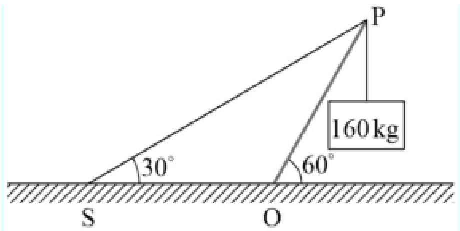
說明：靜力平衡之應用。

Application of static equilibrium to solve problems.

The system in Figure on the right is in state of equilibrium. The angle between the crane arm PO and the horizontal line is 60°. The crane arm is free to pivot around point O and has a mass of 200kgs distributed uniformly along the length. The string PS is at an angle of 30° to the horizontal line and remains taut, while the crane PSO is standing vertically with a mass of 160 kg hanging from the end of the crane arm. Assuming the mass of the cable can be negligible, find the tension in the cable (Let the acceleration of gravity be g=10 m/s^2).

(A) 1600　　**(B) 2600**　　(C) 3200　　(D) 3600　　(E) 5200

右圖為起重機示意圖，起重機臂 PO 和水平線的夾角為 60°，且可繞 O 點自由轉動，其質量為 200 kg 且分布均勻，鋼索 PS 段和水平線的夾角為 30°，PSO 位於垂直面。起重機臂右端懸掛一質量為 160 kg 的重物，若此時處於平衡狀態，且整條鋼索質量可忽略不計，則鋼索上的張力是多少牛頓？(取重力加速度 g=10 m/s^2)



(A) 1600　　**(B) 2600**　　(C) 3200　　(D) 3600　　(E) 5200

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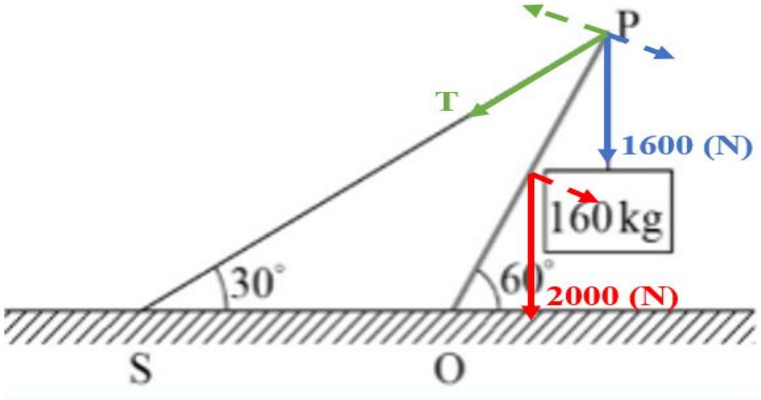


紅線：起重機臂受力、藍線：重物受力、綠線：鋼索上的張力 T
(虛線：以 O 點為支點時各力的有效分量)

Blue line= force on the load;

Green line: tension in the cable T

(Dotted line = the balanced, vertical components of the three forces at the point of rotation O)

解題 Solution：

Because the entire system is at rest, it achieves static equilibrium. As a result, the resultant force = 0 and the resultant torque = 0.

Starting a free-body diagram of the crane from point O as the axis of rotation. As the concept of static equilibrium states that the sum of clockwise torque is equal to the sum of counter-clockwise torque acting on a body, this gives:

$T \times \cos 60^\circ \times L = 200 \times 10 \times \cos 60^\circ \times \left(\frac{L}{2}\right) + 160 \times 10 \times \cos 60^\circ \times L$ so the tension T=2600(N).

因為整個系統維持靜止，所以滿足靜力平衡，因此合力=0，合力矩=0。以 O 點為支點畫出力圖，利用轉動平衡時，逆時針方向力矩等於順時針方向力矩可得：

$T \times \cos 60^\circ \times L = 200 \times 10 \times \cos 60^\circ \times \left(\frac{L}{2}\right) + 160 \times 10 \times \cos 60^\circ \times L$，故張力 T=2600(N)。

Teacher: How do we determine the tension in the cable?

Student: Using the principle of force equilibriums.

Teacher: That's correct! Because this system does not translate or rotate, it achieves static equilibrium.

Student: Well, with point O as the axis of rotation, we can draw a free-body diagram to visualize all the forces acting on the crane. But how do we calculate the torque on the weight of the crane arm?

Teacher: To do that, we can treat the axis of rotation as the point where all forces are concentrated. The distance from that point to the fulcrum point is the moment arm.

Student: That also means the center of gravity is in the middle of the crane arm.

Teacher: Very good. As a result, all three forces cancel each other out. This includes the torque due to the force of T in the counterclockwise direction, and the 2000-N force along with the 1600-N force in the clockwise direction. This system maintains the state of equilibrium in terms of torques and forces.

老師： 我們要如何知道鋼索上的張力呢？

學生： 使用力矩的平衡。

老師： 對喔！此裝置沒有平移也沒有發生轉動，屬於靜力平衡。

學生： 那我們以 O 點為支點，畫力圖平衡力矩，但起重機臂重量的力矩怎麼算呢？

老師： 我們可以取其重心位置，當成所有重力的集中點，重心到支點的距離就是力臂。

學生： 所以其重心在起重機臂中間。

老師： 師很好，所以三個力的力矩平衡，包含 T 所造成的逆時針力矩，及 2000(N)與 1600(N)的重力所造成的順時針力矩，兩方向之力矩達到抗衡。

例題二

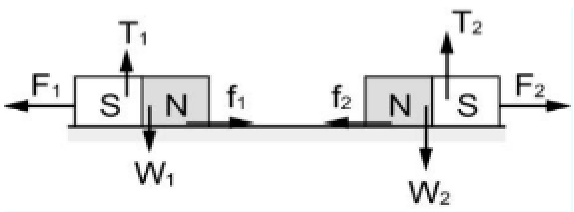
說明：靜力平衡分析。

Analysis of static equilibrium.

Two magnets with different masses are placed at rest on a horizontal table. Their north poles are facing each other and at static equilibrium. Figure 10 displays the forces acting on the two magnets. F_1 and F_2 are magnetic forces, f_1 and f_2 are frictions, w_1 and w_2 are the forces of gravity, and T_1 and T_2 are the normal forces.

Let the gravitational acceleration as g, if we only consider the magnitude of the forces, which of the following equations is correct?

兩塊質量不同的磁鐵靜置於水平桌面，同性磁極 N 相向，達靜力平衡後，圖 10 為它們受力情形的示意圖，F_1、F_2 為磁力，f_1、f_2 為摩擦力，w_1、w_2 為重力，T_1、T_2為正向力。設重力加速度為 g，若僅考慮力的量值，則下列關係式何者正確？



圖（10）

(A) $F_1=w_1g$　　(B) $T_2=w_2g$　　(C) $w_1=w_2$　　(D) $T_1=T_2$　　**(E)** $f_1=f_2$

（109 年學測 47）

解題 Solution：

The system reaches a state of static equilibrium, so: (A) $F_1 = w_1g$ cannot be proven; (B) the correct equation for (B) should be $T_2=w_2$; (C) It has been given that the masses of the magnets shown are not the same; (D) as explained in (C); (E) $F_1 = F^2$ is correct because F_1 and F_2 are the action and reaction forces. They cancel each other out. It is also given that they are at static equilibrium, so $F_1 = f_1$ and $F_2 = f_2$. Thus, $f_1 = f_2$.

系統已達靜力平衡，因此：(A) F_1、w_1是否相等無法證明；(B) $T_2=w_2$才對；(C) 已說兩塊質量不同；(D) 如 (C) ；(E) 因為F_1和F_2本是作用力與反作用力 $F_1=F_2$，因靜力平衡，又推得 $F_1=f_1$, $F_2=f_2$，故 $f_1=f_2$，正確答案是(E)。

Teacher: What is wrong with (A)(B)(C)(D)?

Student: (A) $F_1 = w_1 g$ cannot be proven; (B) the correct equation for (B) should be $T2=w2$; (C) It has been given that the masses of the magnets shown are not the same; (D) as explained in (C).

Teacher: That is right. Then, why is (E) correct?

Student: Because $F1$ and $F2$ are the action and reaction forces. It is also given that they are at static equilibrium, so $F1 = f1$ and $F2 = f2$. Thus, $f1 = f2$.

Teacher: Great job!

老師： (A)(B)(C)(D)錯在哪呢？

學生： (A) F_1、w_1是否相等無法證明；(B) $T_2=w_2$ 才對；(C) 已說兩塊質量不同；(D) 如 C。

老師： 沒錯，那(E)為什麼對呢？

學生： 因為F_1和F_2本是作用力與反作用力$F_1=F_2$，因靜力平衡，又推得$F_1=f_1$, $F_2=f_2$，故$f_1=f_2$。

老師： 很棒喔！

3-3 碰撞 Collision

■ 前言 Introduction

根據牛頓第二運動定律，$\vec{F} = \Delta\vec{p}/\Delta t$，兩物碰撞前後，若無受到其它外力，則兩物的總動量不變。但是，在碰撞的過程，可能因碰撞的力作負功，而造成碰撞前、後兩物體的總動能有損失，此時稱為「非彈性碰撞」或「完全非彈性碰撞」兩種，若是碰撞過程沒有能量損失，則稱為「彈性碰撞」。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
collision	碰撞	kinetic energy	動能
elastic collision	彈性碰撞	momentum	動量
inelastic collision	非彈性碰撞	conservation	守恆
completely inelastic collision	完全非彈性碰撞	center of mass	質心
one-dimensional space	一維空間	loss	損耗
initial velocity	初速度	spring	彈簧
final velocity	末速度	Galilean cannon	伽利略炮

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ If _________, __________ keep the same ______.

例句：**If** there is no external force, the total momentum of a system **keeps the same** before and after collision.

不受外力影響時，系統碰撞前後之總動量相同。

❷ Elastic collisions __________, so __________.

例句：**Elastic collisions** do not consume energy, **so** the total kinetic energy is conserved.

彈性碰撞不消耗能量，故總動能守恆。

❸ We can use _________ to __________.

例句：**We can use** the conservation of total momentum and the conservation of total kinetic energy **to** derive the effect of an elastic collision.

我們可利用總動量守恆，及總動能守恆，來推導彈性碰撞的結果。

❹ Due to _________, when __________, __________.

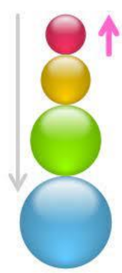
例句：**Due to** the action-reaction law (Newton's 3^{rd} Law), **when** two objects collide, the forces exerted on each object are equal in magnitude and opposite in direction.

因作用力與反作用力定律(牛頓第三定律)，兩物體碰撞時受力大小相等，方向相反。

❺ _______ is a/an application of ______________.

例句：A Galilean cannon **is an application of** collision (shown in the right figure).

伽利略炮為碰撞的應用（如右圖）。



■ 問題講解 Explanation of Problems

☙ 學習目標 ❧

在學習完本單元後，學生應習得以下觀念：

After studying this section, students should be able to know that:

一、完全非彈性碰撞之分析。

Analyses of completely inelastic collisions.

二、彈性碰撞的計算。

Calculation of elastic collision.

❧ 例題講解 ☙

例題一

說明：推算完全非彈性碰撞的能量變化。

To calculate the change of energy in completely inelastic collision.

As shown in Diagram 11, there are two balls A and B, which have mass m_A and m_B. The ratio of which is $\frac{m_A}{m_B} = \alpha < 1$. Both the two balls are hung on a fixed-point O with a rope, which has length L, separately. Ball A remains still, and ball B is pulled rightwards horizontally. Later, ball B is released to collide with ball A, and the two balls immediately stick together. Afterwards, the largest height of the oscillation of the two balls is L/2, as shown in Diagram 12. The mass of the rope and the air resistance are ignored. Assume that the potential energy when the two balls are falling at the lowest point is the zero, what is the ratio of the loss of the mechanical energy caused by the collision and what is the final mechanical energy?

如圖 11 所示，今有質量分別為$m_甲$與$m_乙$之甲、乙兩小球，其質量比值$\frac{m_甲}{m_乙} = \alpha < 1$，將兩小球分別用長度為 L 的細線懸掛於同一固定點 O，甲球靜止懸掛，而乙球向右拉直至細線呈現水平，然後靜止釋放乙球使其與甲球產生碰撞，且兩球立即黏在一起，黏在一起後盪起的最大高度為 L/2，圖 12 為其示意圖，細繩質量及空氣阻力均可忽略。

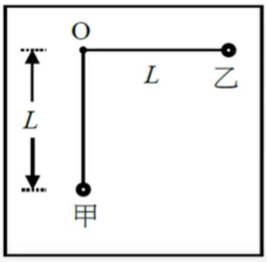


圖 11

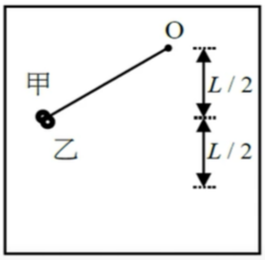


圖 12

假設以兩球自然下垂時的最低點為位能零點，則因碰撞而損失的力學能佔原來總力學能的比例為下列何者？

(A)$\frac{1}{\alpha}$ (B) $\frac{\alpha}{2}$ **(C)** $\frac{\mathbf{1-\alpha}}{\mathbf{2}}$ (D) $\frac{1+\alpha}{2}$ (E) 0

（109 年學測 48）

解題 Solution：

學能必有耗損。但是因兩者之間為碰撞，忽略碰撞過程所受的外力(重力)，則碰撞前、後動量維持守恆。

要計算力學能的變化，可由(圖 11)的總力學能，再扣掉(圖 12)的總力學能。總力學能=重力位能+動能。(因題目只求比列關係，可令$m_{甲}$為 α，$m_{乙}$為 1) (重力加速度:g)算式：

(圖 11)的總力學能(原來總力學能): $m_{甲}$ × g × 0+$m_{乙}$ × g × L=0+gL=gL

(圖 12)的總力學能: $m_{甲}$ × g × L/2+$m_{乙}$ × g × L/2=(α+1) × g × L/2

碰撞而損失的力學能: gL-(α+1) × g × L/2

碰撞而損失的力學能佔原來總力學能的比例:

[gL-(α+1) × g × L/2]/(gL)= (1-α)/2

According to the diagrams, the phenomena is that ball A and ball B stick together when there is a completely inelastic collision, and the total mechanical energy of the two balls must lose. However, if neglect the gravitational force at the moment of the collision, the summation momentum would keep constant.

The change in mechanical energy can be calculated through the total mechanical energy in diagram 11 minus that of diagram 12. Total mechanical energy = gravitational potential energy + kinetic energy. The question only requires the ratio, so m_A can be assumed as α, and m_B as 1. (gravitational acceleration: g)

Teacher: What collision is this question?

Student: Inelastic collision!

Teacher: That's right! Balls A and B eventually stick together, so it is a completely inelastic collision.

Student: So, there will be energy loss.

Teacher: Yes. However, regard the total momentum, the momentum of the whole system will remain still before and after the collision. (If we neglect the gravitational force at the moment of collision.)

Teacher: We can assume that m_A is α, and m_B is 1. Find the total mechanical energy in diagram 11, then minus that of diagram 12. This is the loss of energy.

Student: Finally, we do the ratio.

Teacher: Good for you! Remember that the gravitational potential energy of ball A and the momentum of ball A and B are all 0 in diagram 11. Therefore, the gravitational potential energy of ball B = total mechanical energy.

Teacher: The total mechanical energy in diagram 11 (the initial total mechanical energy): $m_A \times g \times 0 + m_B \times g \times L = 0 + gL = gL$

The total mechanical energy in diagram 12: $m_A \times g \times L/2 + m_B \times g \times L/2 = (\alpha+1) \times g \times L/2$

The loss of mechanical energy caused by collision: $gL-(\alpha+1) \times g \times L/2$

Therefore, the ratio of the loss of mechanical energy caused by collision and the initial total mechanical energy is: $[gL-(\alpha+1) \times g \times L/2]/(gL) = (1-\alpha)/2$

Student: I see.

老師：請問同學這是什麼碰撞呢？

學生：非彈性碰撞！

老師：沒錯！甲、乙兩球最終黏在一起，所以是完全非彈性碰撞歐！

學生：所以會有能量的損失。

老師：是的，但若是考慮整體的動量，則碰撞前、後，整個系統的動量總合，仍然守恆，不會減少。(如果忽略碰撞瞬間的重力作用的話。)

老師：我們可假設 $m_{甲}$ 為 α，$m_{乙}$為 1，然後算出(圖 11)的總力學能，再扣掉(圖 12)的總力學能，就等於耗損的能量。

學生：最後再算比例。

老師： 很棒耶！要記得(圖 11)的甲球重力位能和甲乙兩球動能為 0，所以乙球的重力位能=總力學能喔！

老師：

(圖 11)的總力學能(原來總力學能): $m_{甲}$ × g × 0+$m_{乙}$ × g × L=0+gL=gL

(圖 12)的總力學能: $m_{甲}$ × g × L/2+$m_{乙}$ × g × L/2=(α+1) × g × L/2

碰撞而損失的力學能: gL-(α+1) × g × L/2

碰撞而損失的力學能佔原來總力學能的比例:

[gL-(α+1) ╳g╳L/2]/(gL)= (1-α)/2。

學生： 了解。

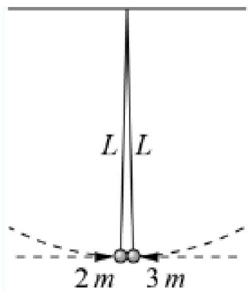
例題二

說明：彈性碰撞過程力學能的分析。

Analysis of the mechanical energy of elastic collision.

There are two same-sized balls hung on the same point on the ceiling. The length of both of the strings is L, and the mass can be ignored. The mass of the left ball is 2m, and the mass of the right ball is 3m. Someone lifts the two balls at h_L and h_R height and releases the balls from stationary state to let the balls move towards each other. An elastic collision happens when the two balls are at the lowest point, as shown in the diagram. After the collision, if the highest height that the left ball reaches is still h_L, what is $h_L : h_R$?

有兩顆大小相同的小球，各以長度為 L、質量可忽略不計的擺繩掛在天花板同一點，左邊小球的質量為 2 m，右邊小球的質量為 3 m。某生拉起兩小球至高度分別為 h_L 和 h_R，將小球由靜止釋放，讓小球擺向中間，使兩小球恰在最低點時發生正向彈性碰撞，如右圖所示。碰撞後，若左邊的小球擺回到最高點的高度仍然為 h_L，則 h_L: h_R 為何？



(A) 9:4　(B) 3:2　(C) 1:1　(D) 2:3　(E) 4:9

（110 年指考 7）

解題 Solution：

Solution: This question is about one-dimensional elastic collision (at the moment od collision). Since the left ball returns to the initial point, it means that velocity is the same before and after the collision, but in different directions. According to the conservation of total momentum as well as the conservation of kinetic energy, the velocity after the collision can be inferred.

此題為一維彈性碰撞（在一剎那間），由於左邊小球碰撞後又回到釋放點，代表碰撞前、後的速度大小一樣，方向相反。隨後根據總動量守恆，及總動能守恆，推算出碰撞後的速度。

v_L末=$-v_L$=(2m-3m/2m+3m)，v_L+(2 × 3m/2m+3m)v_R (v:速度，L:左，R:右，末:碰撞後)

算出v_L和v_R比=$-$3:2，再由此因 1/2mvv=mgh,推得 v(速度)=$\sqrt{2gh}$，可知 h_L: h_R=9:4。

Teacher: What does it mean that the left ball returns to the point it is released? What physical quantities may conserve?

Student: The total mechanical energy of the left ball doesn't lose.

Teacher: Good. Besides, because of action and reaction force, the forces of the two balls are the same, but in different directions. So, what physical quantities are equivalent before and after the collision of the two balls?

Student: The total momentum before and after the collision is conserved.

Teacher: Great. Since the collision causes no loss of total mechanical energy to the left ball, it does not cause any loss to the right ball. Therefore, what collision is it?

Student: Elastic collision.

Teacher: Exactly. So, we can infer the result through the conservation of momentum and conservation of kinetic energy.

Student: Sure.

Teacher: v_L F=$-v_L$ =(2m-3m/2m+3m), v_L +(2 × 3m/2m+3m)v_R (v: velocity, L: left, R: right, F: after collision)

Teacher: And we get the ratio v_L : v_R = $-$3:2. Later, according to 1/2mvv=mgh, we can infer that v (velocity) = $\sqrt{2gh}$. Therefore, h_L : h_R = 9 : 4.

老師： 如果左邊小球碰撞後又回到釋放點，代表什麼現象？那些物理量會守恆？

學生： 左邊小球的總力學能無耗損。

老師： 很好，另外，由於作用力與反作用力的關係，兩者大小相等方向相反，所以兩球在碰撞前後的什麼物理量總和，會相等？

學生： 碰撞前後的總動量會相等。

老師： 很棒，既然碰撞時左邊小球的總力學能無耗損，右邊小球也不會，所以是什麼碰撞？

學生： 彈性碰撞。

老師： 沒錯，因此可以透過總動量守恆，以及動能不變，兩個結論來推導得解。

學生： 好的。

老師： v_L末=$-v_L$=(2m-3m/2m+3m), v_L+(2×3m/2m+3m)v_R (v:速度、L:左、R:右、末:碰撞後)。

老師： 算出v_L和v_R比=−3:2，再由此因 1/2mvv=mgh，推得 v(速度)=$\sqrt{2gh}$，可知 h_L: h_R =9:4。

★主題四 熱學★
Thermodynamics

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■ 前言 Introduction

本章先以巨觀的角度來探討氣體狀態的關係，包含：壓力(P)、體積(V)、溫度(T)、及莫爾數(n)，接著，根據力學原理來探討微觀的氣體分子運動情況，包含平均速率、分子平均動能、碰撞容器之力及壓力，並與巨觀之物理量相結合。最後，也介紹氣體的特性：熱平衡。

4-1 理想氣體狀態方程式
The Ideal Gas Equation of State

■ 前言 Introduction

設在理想氣體之條件下，觀察巨觀的氣體性質，包含波以耳所探討氣體壓力(P)與體積(V)之關係、給呂薩克觀察氣體壓力(P)和溫度(T)之關係、查爾斯探討體積(V)與溫度(T)之關係，最後再由亞佛加厥統整以上關係，得出理想氣體狀態方程式：PV=nRT。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
thermal equilibrium	熱平衡	ideal gas equation of state	理想氣體狀態方程式
thermometer	溫度計	mercury	水銀
kelvin	克氏溫標	gas	氣體
pressure	壓力	macroscopic scale	巨觀
volume	體積	molecule	分子
density	密度	physical quantity	物理量
barometer	氣壓計	ideal gas assumption/hypothesis ideal gas law	理想氣體假設
atmospheric pressure	氣壓	gas constant	氣體常數

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ ______ is valid only for _______.

例句：The Ideal gas equation of state **is valid only for** ideal gases.
理想氣體狀態方程式，只適用於理想氣體。

❷ ________ is inversely proportional to ____ at______.

例句：The volume of an ideal gas **is inversely proportional to** its pressure **at** a fixed temperature.
理想氣體在固定溫度下，其體積與壓力成反比。

❸ Observing ______ at ________, we compare ________.

例句：**Observing** the properties of gases **at** the macroscopic scale, **we compare** the relationships among their pressure, volume, and temperature.
藉由觀察巨觀的氣體性質，我們比較其壓力、體積及溫度的相互關係。

■ 問題講解 Explanation of Problems

☙ 學習目標 ❧

在學習完本單元後，學生應習得以下觀念：

At the end of learning the chapter, students are able to acquire the following concept:

應用理想氣體狀態方程式，並進行概念推理與計算。

Apply the ideal gas state equation to make conceptual inferences and manipulations.

❧ 例題講解 ☙

例題一

說明：結合理想氣體方程式(PV=nRT)，以及浮力公式:$F_B=\rho_{大氣}\cdot V_{氣球}\cdot g$，推算熱氣球升空的溫度。

Two formulae of ideal gases and buoyant force are needed to figure out the heated air temperature when the hot air balloon climbs: the ideal gas equation (PV=nRT) and the buoyancy formula ($FB=\rho_{outside\ air}\times V_{balloon}\times g$).

The Taiwan International Balloon Festival at Taitung is a popular leisure activity. To fly a hot air balloon, the air inside the envelope—the largest part of the balloon, needs to be heated. This causes the volume of the heated air inside the envelope to expand, making the balloon lighter (as any object is buoyed up by a force that is equal to the weight of the air displaced by the object). Now, there are four people riding a hot air balloon with a total mass of 6.0×10^2 kgs (not including the air inside). When the air is heated, its volume expands to $3.0\times10^3\ m^3$ and the balloon rises. At this point, the buoyant force in the air is equal to the total weight of the hot air balloon (including the air inside). Assuming that the air outside the envelope remains at a constant temperature of 22.0°C and density of 1.2 kg/m^3, and that the air inside and outside behaves as an ideal gas under these conditions. Determine the temperature inside the envelope when the balloon climbs.

(A)81 (B) 72 (C) 57 (D) 42 (E) 22

臺東的臺灣國際熱氣球嘉年華是很受歡迎的休旅活動。要讓熱氣球升空，必須加熱氣球裡的空氣，使氣球體積變大，以增加空氣浮力（物體所受的空氣浮力等於物體在空氣中所排開同體積空氣的重量）。有一熱氣球乘載四人後的總質量為 6.0×10^2 kg（不含球內空氣）。當加熱其內空氣，使其體積膨脹至 $3.0\times10^3\ m^3$，即可升空，此時空氣浮力等於熱氣球載人後的總重量（含球內的空氣），則熱氣球內的空氣溫度是多少°C？（設當時外界氣溫為 22°C，空氣密度為 1.2 kg/m^3，氣球內、外的空氣都視為理想氣體，且加熱時球外空氣的溫度、壓力不變。

(A) 81 (B) 72 (C) 57 (D) 42 (E) 22

（110 指考 18）

Teacher: We are given two variables in this word problem. One is the gas volume and the other is the air temperature. Can you think of the ideal gas equation needed to solve this problem?

Student: A free-body diagram also! This diagram can help us map out all the forces, including the buoyant force and the ones that exist for the balloon.

Teacher: With the information, we can figure out the relationship between the amount and temperature of the air inside the envelope. The Ideal Gas Law states PV = nRT, which means if the pressure of a gas (P) and its volume (V) are kept constant, the number of the moles of the gas (n) is inversely proportional to its temperature (T). V is proportional to n, so V is inversely proportional to T.

Student: In this case, the moment the hot air balloon climbs, the buoyant force exerting on it is equal to gravity. We can find out the volume of the air inside the envelope by subtracting the total weight of those four passengers.

Teacher: Buoyant force = $\rho_{\text{outside air}} \times V_{\text{balloon}} \times g$

$= 1.2\ \text{kg}/m^3 \times 3.0 \times 10^3 m^3 \times 10\text{m}/s^2 = (6.0 \times 10^2\ \text{kg} +$ the volume of the heated air inside the envelope$) \times 10\text{m}/s^2$. This gives us 3,000 kg.

However, we still need to figure out the original air temperature prior to take-off. Assuming the pressure and volume of the balloon remains constant and the outside air temperature is 22°C, what is the volume of the air inside the envelope?

Student: 1.2 kg/m^3 × 3.0 × $10^3 m^3$ = 3600 kg. The volume of the air inside the envelope at 22°C, / the volume of the heated air inside the envelope = 6/5

Teacher: You can see that we can use the ideal gas equation to figure out the heated air inside the envelope of a hot air balloon in Kelvin. Remember the temperature should be based on the Kelvin scale.

Student: (273+22)/(273+ the heated air temperature) = 5/6. So, the heated air temperature inside the envelope is 81°C.

老師： 這題有提到氣體體積還有溫度，大家是否能聯想到理想氣體狀態方程式？

學生： 還有力圖分析，浮力和熱氣球的全部重力相等。

老師： 因此會算出質量與溫度的關係，由 PV=nRT 知道，當 PV 固定，n 與溫度成反比，又因質量正比於莫爾數(n)，所以，氣體質量與溫度成反比。

學生： 所以設想熱氣球上升的一瞬間，浮力等於熱氣球重力，這樣在減去乘載四人後的總質量後，就可知道加熱球內空氣的質量。

老師： 浮力 = $\rho_{大氣} \times V_{氣球} \times g$=1.2 kg/ m^3 ×3.0×10^3 m^3×10^3 m^3×10m/ s^2
= (6.0×10^2 kg+加熱後氣球內的空氣質量) ×10m/ s^2
加熱球內空氣質量 = 3000kg。我們還需要一個對照組，假設在熱氣球體積壓力不變狀況下，氣溫為 22°C 時，球內空氣質量為多少？

學生： 1.2 kg/m^3 × 3.0 × $10^3 m^3$ = 3600 kg，22°C 時球內空氣質量 / 加熱球內空氣質量 = 6/5。

老師： 所以利用反比關係，得到加熱後熱氣球內的空氣溫度，記得要用克氏溫標喔！

學生： (273+22) / (273+末溫) = 5/6，所以末溫 = 81°C。

例題二

說明：應用理想氣體狀態方程式：PV=nRT。

Use the ideal gas law, PV=nRT, to solve the following problem.

Given the safety concern, a 10.0-liter oxygen cylinder is installed with an air-relief valve to vent air automatically when the internal pressure exceeds 12 atm. This 10.0-liter cylinder is filled with oxygen at a pressure of 10 atm and a temperature of 300 K. The cylinder is carried in the back of a car for transit. However, due to the hot summer day, the sun heats the car to the point that the valve releases a small pocket of air. Upon arrival at the destination, the oxygen in the cylinder has a new pressure of 12 atm at a temperature of 400 K. Assuming R = 0.082 atm·L/(mol·K), how many moles of gas does the cylinder discharge?

(A) 1.3 **(B) 0.41** (C) 0.23 (D) 0.11 (E) 0.051

基於安全考量，一個容量為 10 公升的氧氣瓶，裝了一個當壓力大於 12 大氣壓時，就將氣體排出的洩氣閥，此氧氣瓶裝有溫度 300K、壓力 10 大氣壓的氧氣。在運送時，氧氣瓶被裝載在車廂中，但炎炎夏日下，車廂內溫度變高,此時洩氣閥正常工作，排出部分氣體,當運送到目的地時，氧氣瓶的氧氣壓力為 12 大氣壓、溫度為 400 K。取理想氣體常數為 0.082 atm·L/(mol·K)，則排出的氣體約為多少莫耳？

(A) 1.3 **(B) 0.41** (C) 0.23 (D) 0.11 (E) 0.051

（108 指考 15）

Teacher: The ideal gas law, PV = nRT, comes to our minds the moment we see this problem.

Student: The volume of the oxygen in the cylinder is constant, which tells us what only P, n, and T change.

Teacher: The initial moles of gas (n) is PV = 10 atm × 10 liters = nRT = n × 0.082 × 300K = 4.16 moles.

Student: The new moles of the gas is 12 atm × 10 liters = n_1 × 0.082 × 400K = 3.65 moles.

Teacher: The new n minus the initial n is 4.16 moles − 3.65moles = 0.41moles. So, the total air released is 0.41 moles.

老師： 看到這題我們就想到 $PV = nRT$。

學生： 氣瓶體積固定，所以只有 P、n、T 產生變動。

老師： 所以由公式，$PV = 10$ 大氣壓 $\times 10$ 公升 $= nRT = n \times 0.082 \times 300K$，300 K 時氧氣之莫爾數 $n = 4.16$mole。

學生： 12 大氣壓力 $\times 10$ 公升 $= n_1 \times 0.082 \times 400K = 3.65$ moles，所以，400 K 時氧氣之莫爾數 $n_1 = 3.65$mole。

老師： 所以相減 4.16mole − 3.65mole = 0.41mole，此為排出的氣體莫耳。

4-2 氣體動力論
The Kinetic Theory of Gases

■ 前言 Introduction

本節從微觀之氣體分子運動，及所涉及的力學原理出發，來解釋氣體的巨觀性質，微觀探討氣體分子的平均動能，和巨觀的絕對溫度間的關係，再透過理想氣體方程式，來推出壓力、體積、莫耳數等資訊，確認理想氣體之分子具備自由運動，不彼此束縛的“動態模型”(kinetic model)，與固體及液體不同，故稱為“氣體動力論”。

■ 詞彙 Vocabulary

單字	中譯	單字	中譯
microscopic	微觀的	temperature	溫度
macroscopic	巨觀的	kinetic energy	動能
pressure	壓力	molecular	分子
root-mean-square speed	方均根速率	Maxwell speed distribution	馬克士威速率分布
internal energy	內能	molar mass	莫耳質量

■ 教學句型與實用句子 Sentence Frames and Useful Sentences

❶ The higher_____, the greater______.

例句：**The higher** the temperature, **the greater** the average kinetic energy of gas molecules **(is)**.
溫度愈高時，氣體分子的平均動能就愈大。

❷ As long as the _______ (is/are) known, ________.

例句：As long as the molecular mass of the gas and the temperature of the gas are known, the root-mean-square velocity of the gas molecule can be calculated.
只要知道氣體的分子量和氣體的溫度，就可以計算出氣體分子的方均根速率。

❸ We must clarify that ______.

例句：We must clarify that due to the random collisions between gas molecules, the speed of motion of each molecule is not constant.
我們必須釐清，由於氣體分子間的隨機碰撞，每個分子的運動速率並不會一成不變。

■ 問題講解 Explanation of Problems

學習目標

在學習完本單元後，學生應習得以下觀念：

At the end of learning the chapter, students are able to acquire the following concept:

一、理解氣體動力論中，物理變數之間的相關性。
Understand the correlation between physical variables in the kinetic theory of gases.

二、學習氣體動力論的分析方式。
Learn how to analyze the gas kinetic theory.

꧁ 例題講解 ꧂

例題一

說明：幫助學生理解氣體動力論中，相關變因的計算。

Help students understand the calculations of relevant variables in the Kinetic theory of Gases.

The same ideal gas is put into two different closed containers. Which of the following statements about the features of the gas in the two containers is correct when the two gas systems reach thermal equilibrium?

(A) The pressure must be higher when the temperature is higher.

(B) The pressure must be lower when the volume is bigger.

(C) The average kinetic energy of gas molecules must be higher when the pressure is higher.

(D) The total kinetic energy of gas molecules must be higher when the gas has more moles.

(E) The root-mean-square speed of gas molecules must be higher when the temperature is higher.

將相同種類的理想氣體分別灌入兩個不同的密閉容器中，當氣體達到熱平衡後，下列關於兩容器內氣體性質的敘述，何者正確？

(A) 溫度較高者，壓力必定較大。

(B) 體積較大者，壓力必定較小。

(C) 壓力較大者，氣體分子的平均動能必定較大。

(D) 莫耳數較大者，氣體分子的總動能必定較大。

(E) 溫度較高者，氣體分子的方均根速率必定較大。

Teacher: Do you remember the States of Ideal Gases Equation?

Student: Yes, we do. It is PV = nRT.

Teacher: Exactly. Let's look at option A first. It says the pressure will be higher when the temperature is higher. Is that right?

Student: Not really. Because n and V are unknown.

Teacher: Yes. Both mole number (n) and volume (V) affect the magnitude of pressure, so we cannot be sure that higher temperature necessarily implies greater pressure simply because the temperature is higher. How about option B?

Student: We cannot be sure about that, either. The n and T are unknown.

Teacher: Correct. Pressure and the average kinetic energy of gas molecules are mentioned in option C. Can you tell us how pressure and the average kinetic energy of gas molecules are related?

Student: Sure, $P = \frac{2N}{3V}\overline{E}_k$.

Teacher: Yes. In this case, we know that due to the unknown number of molecules of the volume of a unit, we can't tell if the average kinetic energy of the molecule is larger. Let's take a look at D. Can you tell us the relationship between the final kinetic energy of molecules and the number of moles?

Student: According to $\overline{E}_{KT} = \frac{3}{2}$nRT, since we don't know the temperature, we can't tell if the final kinetic energy of the molecule is larger.

Teacher: Great. The last one is E. Root-mean-square speed is mentioned, and the formula of it is $v_{rms} = \sqrt{\frac{3RT}{M}}$. Could you tell us why the higher the temperature is, the higher the root-mean-square speed must be?

Student: Because the question says the types of the gas are the same, the molecular weight (M) must be the same. It is known that R is a constant, so we can be sure that the higher the temperature is, the higher the root-mean-square speed will be.

老師： 大家還記得理想氣體方程式怎麼寫嗎？

學生： 記得！是 PV=nRT。

老師： 沒錯，那我們首先看 A 選項，A 選項說溫度較高，壓力一定較大，對嗎？

學生： 不一定，因為 n 和 V 都是未知數。

老師： 是的，由於莫耳數 n 和體積 V 都會影響到壓力的大小，所以我們無法確定溫度較高者壓力一定較大，那麼 B 選項呢？

學生： 也是無法確定，這次是 n 和 T 未知。

老師： 說得好，接者 C 選項談到壓力和氣體分子的平均動能，能請同學們列出兩者之間的關係式嗎？

學生： 可以，是 $P = \frac{2N}{3V}\overline{E}_k$。

老師： 沒錯，那我們就可以知道由於單位體積的分子個數未知，所以無法判定分子的平均動能是否較大，接著我們看 D 選項，能請同學列出分子總動能與莫耳數之間的關係嗎？

學生： 由 $\overline{E}_{KT}=\frac{3}{2}nRT$ 我們知道因為溫度未知，所以無法判斷分子總動能是否必定較大。

老師： 很棒，那剩下 E 選項，選項中提到了方均根速率，那方均根速率的公式為 $v_{rms}=\sqrt{\frac{3RT}{M}}$，你們能告訴我為什麼溫度較高者，氣體分子的方均根速率必定較大嗎？

學生： 因為題目說氣體種類相同，故分子量 M 相同，而 R 為已知的常數，因此可以確定溫度越高者，方均根速率必定較大。

例題二

說明：氣體動力論的應用與計算。

The application and calculation of Kinetic theory of Gases.

There is a sealed container with an ideal gas which reaches thermal equilibrium and has two monatomic molecules. The gas is mixed with 2 moles of Gas X and 1 mole of Gas Y. It is also known that Gas Y has twice as many molecules as Gas X. Which of the following statements is true?

(A) The kinetic energy of Gas X is different from that of Gas Y.

(B) Gases X and Y share the same root-mean-square speed.

(C) The partial pressure of Gas X is half of that of Gas Y.

(D) The final kinetic energy of molecules of Gas X is twice as much as that of Gas Y.

(E) The average kinetic energy of molecules of Gas X is twice as much as that of Gas Y.

一靜止且密封容器內有處於熱平衡的兩種分子的理想氣體，分別是 2 莫耳的氣體 X 和 1 莫耳的氣體 Y。已知 Y 的分子量是 X 的分子量的 2 倍，則下列敘述何者正確？

(A) 兩種氣體分子的總動量不相等。

(B) 兩種氣體分子的方均根速率相等。

(C) X 氣體的分壓，是 Y 氣體分壓的 $\frac{1}{2}$ 倍。

(D) X 氣體分子總動能，是 Y 氣體分子總動能的 2 倍。

(E) X 氣體分子平均動能，是 Y 氣體分子平均動能的 2 倍。

Teacher: According to the description of this question, the variables of the gas are the number of moles and the molecular weights. What do you think about the final kinetic energy of molecules in Option A?

Student: We only know that momentum equals the mass of the substance times its speed, but we don't how this will help us tell whether Option A is right or wrong.

Teacher: Sure. The formula of final momentum of molecules is $\Sigma\vec{P} = \Sigma m\vec{v}$. We can imagine that there are a lot of molecules moving in all different directions in the container. Each molecule has the same mass but in random speeds and directions.
Generally, there is no significant difference in the number of molecules moving in different directions. Given this, what conclusion can we reach?

Student: Regarding the gas molecules, the final momentum is zero.

Teacher: Exactly. Option A, therefore, is wrong. Next, Option B mentioned root-mean-square speed, and its formulas is $v_{rms} = \sqrt{\frac{3RT}{M}}$. The question also mentioned that the gas is in thermal equilibrium inside the container. Given these, can we tell if B is wrong or right?

Student: We know that Gases X and Y have the same temperature because of thermal equilibrium. However, the question says that the two gases have different molecular weights (M), so the two have different root-mean-square speeds. Option B is therefore incorrect.

Teacher: Well done. Let's look at Option C. It talks about partial pressure. We have learned that partial pressure is equivalent to its molar ratio. As a result, the partial pressure of Gas X is twice as high as that of Gas Y. The answer is incorrect.

Student: Okay.

Teacher: Next, D talks about the final kinetic energy of molecules, and the related formula is $\overline{E}_{KT} = \frac{3}{2}\mathrm{nRT}$. How does this formula apply in this option?

Student: According to this formula, we know that under the circumstance of thermal equilibrium, the final kinetic energy of molecules is proportional to the number of moles (n). Therefore, the final kinetic energy of molecules of Gas X is twice as much as that of Gas Y. So the answer is correct.

Teacher: Yes. The last one is Option E. It mentioned the average kinetic energy of molecules. Can anybody try to explain this by what we've learned before?

Student: Sure. The formula of the average kinetic energy of molecules is $\bar{E}_K = \frac{3}{2}kT$.

In this formula, k is a constant, and since the gas has reached thermal equilibrium, Gases X and Y have the same k and T. As a result, the average kinetic energy of molecules of Gas X equals that of Gas Y. The option is therefore incorrect.

老師： 根據題目的敘述，兩種氣體的變因，分別是莫爾數及分子量。選項 A 提到分子的總動量，大家對於動量有什麼見解嗎？

學生： 只知道動量等於物質的質量乘以其速度，但不清楚怎麼用來判斷 A 選項是否正確？

老師： 是的，分子總動量 $\Sigma\vec{P} = \Sigma m\vec{v}$，我們可以想像容器中有非常多的分子往四面八方運動，每顆分子的質量一樣，但速度方向隨機分配。平均來說，往各個方向運動的分子數目並無明顯差異，這樣可以得到什麼結論呢？

學生： 對氣體分子而言，總動量均為零。

老師： 沒錯，故 A 選項是錯的，再來看選項 B，選項中提到了方均根速率，而其公式為 $v_{rms} = \sqrt{\frac{3RT}{M}}$，題目又說到容器內是熱平衡的狀態，那怎麼判斷 B 選項是否正確呢？

學生： 由熱平衡知道兩種氣體的溫度 T 一樣，但由題目知道兩種氣體的分子量 M 不一樣，所以兩者的方均根速率不一樣，故 B 選項是錯的。

老師： 說的好，接著我們看選項 C，此選項談到氣體的分壓，我們之前有學到氣體的分壓比就是其莫耳數比，故 X 氣體的分壓應為 Y 氣體的兩倍，此選項是錯誤的。

學生： 好的。

老師： 接著 D 選項談到分子的總動能，而與此相關的公式是 $\bar{E}_{KT} = \frac{3}{2}$nRT，怎麼利用此公式解釋這個選項呢？

學生： 由此公式我們可知在熱平衡的環境下，分子的總動能正比於莫耳數 n，故 X 氣體分子的總動能為 Y 氣體分子總動能的 2 倍，所以此選項是正確的。

老師： 是的，現在剩最後的 E 選項，此選項提到分子的平均動能，能請同學根據之前所學的來試著解釋看看嗎？

學生： 可以，分子的平均動能的公式為$\bar{E}_K = \frac{3}{2}kT$，其中 k 是常數而溫度 T 因為熱平衡所以兩者一樣，因此 X 氣體分子平均動能與 Y 氣體分子平均動能相等，此選項不正確。

國內外參考資源 More to Explore

PBS LearingMedia

有科學類的影片，分年級分類別，推薦影片及提供影片內可詢問學生的問題，部分影片有閱讀材料。
https://www.pbslearningmedia.org/

MIT opencourseware

此網站為 MIT 的開放式課程，包含講義及課程設計及實驗設計。
https://ocw.mit.edu/

Khan Academy

可汗學院，有分年級的物理教學影片及有問題的討論。
https://www.khanacademy.org/

Interactive Simulations, University of Colorado Boulder

互動式電腦模擬，除了物理，還有其他自然科。
https://phet.colorado.edu/

Collection of Physics Experiments, Charles University in Prague

探究物理實驗設計及結果，並包含原理解說。
https://physicsexperiments.eu/en/physics

PhysPort, PER	(QR code)
物理教育研究資源庫，分享評量相關工具，包含迷思概念，情意成效，學習觀等。 https://www.physport.org/assessments/	
泛科學	(QR code)
介紹自然科學相關的知識。 https://pansci.asia/	
ISLE Physics	(QR code)
此網站是以設計給學生學習物理相關知識為目的。 https://www.islephysics.net/	

自然領域雙語教學資源手冊：物理科英語授課用語

[高中選修(II)]

A Reference Handbook for Senior High School Bilingual Teachers in the Domain of Natural Sciences (Physics): Instructional Language in English

[Elective Physics (II)]

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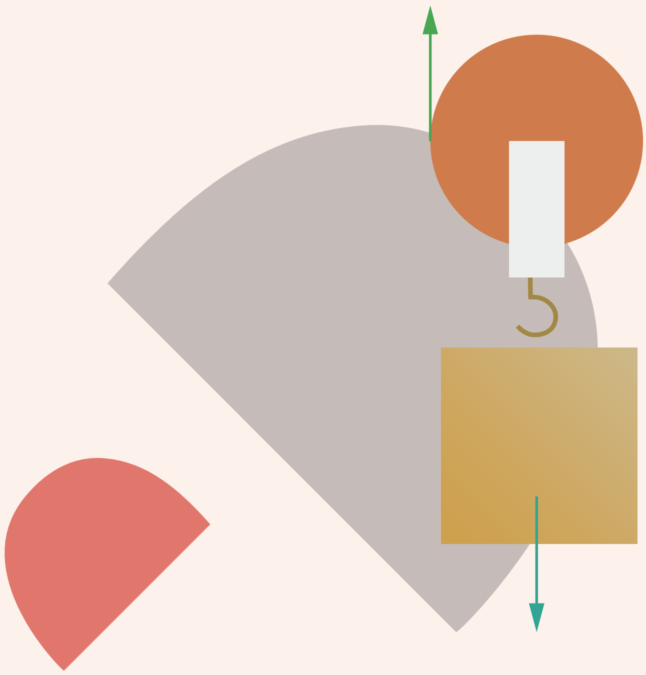

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